

# **Radiative capture study by combining EFT with ab initio calculations:**



Xilin Zhang (Ohio University)

*Nuclear Theory Group Seminar, LANL, Jan 15, 2014*

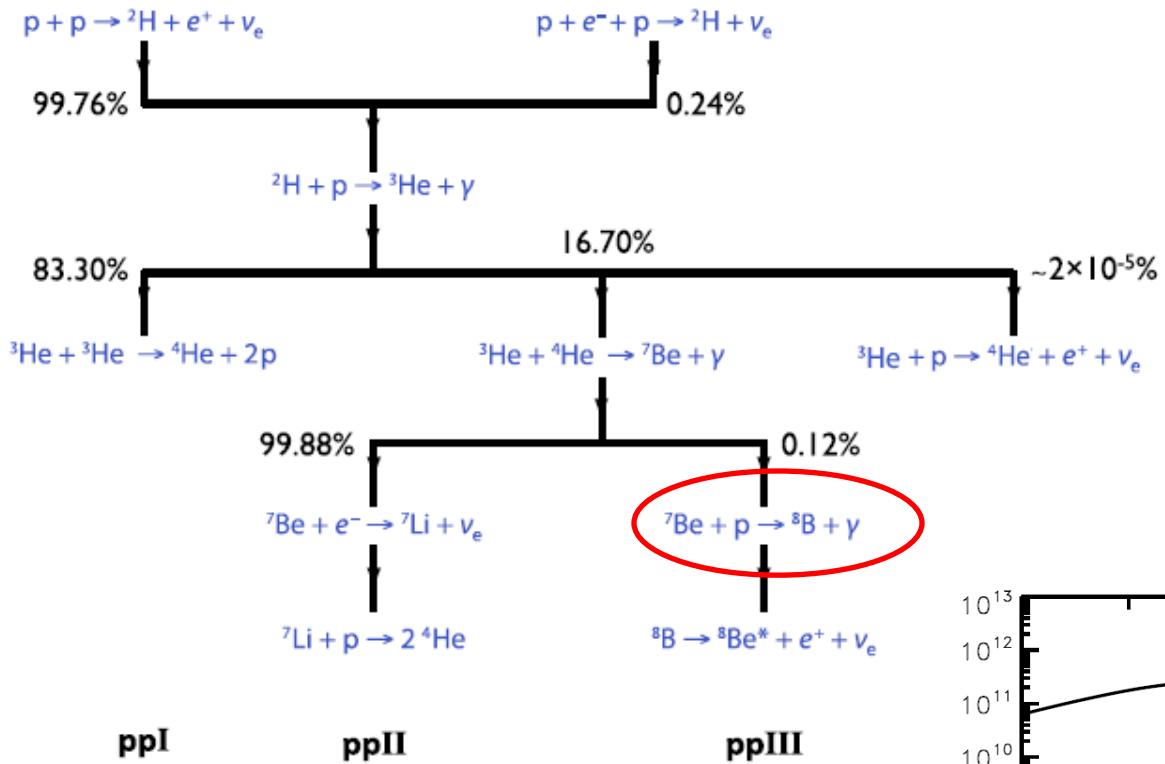
X. Z, K. M. Nollett and D. R. Phillips, arXiv:1311.6822; 1401.xxxx

# Outline

- Motivations
- A toy model: spinless nucleon and core
- Li7 capture: spins, core excitation, leading order (LO) results
- Be7 capture: nonperturbative Coulomb, LO results
- Outlook: Next-to-LO
- My other works: neutrino-nucleus, jet quenching in heavy ion collision, cold nuclear matter

