$J/\psi$ PRODUCTION AT HADRON COLLIDERS

Hee Sok Chung
Argonne National Laboratory

Based on
Geoffrey T. Bodwin, HSC, U-Rae Kim, Jungil Lee, Yan-Qing Ma, Kuang-Ta Chao, in preparation
OUTLINE

• Leading-power fragmentation in quarkonium production

• Cross section and polarization of
  • direct $J/\psi$
  • $\psi(2S)$ and $\chi_{cJ}$
  • prompt $J/\psi$

• Summary
HEAVY QUARKONIUM

• Bound states of a heavy quark and a heavy antiquark:
  e.g. $J/\psi, \psi', \eta_c, h_c, \chi_{cJ}, \Upsilon(nS), \eta_b, \chi_{bJ}$ ...

• $2m_b > 2m_c \gg \Lambda_{QCD}$

• $m_{J/\psi} \approx m_{\eta_c} \approx 2m_c, m_{\Upsilon(1S)} \approx m_{\eta_b} \approx 2m_b$,
  which allow nonrelativistic description:
  $v^2 \approx 0.3$ for charmonia, $v^2 \approx 0.1$ for bottomonia

• Typical energy scales $m > mv > mv^2 \approx \Lambda_{QCD}$,
  Ideal for studying interplay between perturbative and nonperturbative physics
HEAVY QUARKONIUM

- Quark model assignments of some heavy quarkonia

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INCLUSIVE $J/\psi$ PRODUCTION

- The $p_T$-differential cross section has been measured at hadron colliders like RHIC, Tevatron and the LHC.
- $J/\psi$ is usually identified from its leptonic decay.
- Large contributions from B hadron decays are subtracted to yield the “prompt” cross section, which includes contributions from direct production and from decays of heavier charmonia.
INCLUSIVE $J/\psi$ PRODUCTION

• Color-singlet model (CSM): A $c\bar{c}$ pair with same spin, color and C P T is created in the hard process, which evolves into the $J/\psi$.

The universal rate from $c\bar{c}$ to $J/\psi$ (color-singlet long-distance matrix element) is known from models, lattice measurements, and fits to experiments.
INCLUSIVE $J/\psi$ PRODUCTION

- CSM is incomplete:
  - A color-octet (CO) $c\bar{c}$ pair can evolve into a color singlet meson by emitting soft gluons.
  - In the effective theory nonrelativistic QCD, CO LDMEs are suppressed by powers of $\nu$.
  - In many cases, color-octet channels are necessary:
    - For S-wave vector quarkonia ($J/\psi, \psi(2S), \Upsilon(nS)$), CSM severely underestimates the cross section.
    - For production or decay of P-wave quarkonia ($\chi_{cJ}, \chi_{bJ}$), CS channel contains IR divergences that can be cancelled only when the CO channels are included.
INCLUSIVE $J/\psi$ PRODUCTION

- CSM prediction vs. measurement at Tevatron

- LO CSM ($\sim 1/p_T^8$) is inconsistent with both shape and normalization.

- Radiative corrections are larger than LO and has different shape ($\sim 1/p_T^4$), but still not large enough
INCLUSIVE $J/\psi$ PRODUCTION

- NRQCD can be used to describe the physics of scales smaller than the quarkonium mass.

- NRQCD factorization conjecture for production of $H$

\[
d\sigma_{A+B\rightarrow H+X} = \sum_n d\sigma_{A+B\rightarrow Q\bar{Q}(n)+X} \left\langle O^H(n) \right\rangle \]

- Short-distance cross sections are essentially the production cross section of $Q\bar{Q}$ that can be computed using perturbative QCD

- The LDMEs are nonperturbative quantities that correspond to the rate for the $Q\bar{Q}$ to evolve into $H$

- LDMEs have known scaling with $\nu$
INCLUSIVE $J/\psi$ PRODUCTION

- NRQCD factorization conjecture for production of $H$

$$d\sigma_{A+B\rightarrow H+X} = \sum_n d\sigma_{A+B\rightarrow Q\bar{Q}(n)+X} \langle O^H(n) \rangle$$

