Parton Distribution Functions on the Lattice

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§ Introduction to PDFs
   ✩ A brief overview on global analysis

§ Lattice QCD
   ✩ Difficulties: why seek a new idea?

§ New Approach on the Lattice
   ✩ Preliminary results on nucleon quark, helicity and transversity distributions
Parton Distribution Functions

§ Structure functions studied through scattering processes

⇒ Deep inelastic scattering beginning in 1960s at SLAC
⇒ Depend on energy scale ($Q^2$) and quark momentum fraction ($x$)

§ “Parton”
⇒ 1969 by Feynman: pointlike constituents inside hadron → now known to be quarks and gluons

§ Still limited knowledge
⇒ Many ongoing/planned experiments (EIC, LHeC, ...)

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Parton Distribution Functions

§ Quark distribution

Process: DIS ($F_2, \sigma$), Drell-Yan, $W$-asymmetry, $Z$-rapidity, ($\gamma+$) jet, ...

Experiment: BCDMS, NMC, SLAC, JLab, HERA, E866, CDF, DØ,...

§ Helicity distribution

Process: polarized DIS, semi-inclusive DIS, photo- and electroproduction of hadrons and charm, $pp$ collisions

Experiment: EMC, HERMES, Hall A, CLAS, COMPASS, STAR, PHENIX, ...

§ Transversity distribution

Process: single-spin asymmetry in SIDIS, ...

Experiment: HERMES, COMPASS, Belle...
§ Experiments cover diverse kinematics of parton variables

Global analysis takes advantage of all data sets
Global Analysis

§ Experiments cover diverse kinematics of parton variables
☞ Global analysis takes advantage of all data sets

PDFs

Applications

Predictions

§ Important fundamental QCD property
☞ Exploration of the valence and sea-quark content of the nucleon

§ Important for BSM searches
☞ Provides SM cross-section prediction for LHC new-physics search
☞ IceCube PeV neutrinos can be explained by PDF uncertainties
☞ Proton weak charge (medium-modification effects)
Global Analysis

Some choices made for the analysis

- Choice of data sets and kinematic cuts
- Strong coupling constant \( \alpha_s(M_Z) \)
- Uncertainties in perturbation theory (depends on process whether LO, NLO or NNLO is known)
- Evolution of PDFs to different scales
- Parametrization assumptions

\[
f(x, \mu_0) = a_0 x^{a_1} (1 - x)^{a_2} P(x) \\
P(x) = e^{a_3 x} (1 + e^{a_4 x})^{a_5}
\]

Discrepancies appear when data is scarce
Global Analysis

§ Some choices made for the analysis

☞ Choice of data sets and kinematic cuts
☞ Strong coupling constant $\alpha_s(M_Z)$
☞ Uncertainties in perturbation theory (depends on process whether LO, NLO or NNLO is known)
☞ Evolution of PDFs to different scales
☞ Parametrization assumptions

§ Sum rules to constrains the fit

☞ Quark number, momentum, $g_A$, SU(3) flavor symmetry...

§ Assumptions imposed where theory and exp’t are lacking

☞ Charge symmetry, (anti-)strange, “sea” (antiquark) distribution...
Global Analysis

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Sum rules to constrain the fit:

- Quark number, momentum, $g_A$, SU(3) flavor symmetry...

Assumptions imposed where theory and experimental input are lacking:

- Charge symmetry, (anti-)strange, “sea” (antiquark) distribution...

For example, $s + \bar{s} = \kappa (\bar{u} + \bar{d})$ or symmetric sea in helicity...
Many groups have tackled the analysis 

CTEQ, MSTW, ABM, JR, NNPDF, etc.

Many groups have tackled the analysis:

- CTEQ, MSTW, ABM, JR, NNPDF, etc.