# Motivations

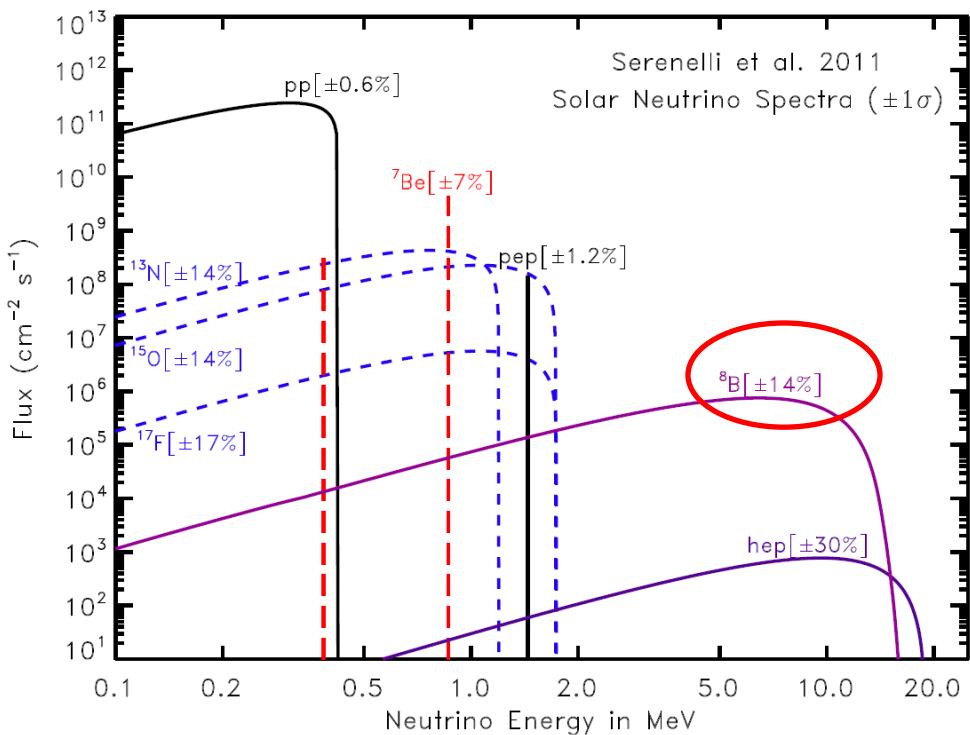
## Astrophysics



# Solar neutrino generation

Not experimentally accessible

W. C. Haxton et.al., arXiv:1208.5723



# SENSITIVITY OF *r*-PROCESS NUCLEOSYNTHESIS TO LIGHT-ELEMENT NUCLEAR REACTIONS

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G. J. MATHEWS AND K. OTSUKI<sup>2</sup>

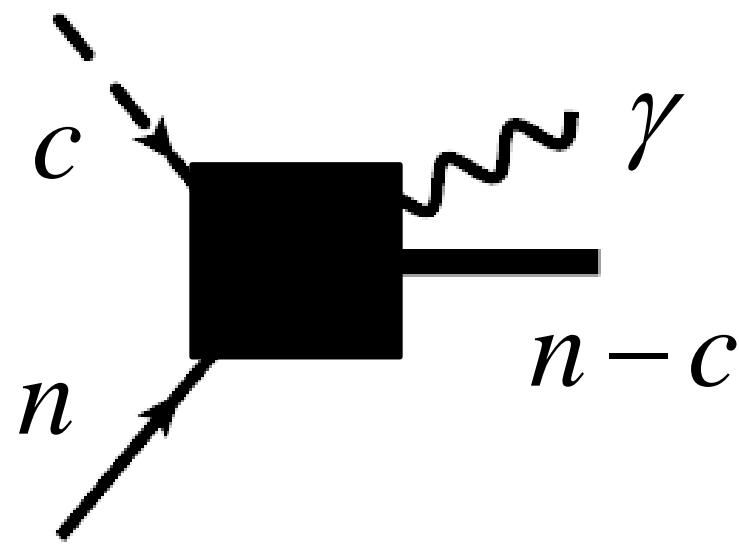
Center for Astrophysics, Department of Physics, University of Notre Dame, Notre Dame, IN 46556

<sup>AND</sup>  
MOST IMPORTANT 18 LIGHT-MASS NUCLEAR REACTIONS, ADOPTED “STANDARD” THERMONUCLEAR REACTION RATES  $\lambda_i(0)$ , AND UNCERTAINTIES