- Usually truncated at relative order $n^4$: $S_0^{[8]}$, $S_1^{[8]}$, $P_j^{[8]}$, $S_1^{[1]}$ channels for $J/\psi$

- The short-distance cross sections have been computed to NLO in $\alpha_s$ by three groups:
  Kuang-Ta Chao’s group (PKU): Ma, Wang, Chao, Shao, Wang, Zhang
  Bernd Kniehl’s group (Hamburg): Butenschön, Kniehl
  Jianxiong Wang’s group (IHEP): Gong, Wan, Wang, Zhang

- It is not known how to calculate color-octet LDMEs, and are usually extracted from measurements
INCLUSIVE $J/\psi$ PRODUCTION

- In order to extract CO LDMEs from measured cross sections we need to determine the short-distance cross sections as functions of $p_T$

- NLO corrections give large K-factors that rise with $p_T$; this casts doubt on the reliability of perturbation theory

Ma, Wang, Chao, PRL106, 042002 (2011)
$J/\psi$ POLARIZATION PUZZLE

\[ \lambda_\theta = \begin{cases} 
+1 & \text{Transverse} \\
0 & \text{Unpolarized} \\
-1 & \text{Longitudinal} 
\end{cases} \]

- NRQCD at LO in $\alpha_s$ predicts transverse polarization at large $p_T$
- Disagrees with measurement
- NLO corrections are large in the $^1S_0^{[8]}$ and $^3P_J^{[8]}$ channels
- NRQCD at NLO still predicts transverse polarization

CDF, PRL99, 132001 (2007)
LP FRAGMENTATION

• Large NLO corrections arise because new channels that fall off more slowly with $p_T$ open up at NLO

• The leading power (LP) in $p_T \left(1/p_T^4\right)$ is given by single-parton fragmentation

\[
\frac{d\sigma}{dp_T^2}[ij \rightarrow c\bar{c} + X] = \sum_k \int_0^1 dz \frac{d\sigma}{dp_T^2}[ij \rightarrow k + X] D[k \rightarrow c\bar{c} + X] + O(m_c^2/p_T^6)
\]

- $z$: fraction of momentum transferred from parton $k$ to hadron
- $i, j, k$: run over quarks, antiquarks, and gluon

• Corrections to LP fragmentation go as $m_c^2/p_T^2$
Fragmentation functions (FFs) for production of $c\bar{c}$ can be computed using perturbative QCD.

- A gluon can produce a $c\bar{c}$ pair in $^3 S_1^{[8]}$ state directly: gluon FF for this channel starts at order $\alpha_s$, involves a delta function at $z = 1$.

- A gluon can produce a $c\bar{c}$ in $^3 P_j^{[8]}$ state by emitting a soft gluon: gluon FF for this channel starts at order $\alpha_s^2$, involves distributions singular at $z = 1$.

- A gluon can produce a $c\bar{c}$ in $^1 S_0^{[8]}$ state by emitting a gluon: gluon FF for this channel starts at order $\alpha_s^2$, does not involve divergence at order $\alpha_s^2$.

(Collins and Soper, NPB194, 445 (1982))
FRAGMENTATION FUNCTIONS

- For the $^3S_1^{[8]}$ and $^3P_J^{[8]}$ channels, gluon polarization is transferred to the $c\bar{c}$ pair, and therefore the $c\bar{c}$ is mostly transverse.

- For the $^1S_0^{[8]}$ channel, the $c\bar{c}$ is unpolarized because it is isotropic.
LP FRAGMENTATION

- LP fragmentation explains the large, $p_T$-dependent K-factors that appear in fixed-order calculations.

- $^{3}S_{1}^{[8]}$ channel is already at LP at LO: NLO correction is small.