Many groups have tackled the analysis

DSSV, ACC, BB, LSS, JAM, etc.
There have only been 2 attempts (still very preliminary)

- Requires more theory input and experimental data
- More assumptions are made to extract the distribution


PDFs on the Lattice

Lattice QCD is an ideal theoretical tool for investigating strong-coupling regime of quantum field theories

Ideal tool for studying nonperturbative hadron structure

Physical observables are calculated from the path integral

$$\langle 0 | O(\bar{\psi}, \psi, A) | 0 \rangle = \frac{1}{Z} \int DA \ D\bar{\psi} \ D\psi \ e^{iS(\bar{\psi}, \psi, A)} \ O(\bar{\psi}, \psi, A)$$

Lattice QCD

Impose a UV cutoff
discretize spacetime

Impose an Infrared cutoff
finite volume

Wick rotate to Euclidean

Use compact gauge group

$$U_\mu(x) = e^{iagt_c \ A_\mu^c(x)}$$
PDFs on the Lattice

§ Many lattice calculations of the moments of the PDFs

\[ \langle x^{n-1} \rangle_q = \int_{-1}^{1} dx x^{n-1} q(x) = \int_{0}^{1} dx x^{n-1} \left[ q(x) - (-1)^{n-1} \bar{q}(x) \right] \]

\[ \langle x^{n-1} \rangle_{\Delta q} = \int_{-1}^{1} dx x^{n-1} \Delta q(x) = \int_{0}^{1} dx x^{n-1} \left[ \Delta q(x) + (-1)^{n-1} \Delta \bar{q}(x) \right] \]

\[ \langle x^{n-1} \rangle_{\delta q} = \int_{-1}^{1} dx x^{n-1} \delta q(x) = \int_{0}^{1} dx x^{n-1} \left[ \delta q(x) - (-1)^{n-1} \delta \bar{q}(x) \right] \]

⚠ Limited to the lowest few moments
⚠ Might provide constraints on models or tests of experiment

§ Also applies to GPDs: limited to 3rd moment

§ Most progress made in quark contributions
⚠ Very costly to obtain useful gluon signal
⚠ Limited by available computational resources
§ Leading moment $\langle x \rangle$, hypercubic decomposition

\[ 4_1 \otimes 4_1 = 1_4 \oplus 3_3 \oplus 6_6 \]  

- Both operators go to same continuum limit

§ No mixing with operators of same or lower dimension

§ To improve to $O(a)$

Consider all irrelevant operators of same symmetry:

§ Higher moments $\langle x^n \rangle$

- $4_1: O_{111}$ mixes with $\bar{q} \gamma \sigma_{\mu \rho} \overleftrightarrow{D} \gamma \sigma_{\mu \rho} q$
- $4_2: O_{\{123\}}$ requires all momentum components to be nonzero
- $8_1: O_{\{441\}} - \left(O_{\{221\}} + O_{\{331\}} \right)/2$ mixes under renormalization

§ For higher spin, all ops mix with lower-dimension ops
§ Leading moment $\langle x \rangle$, hypercubic decomposition

$4_1 \otimes 4_1 = 1_1 \oplus 3_1 \oplus 6_1 \oplus 6_3$:

$0_{44} - (O_{11} + O_{22} + O_{33})/3$ $O_{14} + O_{41}$, (requires $p \neq 0$)

Both operators go to same continuum limit

§ No mixing with operators of same or lower dimension

§ To improve to $O(a)$

Consider all irrelevant operators of same symmetry:

$O_{\{\mu \nu\}} \rightarrow (1 + a m_q c_0) O_{\{\mu \nu\}} + i a c_1 \overline{q} \sigma_{\mu \rho} D^{[\nu} D^{\rho]} q$

$+ a c_2 \overline{q} D_{[\mu} D_{\nu]} q + i a c_3 \partial_{\rho} \left( \overline{q} \sigma_{\mu \rho} D_{\nu} q \right)$

§ Higher moments $\langle x^2 \rangle$

$4_1$: $O_{111}$ mixes with $\overline{q} \gamma_1 q$ with coefficient $\sim 1/a^2$

$4_2$: $O_{\{123\}}$ requires all momentum components to be nonzero

$8_1$: $O_{\{441\}} - (O_{\{221\}} + O_{\{331\}})/2$ mixes under renormalization

§ For higher spin, all ops mix with lower-dimension ops
\( \langle x^n \rangle \) Moments

§ For higher spin, all ops mix with lower-dimension ops

☞ Tricks: subtraction to remove divergent terms, heavy fields, four-point functions... None is practical enough

§ Relative error grows in higher moments

☞ Calculation would be costly

Dolgov et al. PRD66, 034506 (2002)
Göckeler et al. PRD71, 114511 (2005)

LHPC (SCRI, SESAM): 2f, Wilson and clover
QCDSF: 0f

\( \langle x^2 \rangle_q \)

\( \langle x^3 \rangle_q \)
§ What can we learn about the $x$-distribution?