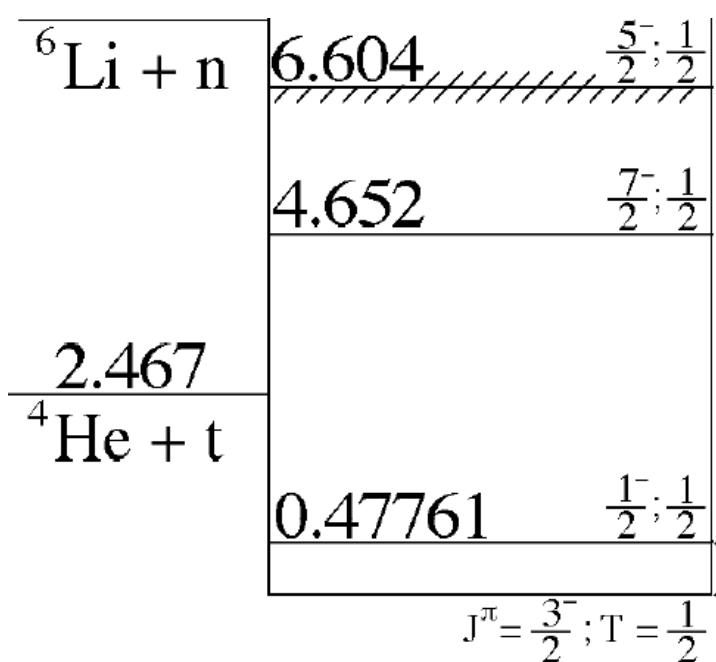
No.	Reaction	$N_{\text{Av}} \langle \sigma v \rangle$	$1 \sigma^{\text{a}}$	Reference <sup>b</sup>
(1).....	$\alpha(\alpha n, \gamma)^9\text{Be}$	$N_{\text{Av}}^2 \langle \alpha \alpha n \rangle = 2.43 \times 10^9 T_9^{-2/3} \exp[-13.490 T_9^{-1/3} - (T_9/0.15)^2] (1 + 74.5 T_9)$ $+ 6.09 \times 10^5 T_9^{-3/2} \exp(-1.054/T_9) (1 - 58.80 T_9 - 1.794 \times 10^4 T_9^2$ $+ 2.969 \times 10^6 T_9^3 - 1.535 \times 10^8 T_9^4 + 2.610 \times 10^9 T_9^5)$	$\pm 35\%$	1
(2).....	$\alpha(t, \gamma)^7\text{Li}^{\text{c}}$	$3.032 \times 10^5 T_9^{-2/3} \exp(-8.09/T_9^{1/3}) (1.0 + 0.0516 T_9^{1/3} + 0.0229 T_9^{2/3}$ $+ 8.28 \times 10^{-3} T_9 - 3.28 \times 10^{-4} T_9^{4/3} - 3.01 \times 10^{-4} T_9^{5/3})$ $+ 5.109 \times 10^5 T_{9*}^{5/6} T_9^{-3/2} \exp(-8.068/T_{9*}^{1/3})$	$\pm 30\%$	2
(3).....	$^7\text{Li}(n, \gamma)^8\text{Li}$	$4.90 \times 10^3 + 9.96 \times 10^3 T_9^{-3/2} \exp(-2.62/T_9)$	$\pm 35\%$	3

*Li7 capture is used to constrain models of Be7 capture.*

# A toy model

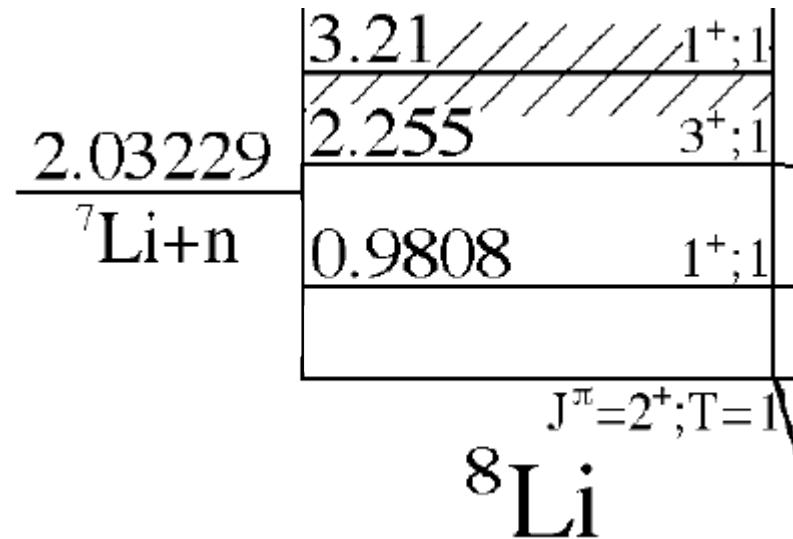


# Gross features: p-wave



${}^7\text{Li}$

*Shallow p-wave  
bound state*



$$\Lambda \sim \sqrt{2M_{43}B_{Li7}} \sim 100 \text{ MeV}$$

$$\gamma \sim \sqrt{2M_{71}B_{Li8}} = 57.8 \text{ MeV}$$

$$\frac{\gamma}{\Lambda} \sim 0.5$$

# Gross features: s-wave

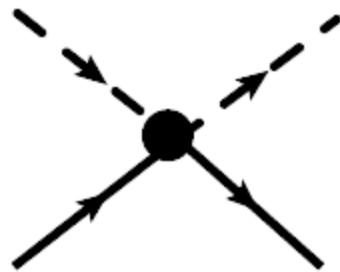
Parameter	Channel	Value	Assigned scaling
$a_{(5S_2)}$	S-wave, $S = 2$	$-3.63(5)$ fm	$1/\gamma$
$a_{(3S_1)}$	S-wave, $S = 1$	$0.87(7)$ fm	$1/\Lambda$

$$\Lambda \approx 100 \text{ MeV}$$

*Large s-wave scattering length*

L. Koester, K. Knopf, and W. Waschkowski, Z. Phys. A 312, 81 (1983)

# S-wave in EFT



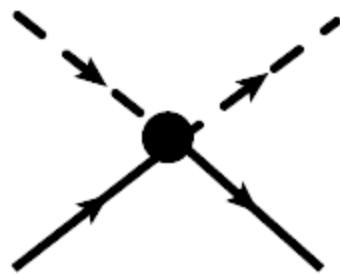
$$T = \frac{2\pi}{M_R} \frac{1}{-k \cot \delta_0 + ik}$$