- $^{1}S_{0}^{[8]}$ and $^{3}P_{J}^{[8]}$ channels do not receive an LP contribution until NLO: NLO corrections are large.
LP FRAGMENTATION

- LP fragmentation reproduces the fixed-order calculation at NLO accuracy at large $p_T$

\[ \frac{d\sigma_{NLO}}{dp_T} \]

The difference is suppressed by $m_c^2/p_T^2$

- The slow convergence in $^{1}S_0^{[8]}$ channel is because the fragmentation contribution is small (no $\delta$ function or plus distribution from IR divergence)
LP+NLO

- We combine the LP fragmentation contributions with fixed-order NLO calculations to include corrections of relative order $m_c^2/p_T^2$.

$$
\frac{d\sigma_{\text{LP+NLO}}}{dp_T} = \frac{d\sigma_{\text{LP}}}{dp_T} + \frac{d\sigma_{\text{NLO}}}{dp_T} - \frac{d\sigma_{\text{LP, NLO}}}{dp_T}
$$

- We take $p_T > 3 \times m_{\text{quarkonium}}$ in order to suppress possible non-factorizing contributions.
LP+\text{NLO}

• Alternatively, one can consider the LP fragmentation to supplement the fixed-order NLO calculation

\[ \frac{d\sigma^{\text{LP+NLO}}}{dp_T} = \frac{d\sigma^{\text{LP}}}{dp_T} - \frac{d\sigma^{\text{LP}}_{\text{NLO}}}{dp_T} + \frac{d\sigma^{\text{NLO}}}{dp_T} \]

\text{LP fragmentation resummed leading logs} \quad \text{LP fragmentation to NLO accuracy}

\text{Additional fragmentation contributions}

\text{fixed-order calculation to NLO}
LP CONTRIBUTIONS
THAT WE COMPUTE

$^{3}S_{1}^{[8]}$ channel

$^{1}S_{0}^{[8]}$ and $^{3}P_{J}^{[8]}$ channels

$\alpha_s$, $\alpha_s^2$, $\alpha_s^3$ \ldots

**Parton production cross sections**

- $\alpha_s$: LO, NLO, NNLO
- $\alpha_s^2$: NLO, NNLO
- $\alpha_s^3$: NNLO

**Fragmentation functions**

- $\alpha_s$: - NLO NNLO
- $\alpha_s^2$: - NNLO
- $\alpha_s^3$: -

- Available
- Not yet available
- Leading logarithms only

- We resum the leading logarithms in $p_T/m_c$ to all orders in $\alpha_s$

- Corrections to LP contributions give “normal” K-factors ($\lesssim 2$)
RESUMMATION OF LEADING LOGARITHMS

• The leading logarithms can be resummed to all orders by solving the LO DGLAP equation

\[
\frac{d}{d \log \mu_f^2} \left( \begin{array}{c} D_S \\ D_g \end{array} \right) = \frac{\alpha_s(\mu_f)}{2\pi} \left( \begin{array}{cc} P_{qq} & 2n_f P_{gq} \\ P_{gq} & P_{gg} \end{array} \right) \otimes \left( \begin{array}{c} D_S \\ D_g \end{array} \right)
\]

\[D_S = \sum_q (D_q + D_{\bar{q}})\]

• This equation is diagonalized in Mellin space; the inverse transform can then be carried out numerically

• Because the FFs are singular at the endpoint, the inversion is divergent at \( z = 1 \); special attention is needed for contribution at \( z \approx 1 \)
RESUMMATION OF LEADING LOGARITHMS

• We split the $z$ integral:

$$\int_0^1 dz \, \hat{\sigma}(z) D(z) = \int_0^{1-\epsilon} dz \, \hat{\sigma}(z) D(z) + \int_{1-\epsilon}^1 dz \, \hat{\sigma}(z) D(z)$$

$$\approx \int_0^{1-\epsilon} dz \, \hat{\sigma}(z) D(z) + \hat{\sigma}(z = 1) \int_{1-\epsilon}^1 dz \, z^N D(z)$$

• $N$ is chosen so that $\hat{\sigma}(z) \approx \hat{\sigma}(1) z^N$ near $z \approx 1$

$$\int_{1-\epsilon}^1 dz \, z^N D(z) = \int_0^1 dz \, z^N D(z) - \int_0^{1-\epsilon} dz \, z^N D(z)$$

Well defined in Mellin space (Mellin transform of $D(z)$)

Well behaved numerically
LP+NLO

- The additional fragmentation contributions have important effects on the shapes in the $3P_J^{[8]}$ channel.

- Large corrections to the shape of the $3P_J^{[8]}$ channel because the LO and NLO contributions cancel at about $p_T \approx 7.5$ GeV.
**$J/\psi$ PRODUCTION**

- We obtain good fits to the cross section measurements by CDF and CMS.