Make an ansatz of some smooth form for the distribution and fix the parameters by matching to the lattice moments

$$x q(x) = ax^b (1 - x)^c \left( 1 + \epsilon \sqrt{x} + \gamma x \right)$$

Cannot separate valence-quark contribution from sea

New idea needed to access the sea!

§ Lightcone nucleon quark distribution

Transform lab coordinates to light-cone ones $x_\pm = z \pm t$

$$q(x, \mu) = \int \frac{d\xi_-}{4\pi} e^{-i\xi_- x P_+} \left\langle P \left| \psi(\xi_-) \gamma_+ \exp\left(-i g \int_0^{\xi_-} d\eta_- A_+(\eta_-)\right) \psi(0) \right| P \right\rangle$$

Renormalization scale $\mu$

Lightcone coordinate $\xi_\pm = (t \pm z)/\sqrt{2}$

Nucleon momentum $P_\mu$

Gluon potential $A_+$
§ Lightcone nucleon quark distribution

Transform lab coordinates to light-cone ones $x_\pm = z \pm t$

$$q(x, \mu) = \int \frac{d\xi_-}{4\pi} e^{-i\xi_-x^+} P \left| \bar{\psi}(\xi_-) \gamma_+ \exp \left( -ig \int_0^{\xi_-} d\eta_- A_+(\eta_-) \right) \psi(0) \right| P$$

Renormalization scale $\mu$

Lightcone coordinate $\xi_\pm = (t \pm z)/\sqrt{2}$

Nucleon momentum $P_\mu$

Massive particles lie on hyperboloids invariant under Lorentz transformation
The Idea

§ Lightcone nucleon quark distribution

\[ q(x, \mu) = \int \frac{d \xi_-}{4\pi} e^{-i \xi_- x P_+} \left( P \right) \bar{\psi}(\xi_-) \gamma_+ \exp\left(-i g \int_0^{\xi_-} d \eta_- A_+(\eta_-)\right) \psi(0) \mid P \right) \]

Renormalization scale \( \mu \)

Lightcone coordinate \( \xi_\pm = (t \pm z)/\sqrt{2} \)

Gluon potential \( A_+ \)

Nucleon momentum \( P_\mu \)

§ Approaching lightcone with large \( P \)

\( \therefore \) Just another limit to take, like taking \( a \rightarrow 0 \)

§ Finite-momentum quark distribution

\[
q(x, \mu, P_z) = \int \frac{dz}{4\pi} e^{-izk_z} \left\langle P \right| \overline{\psi}(z) \gamma_z \exp(-ig \int_0^z dz' A_z(z')) \psi(0) \left| P \right\rangle 
\]

\[
= \text{Product of lattice gauge links} + O(\Lambda^2_{QCD} / P_z^2, M_N^2 / P_z^2)
\]

\[x = k_z / P_z\]

Lattice z coordinate

Nucleon momentum \(P_\mu = \{P_0, 0, 0, P_z\}\)

\(\Rightarrow\) In \(P_z \rightarrow \infty\) limit, parton distribution is recovered

\(\Rightarrow\) For finite \(P_z\), corrections are needed

Some Lattice Details

§ Exploratory study

☞ $N_f = 2+1+1$ clover/HISQ lattices (MILC)
  $M_\pi \approx 310$ MeV, $a \approx 0.12$ fm ($L \approx 2.88$ fm)
  Isovector only ("disconnected" suppressed)
  gives us flavor asymmetry between up and down quark
  2 source-sink separation ($t_{\text{sep}} \approx 0.96$ and 1.2 fm) used

§ Properties known on these lattices

☞ Lattice $Z_\Gamma$ for bilinear operator $\sim 1$
  (with HYP-smearing)
  $M_\pi L \approx 4.6$ large enough to avoid finite-volume effects

§ Feasible with today’s computational resources!