Effective range expansion (ERE):  $-k \cot \delta_0 = \frac{1}{a_0} - \frac{1}{2} r_0 k^2 + \dots$

$$T = \frac{2\pi}{M_R} a_0 \left[ 1 - ia_0 k + \left( \frac{a_0 r_0}{2} - a_0^2 \right) k^2 + \dots \right]$$

Natural  
 $a_0 \sim \frac{1}{\Lambda}$  and  $r_0 \sim \frac{1}{\Lambda}$

# S-wave in EFT



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 $a_0 \sim \frac{1}{\Lambda}$  and  $r_0 \sim \frac{1}{\Lambda}$

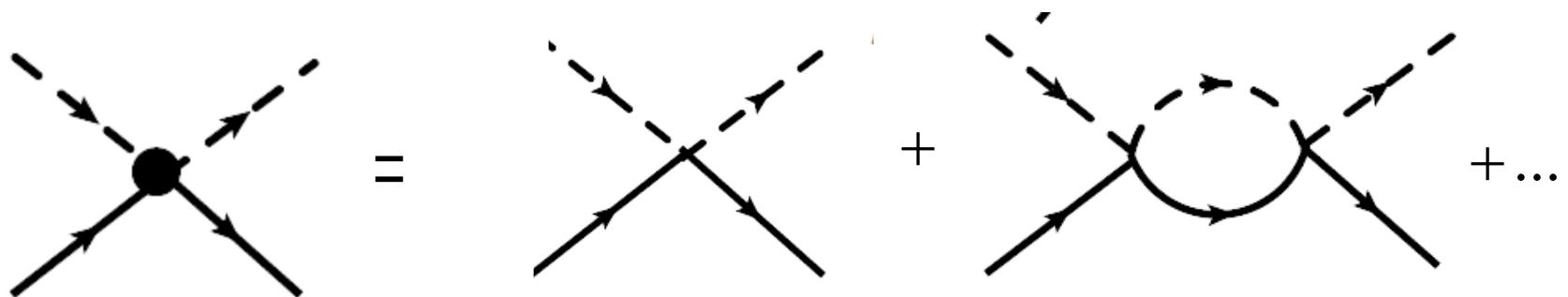
$$T = \frac{2\pi}{M_R} \frac{1}{a_0^{-1} + ik} \left( 1 + \frac{r_0 k^2}{2} \frac{1}{a_0^{-1} + ik} + \dots \right)$$

Unnatural  
 $a_0 \sim \frac{1}{\gamma}$  but  $r_0 \sim \frac{1}{\Lambda}$

# S-wave in EFT

$$\mathcal{L}_0 = n^\dagger \left( i\partial_t + \frac{\nabla^2}{2M_n} \right) n + c^\dagger \left( i\partial_t + \frac{\nabla^2}{2M_c} \right) c$$

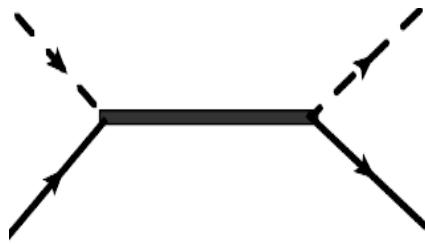
$$\mathcal{L}_S = g c^\dagger n^\dagger c n \quad g(\mu) = \frac{2\pi}{M_R \left( 2\mu - \frac{1}{a_0} \right)} \quad \rightarrow$$



$$T = \frac{2\pi}{M_R} \frac{1}{a_0^{-1} + ik}$$

*One parameter: g (or a0)*

# P-wave in EFT



$$\langle p' | T(E) | p \rangle = \frac{6\pi}{M_R} \frac{p' \cdot p}{a_1^{-1} - \frac{1}{2} r_1 k^2 + ik^3}$$

$$k^3 \cot \delta_1 = -\frac{1}{a_1} + \frac{1}{2} r_1 k^2 + \dots$$

Shallow p-wave bound state:  $\frac{1}{a_1} + \frac{1}{2} r_1 \gamma^2 + \gamma^4 = 0$

$$a_1 \sim \frac{1}{\Lambda \gamma^2} \text{ and } r_1 \sim \Lambda$$



Unnatural

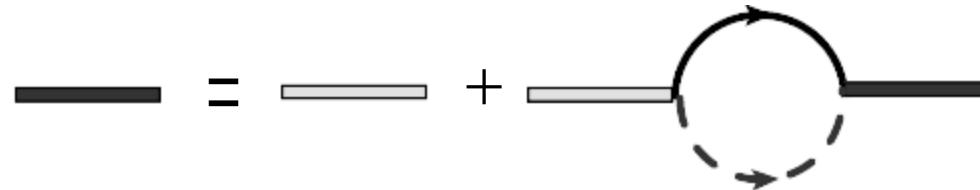
Natural

# P-wave in EFT

$$\mathcal{L}_P = \pi^{\dagger i} \left( i\partial_t + \frac{\nabla^2}{2M_{\text{nc}}} + \Delta \right) \pi_i + h\pi^{\dagger i} n_i (V_n - V_c)_i c + \text{C.C.} .$$