- $p_T > 10$ GeV ($\approx 3 \times m_{J/\psi}$) was used in the fit.

- 25% theoretical uncertainty from varying fragmentation, factorization and renormalization scales.

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CDF, PRD71, 032001 (2005)
CMS, JHEP02, 011 (2012)
Bodwin, HSC, Kim, Lee, PRL113, 022001 (2014)

$$B = \text{Br}[J/\psi \rightarrow \mu^+\mu^-] \quad \chi^2/d.o.f. = 0.085$$
**J/ψ PRODUCTION**

- The data falls off faster than $^3 S_1^{[8]}$ and $^3 P_J^{[8]}$ channels
- The fit constrains the $^3 S_1^{[8]}$ and $^3 P_J^{[8]}$ channels to cancel
- $^1 S_0^{[8]}$ channel dominates the cross section
- This possibility was first suggested by Chao et al.

Chao, Ma, Shao, Wang, Zhang, PRL108, 242004 (2012)

\[
\langle O^{J/\psi} (^1 S_0^{[8]}) \rangle = 0.099 \pm 0.022 \text{ GeV}^3
\]
\[
\langle O^{J/\psi} (^3 S_1^{[8]}) \rangle = 0.011 \pm 0.010 \text{ GeV}^3
\]
\[
\langle O^{J/\psi} (^3 P_0^{[8]}) \rangle = 0.011 \pm 0.010 \text{ GeV}^5
\]
**$J/\psi$ POLARIZATION**

- Because of $^{1}\Sigma_0^{[8]}$ dominance, $J/\psi$ is almost unpolarized

- **FIRST PREDICTION OF UNPOLARIZED $J/\psi$ IN NRQCD**
  
  Bodwin, HSC, Kim, Lee, PRL113, 022001 (2014)

- This is in *good agreement* with CMS data and much *improved agreement with CDF Run II data*
  
  CMS, PLB727, 381 (2013)

- **Caveat:** feeddown ignored
PROMPT $J/\psi$ PRODUCTION

- $J/\psi$ can also be produced from decays of $\psi(2S)$ and $\chi_{cJ}$
- $\psi(2S)$ LDMEs from fit to CMS and CDF cross section data
- $\chi_{cJ}$ LDMEs from fit to ATLAS cross section data
- 30% theoretical uncertainty from scale variation

CMS, JHEP02, 011 (2012)
CDF, PRD80, 031103 (2009)
CMS-PAS-BPH-14-001

ATLAS, JHEP1407, 154 (2014)
ψ(2S) POLARIZATION

- We predict that the ψ(2S) is slightly transverse at the Tevatron

- We predict that the ψ(2S) is slightly transverse at the LHC

Agrees with CMS data within errors

CDF, PRL85, 2886 (2000)
CDF, PRL99, 132001 (2007)
CMS, PLB727, 381 (2013)
\( \chi_c J \) PRODUCTION

- \( ^3 S_1^8 \) and \( ^3 P_J^1 \) channels contribute at leading order in \( \nu \)

- We obtain good fits to ATLAS data \( \text{ATLAS, JHEP} 1407, 154 (2014) \)

- The \( ^3 P_J^1 \) matrix element obtained from fit agrees with the potential model calculation

\[ |R'(0)|^2 = 0.075 \text{ GeV}^5 \]

Eichten and Quigg, PRD 52, 1726 (1995)

\[ \chi^2 / \text{d.o.f.} = 0.080 \]

\( \chi_1, \text{ATLAS Data, } |y| < 0.75 \)
\( \chi_2, \text{ATLAS Data, } |y| < 0.75 \)

PRELIMINARY

\[ B_X = \text{Br}[\chi_c \rightarrow J/\psi \gamma] \times \text{Br}[J/\psi \rightarrow \mu^+ \mu^-] \]

\[ B_X \times d\sigma (\text{pb}/\text{GeV}) \]

\[ p_T \text{ (GeV)} \]

Our fit

\[ |R'(0)|^2 = 0.055 \pm 0.025 \text{ GeV}^5 \]

\( \rightarrow \text{Suggests that NRQCD factorization works} \)
POLARIZATION OF $J/\psi$ FROM $\chi_{cJ}$ DECAY

- We predict that the $J/\psi$ from $\chi_{cJ}$ decay is slightly transverse at LHC.
- We assume $E1$ transition in $\chi_{cJ} \rightarrow J/\psi + \gamma$ (higher-order transitions have little effect).