☞ 8/16 nodes on UW Hyak cluster
Exploratory study

$N_f = 2+1+1$ clover/HISQ lattices (MILC)

$M_\pi \approx 310$ MeV, $a \approx 0.12$ fm ($L \approx 2.88$ fm)

NO SYSTEMATICS YET!
§ Exploratory study

\[ \langle P \mid \bar{\psi}(z) \gamma_z \exp\left(-i g \int_0^z d z' A_z(z')\right) \psi(0) \mid P \rangle \]

How many links are needed?

Lattice momenta discretized by finite size of volume

\[ P_z \in \{1, 2, 3\} \frac{2\pi}{L} \]
Exploratory study

\[ \int \frac{dz}{4\pi} e^{-izk_z} \left| P \right| \bar{\psi}(z) \gamma_z \exp\left( -ig \int_0^z dz' A_z(z') \right) \psi(0) \left| P \right| \]

\[ P_z \in \{1, 2, 3\}^{2\pi/L} \]

Uncorrected bare lattice results

\[ x = \frac{k_z}{P_z} \]
Exploratory study

\[ \int \frac{dz}{4\pi} e^{-izk_z} \left( P \bar{\psi}(z) \gamma_z \exp \left( -ig \int_0^z dz' A_z(z') \right) \psi(0) \right) \]

\[ P_z \in \{1, 2, 3\} \frac{2\pi}{L} \]

Distribution gets sharper as \( P_z \) increases
Artifacts due to finite \( P_z \) on the lattice

Improvement?
Work out leading-\( P_z \) corrections
§ Back to the continuum

\[ q(x, \mu) = q_{FP}(x, \mu, P_z) + O(\Lambda_{QCD}^2 / P_z^2) + O(M_N^2 / P_z^2) + O(\alpha_s) \]

What we want
What we calculate on the lattice

\[ P_z \in \{1, 2, 3\}^{2\pi / L} \]

Smaller \( P_z \) correction but complicated twist-4 operator
(extrapolate it away)

J.-H. Zhang, Y. Zhao, J.-W. Chen et al. (in preparation)

Dominant correction (for nucleon); known scaling form

J.-W. Chen et al. (in preparation)

Finite \( P_z \rightarrow \infty \)

Estimate \( O(20\%) \) effect

X. Xiong, X. Ji, J. Zhang, 1310.7471 [hep-ph]
§ Exploratory study

- Take ratios (partially cancel statistical and systematic errors)

\[ q_{\text{norm}}(x, \mu, P_z) = \frac{q(x, \mu, P_z)}{\int dx \, q(x, \mu, P_z)} \]

Removing \( O(M_N^n/P_z^n) \) errors + \( O(\alpha_s) \)

No significant finite-momentum effect seen for \( P_z > 1 \)

§ Renormalization needed
§ Compare with experiments

\[ q_f(x) = - q_f(-x) \]

Compared with E866
Too good to be true?

Lost resolution in small-x region
Future improvement to have larger lattice volume

\[ \int dx \frac{\bar{u}(x) - \bar{d}(x)}{g_V} \approx -0.16(12) \]

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<table>
<thead>
<tr>
<th>Experiment</th>
<th>( x ) range</th>
<th>( \int_0^1 [\bar{d}(x) - \bar{u}(x)] dx )</th>
</tr>
</thead>
<tbody>
<tr>
<td>E866</td>
<td>( 0.015 &lt; x &lt; 0.35 )</td>
<td>0.118 ± 0.012</td>
</tr>
<tr>
<td>NMC</td>
<td>( 0.004 &lt; x &lt; 0.80 )</td>
<td>0.148 ± 0.039</td>
</tr>
<tr>
<td>HERMES</td>
<td>( 0.020 &lt; x &lt; 0.30 )</td>
<td>0.16 ± 0.03</td>
</tr>
</tbody>
</table>

R. Towell et al. (E866/NuSea), Phys.Rev. D64, 052002 (2001)
Exploratory study

\[
\int \frac{dz}{4\pi} e^{-iz k_z} \left( P \left| \bar{\psi}(z) \gamma_z \gamma_5 \exp\left(-i g \int_0^z dz' A_z(z')\right) \psi(0) \right| P \right)
\]
§ Exploratory study

\[
\int \frac{d\zeta}{4\pi} e^{-izk_z} \left\langle P \left| \bar{\psi}(\zeta) \gamma_z \gamma_5 \exp \left( -ig \int_0^\zeta dz' A_{\zeta}(z') \right) \psi(0) \right| P \right\rangle
\]