# P-wave in EFT

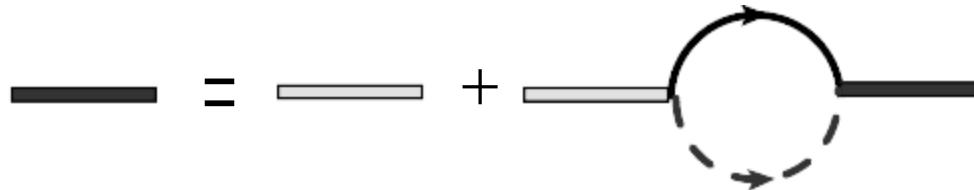
$$\mathcal{L}_P = \pi^{\dagger i} \left( i\partial_t + \frac{\nabla^2}{2M_{\text{nc}}} + \Delta \right) \pi_i + h\pi^{\dagger i} n_i (V_n - V_c)_i c + \text{C.C.} .$$



$$\Sigma_i^j(p^0, \mathbf{p}) \equiv \delta_i^j \Sigma(p^0, \mathbf{p}) = -\delta_i^j \frac{h^2}{6\pi M_R} k^2 (ik + 3\mu)$$

# P-wave in EFT

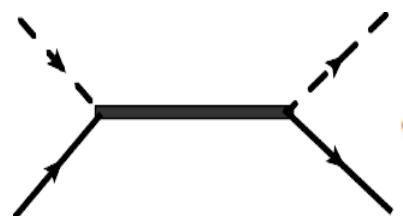
$$\mathcal{L}_P = \pi^{\dagger i} \left( i\partial_t + \frac{\nabla^2}{2M_{\text{nc}}} + \Delta \right) \pi_i + h\pi^{\dagger i} n_i (V_n - V_c)_i c + \text{C.C.} .$$



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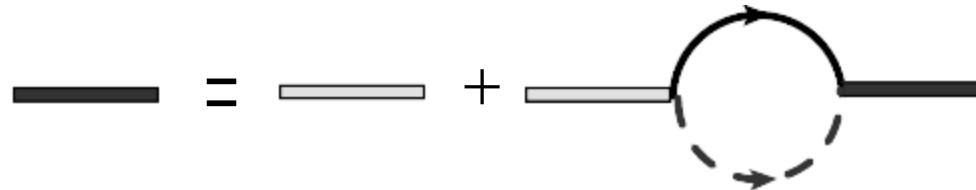
$$\Delta \frac{6\pi M_R}{h^2} = \frac{1}{a_1} \quad \frac{3\pi}{h^2} + 2\mu = -\frac{1}{2}r_1$$



  $\langle p' | T(E) | p \rangle = \frac{h^2}{M_R^2} (\mathbf{p}' \cdot \mathbf{p}) D(E, 0) = \frac{6\pi}{M_R} \frac{\mathbf{p}' \cdot \mathbf{p}}{a_1^{-1} - \frac{1}{2}r_1 k^2 + ik^3}$

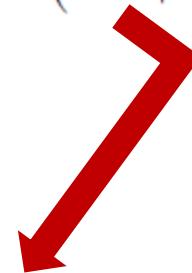
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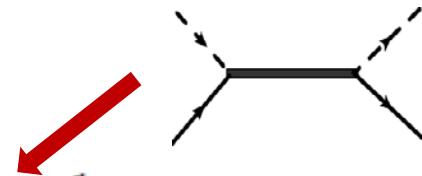


A Feynman diagram for the T-matrix element  $\langle p' | T(E) | p \rangle$ . It shows a horizontal line with two arrows at its ends, representing a particle exchange between two external lines. A red oval highlights the term  $D(E, 0)$  in the equation below.

$$\langle p' | T(E) | p \rangle = \frac{h^2}{M_R^2} (\mathbf{p}' \cdot \mathbf{p}) D(E, 0) = \frac{6\pi}{M_R} \frac{\mathbf{p}' \cdot \mathbf{p}}{a_1^{-1} - \frac{1}{2}r_1 k^2 + ik^3}$$