Faccioli, Lourenco, Seixas, and Wohri, PRD83, 096001 (2011)
PROMPT $J/\psi$ PRODUCTION

- After including feeddown contributions, we again obtain good fits
  \[ \chi^2/\text{d.o.f.} = 0.44 \]

- Again, $p_T > 10$ GeV
  \[ \approx 3 \times m_{J/\psi} \]
  was used in the fit
PROMPT $J/\psi$ PRODUCTION

- The direct cross section falls off faster than $^3S_1^{[8]}$ and $^3P_J^{[8]}$ channels.
- The fit constrains the $^3S_1^{[8]}$ and $^3P_J^{[8]}$ channels to cancel.
- $^1S_0^{[8]}$ channel dominates the direct cross section.

\[
\langle \mathcal{O}^{J/\psi} (^1S_0^{[8]}) \rangle = 0.094 \pm 0.016 \text{GeV}^3 \\
\langle \mathcal{O}^{J/\psi} (^3S_1^{[8]}) \rangle = 0.004 \pm 0.008 \text{GeV}^3 \\
\langle \mathcal{O}^{J/\psi} (^3P_J^{[8]}) \rangle = 0.005 \pm 0.007 \text{GeV}^5 
\]
PROMPT $J/\psi$ POLARIZATION

- Direct $J/\psi$ and $J/\psi$ from feeddown is slightly transverse

- **PROMPT $J/\psi$ HAS SMALL POLARIZATION**

- This is in *reasonably good agreement with CMS data*

CMS, PLB727, 381 (2013)
SUMMARY

• We present new LP fragmentation contributions that have a significant effect on calculations of $J/\psi$ production in hadron colliders.

• When we include LP fragmentation contributions, we predict the $J/\psi$ to have near-zero polarization at high $p_T$ at hadron colliders.

• This is the first prediction of small $J/\psi$ polarization at high $p_T$ in NRQCD.

• Work on higher-order corrections, as well as other quarkonium states is in progress.
BACKUP
GLUON FRAGMENTATION INTO $c\bar{c}$

- Lowest-order diagrams of a gluon producing CO $c\bar{c}$

1. $g \rightarrow c\bar{c}(^3S_1^{[8]})$
   
2. $g \rightarrow c\bar{c}(^1S_0^{[8]})$
   
3. $g \rightarrow c\bar{c}(^3P_j^{[8]})$
GLUON FRAGMENTATION INTO $c\bar{c}$

- Computation of fragmentation functions involve Eikonal lines, and their interaction with gluons
PREDICTIONS AT NLO

Bernd Kniehl’s group

- Used cross section measurements at HERA and Tevatron to fix LDMEs, predicts transverse polarization at large $p_T$
- H1 and ZEUS data are at small $p_T$
- Does not include feeddown

PREDICTIONS AT NLO

• Used CDF and LHCb cross section data to fit LDMEs
• Includes feeddown
• Prediction still more transverse than measurement

PREDICTIONS AT NLO

- Used CDF, ATLAS, CMS, LHCb cross section data to fit LDMEs
- Included feeddown
- Assumed positivity of all LDMEs, although $^3 P^*_J$ LDME has strong factorization scale dependence
- Prediction still more transverse than data

Kuang-Ta Chao’s group

Shao, Han, Ma, Meng, Zhang, Chao, hep-ph/1411.3300 (2014)
PREDICTIONS AT NLO

- Used CDF, ATLAS, CMS, LHCb cross section data to fit LDMEs
- Included feeddown
- Assumed positivity of all LDMEs, although $P_J^{[8]}$ LDME has strong factorization scale dependence
- Prediction still more transverse than data

Kuang-Ta Chao’s group

Shao, Han, Ma, Meng, Zhang, Chao, hep-ph/1411.3300 (2014)