Removing \(O(M_n^n/P_z^n)\) errors + \(O(\alpha_s)\)

Larger \(O(\Lambda_{QCD}^2/P_z^2)\) seen but well fit by extrapolation

\[
\int dx \frac{\Delta u(x) - \Delta d(x)}{g_A} \approx 0.19(5)
\]

Huey-Wen Lin — Los Alamos National Lab
§ Model: large-$N_c$ predicts larger polarized antiquark asymmetry
chiral quark-soliton model

\[ \int dx \left( \Delta \bar{u}(x) - \Delta \bar{d}(x) \right) \approx 0.31 \]

B. Dressler et al, hep-ph/9809487

A. Airapetian et al. (HERMES),

§ Model: large-$N_c$ predicts larger polarized antiquark asymmetry
chiral quark-soliton model \( \int dx \left( \Delta \bar{u}(x) - \Delta \bar{d}(x) \right) \approx 0.31 \)

B. Dressler et al, hep-ph/9809487

§ Experimental comparison

D. De Florian et al.,
Transversity Distribution

§ Exploratory study

\[
\int \frac{d\mathbf{k}}{4\pi} e^{-i\mathbf{k} \cdot \mathbf{z}} \left\langle P \left| \bar{\psi}(z) \sigma_{xy} \exp \left( -i g \int_0^z dz' A_z(z') \right) \psi(0) \right| P \right\rangle
\]

Uncorrected bare lattice results

§ Renormalization needed
§ Exploratory study

\[ \int \frac{dz}{4\pi} \ e^{-i z k_z} \left| P \right| \bar{\psi}(z) \sigma_{xy} \exp \left( -i g \int_0^z dz' A_z(z') \right) \psi(0) \left| P \right| \]

Removing \( O(M_{N/p}^n/P_{z}^n) \) errors + \( O(\alpha_s) \)

§ Renormalization needed
Exploratory study

We found $\delta u < \delta d$ with large sea asymmetry

Chiral quark-soliton model

\[
\int dx \frac{\delta \bar{u}(x) - \delta \bar{d}(x)}{g_T} \approx -0.23(9)
\]

\[
\int dx \left( \delta \bar{u}(x) - \delta \bar{d}(x) \right) \approx -0.082
\]

Preliminary

\[\delta q_f(x) = -\delta q_f(-x), \quad x\]

CQS model

P. Schweitzer et al. PRD 64, 034013 (2001)
§ Exploratory study

\[ \int \frac{dz}{2\pi} e^{-izk_z} \left\langle 0 \left| \bar{d}(z) \gamma_z \gamma_5 \exp(-ig \int_0^z dz' A_z(z')) u(0) \right| \pi^+(P) \right\rangle \]

Only leading mass correction applied

Dominated by \( O(\Lambda_{\text{QCD}}^2/P_z^2) \) errors

\[ P_z \in \{1, 2, 3\} \frac{2\pi}{L} \]
A NEW HOPE

It is a period of war and economic uncertainty. Turmoil has engulfed the galactic republics. Basic truths at foundation of the human civilization are disputed by the dark forces of the evil empire.

A small group of QCD Knights from United Federation of Physicists has gathered in a remote location on the third planet of a star called Sol on the inner edge of the Orion–Cygnus arm of the galaxy.

The QCD Knights are the only ones who can tame the power of the Strong Force, responsible for holding atomic nuclei together, for giving mass and shape to matter in the Universe.

They carry secret plans to build the most powerful
Summary and Outlook

Exciting time for hadron structure on the lattice

§ Overcoming longstanding obstacle to x-distribution
  Û New idea by Ji for studying full x dependence of PDFs
  Û Promising results on unpolarized and polarized sea asymmetry compared with experiments, even at non-physical pion mass

§ Hope this study motivates others to give Ji’s method a try

§ Caveats
  Û Not a precision calculation yet, proper renormalization,…
  Û Systematics due to large momenta (some ideas to improve it)

§ Need improvement for large-momentum sources
  Û Applications: large-q form factors, hadronic and flavor physics, …

§ Many more quantities to study
  Û strange/charm/beauty sea distributions, gluons, TMD…