*Two parameters: Delta and h (or a1 and r1)*

# P-wave in EFT



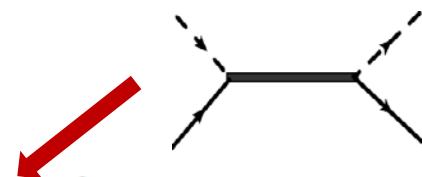
$$\langle r' | \frac{1}{E - H} | r \rangle = \langle r' | \frac{1}{E - H_0} + \frac{1}{E - H_0} T \frac{1}{E - H_0} | r \rangle$$

*Asymptotic  
normalization  
coefficient (ANC)*

$$\xrightarrow{E \rightarrow -B} C^2 \times \sum_j \frac{\phi_j(r') \phi_j^*(r)}{E + B} .$$

$$\phi_j(\mathbf{r}) = \left(1 + \frac{1}{\gamma r}\right) Y_{1j}(\hat{\mathbf{r}}) \frac{e^{-\gamma r}}{r}$$

# P-wave in EFT



$$\langle r' | \frac{1}{E - H} | r \rangle = \langle r' | \frac{1}{E - H_0} + \frac{1}{E - H_0} T \frac{1}{E - H_0} | r \rangle$$

*Asymptotic  
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$$E \xrightarrow{-B} C^2 \times \sum_j \frac{\phi_j(r') \phi_j^*(r)}{E + B} .$$

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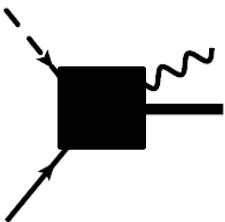
—  $D(E, 0) \rightarrow \frac{Z}{E + B}$  →  $h^2 \frac{\gamma^2 Z}{3\pi} = C^2$

$$C^2 = \sqrt{\frac{-2\gamma^2}{r_1 + 3\gamma}}$$

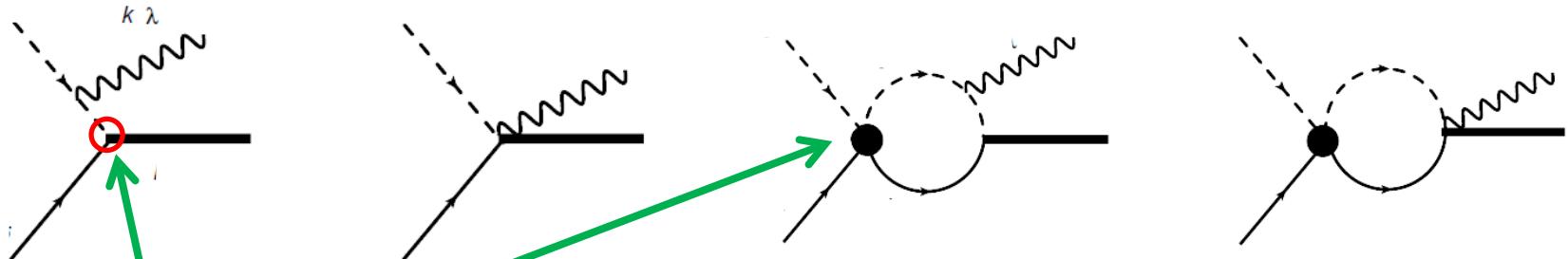
$$\frac{1}{a_1} + \frac{1}{2} r_1 \gamma^2 + \gamma^3 = 0$$

}

$a_1$  and  $r_1$  (or  $h$  and  $\Delta$ )



# Radiative capture: LO

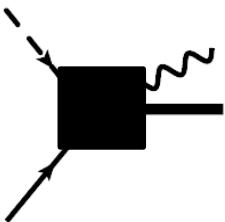


*S wave scattering length*

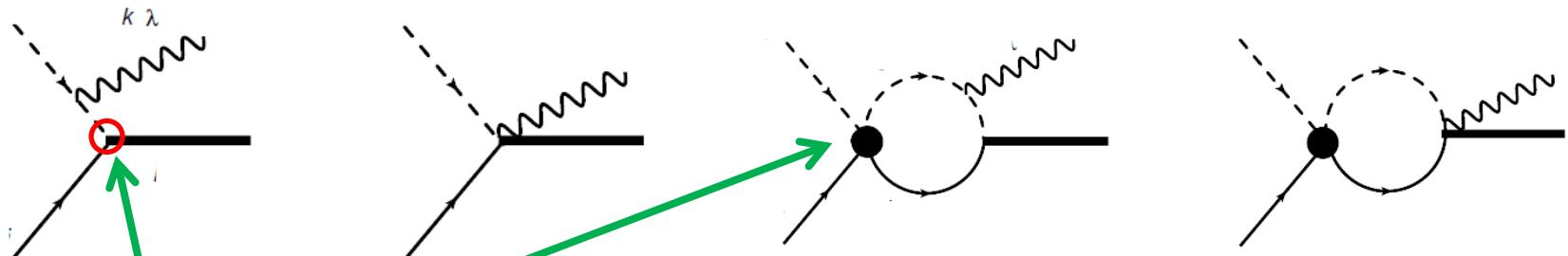
*Halo-EFT parameters*

$$\mathcal{M} \sim ie_c h \sqrt{Z} \left[ \frac{\epsilon^*(\lambda) \cdot V_c}{p_c^0 - \omega - \frac{(p_c - k)^2}{2M_c} + i\epsilon} \left( \frac{p_c}{M_R} - \frac{k}{M_c} \right)_j + (1 + X(p_c; \gamma, a_0)) \frac{\epsilon^*(\lambda)_j}{M_c} \right]$$

$$X(p_c; \gamma, a) \equiv \frac{(-)i}{a^{-1} + ip_c} \left[ p_c - \frac{2}{3}i \frac{\gamma^3 - ip_c^3}{\gamma^2 + p_c^2} \right]$$



# Radiative capture: LO



*S wave scattering length*

*Halo-EFT parameters*

$$\mathcal{M} \sim ie_c \hbar \sqrt{Z} \left[ \frac{\epsilon^*(\lambda) \cdot V_c}{p_c^0 - \omega - \frac{(p_c - k)^2}{2M_c} + i\epsilon} \left( \frac{p_c}{M_R} - \frac{k}{M_c} \right)_j + (1 + X(p_c; \gamma, a_0)) \frac{\epsilon^*(\lambda)_j}{M_c} \right]$$

↓

C

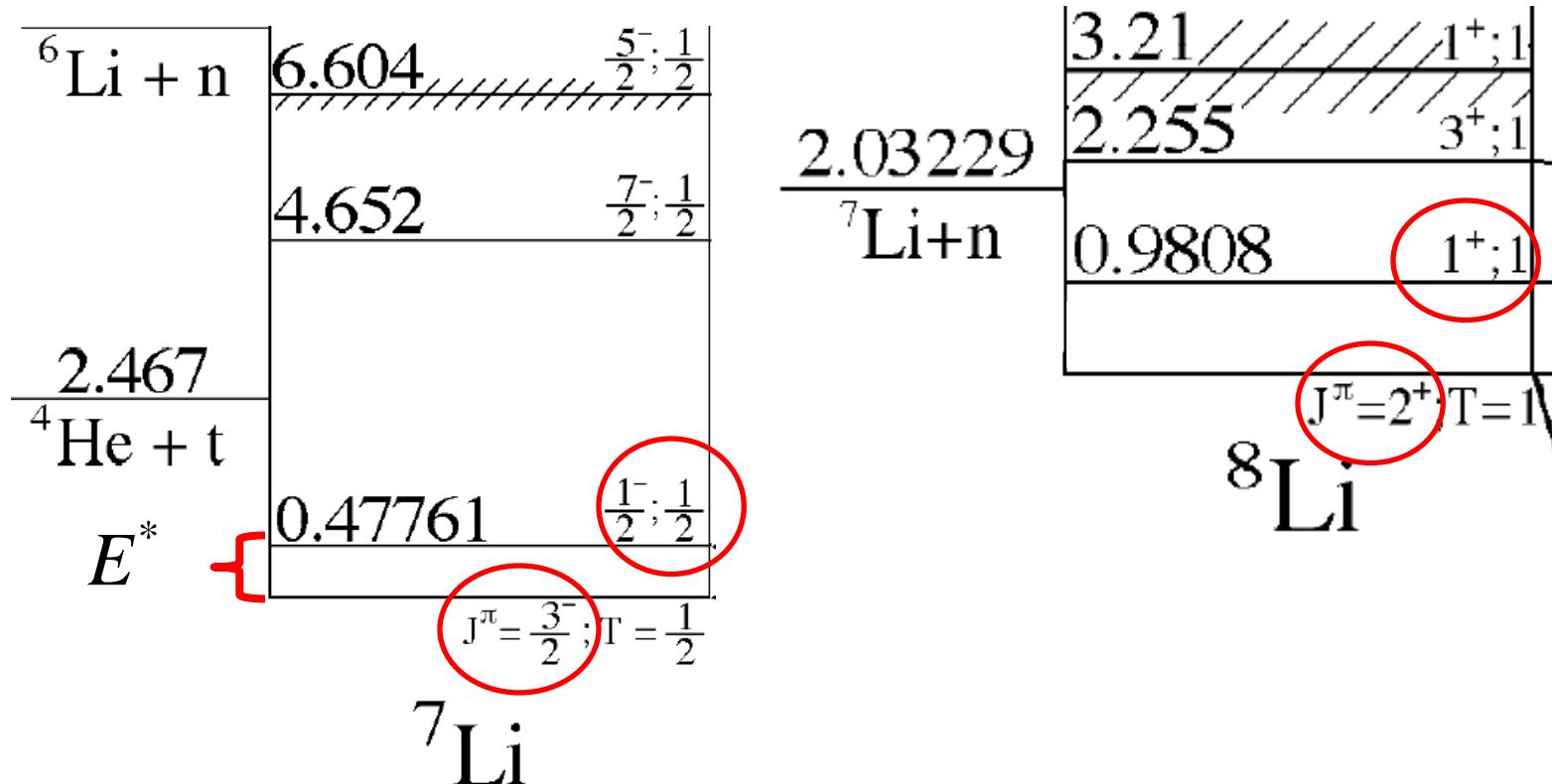
$$X(p_c; \gamma, a) \equiv \frac{(-)i}{a^{-1} + ip_c} \left[ p_c - \frac{2}{3} i \frac{\gamma^3 - ip_c^3}{\gamma^2 + p_c^2} \right]$$

$$a \sim \frac{1}{\gamma} \Rightarrow X \sim 1, \quad a \sim \frac{1}{\Lambda} \Rightarrow X \sim \frac{\gamma}{\Lambda}$$

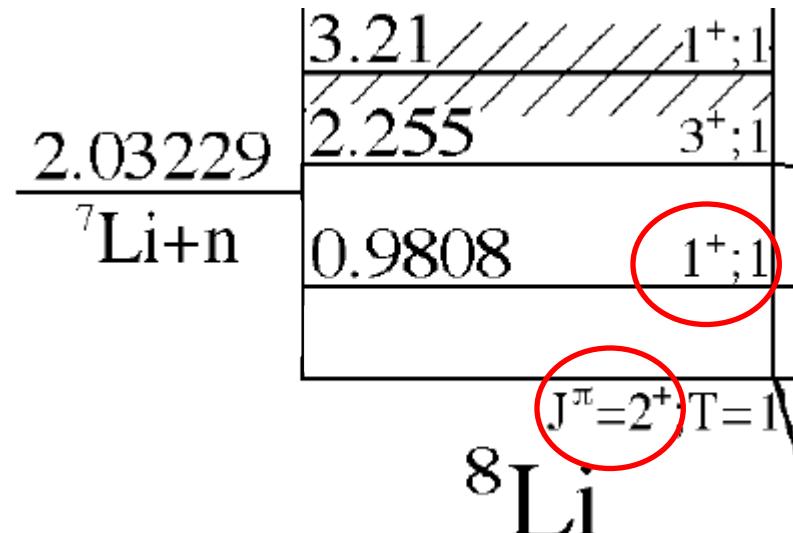
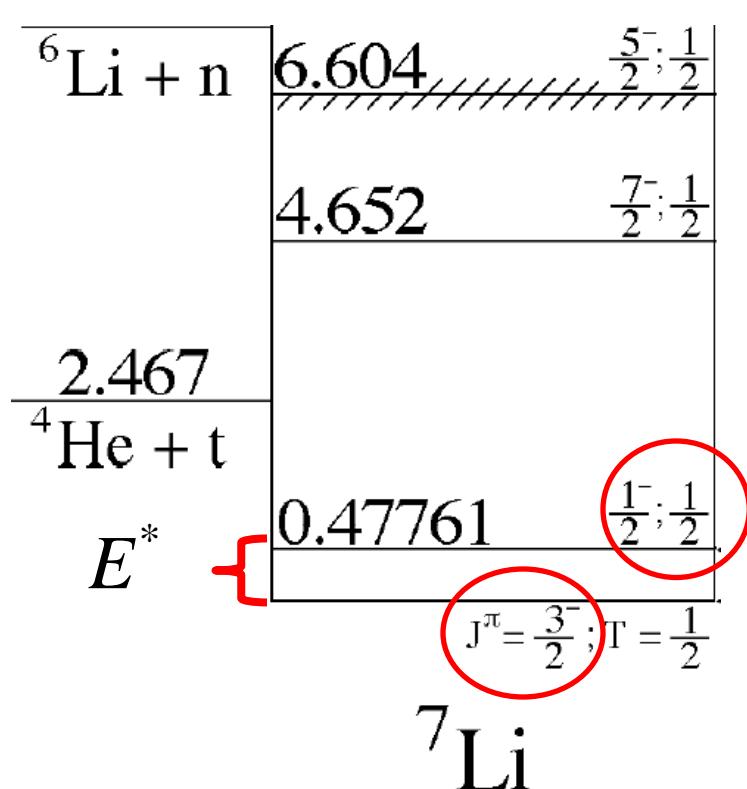


G. Rupak and R. Higa, *Phys. Rev. Lett.* 106, 222501 (2011)

# Scales, spins, core excitations

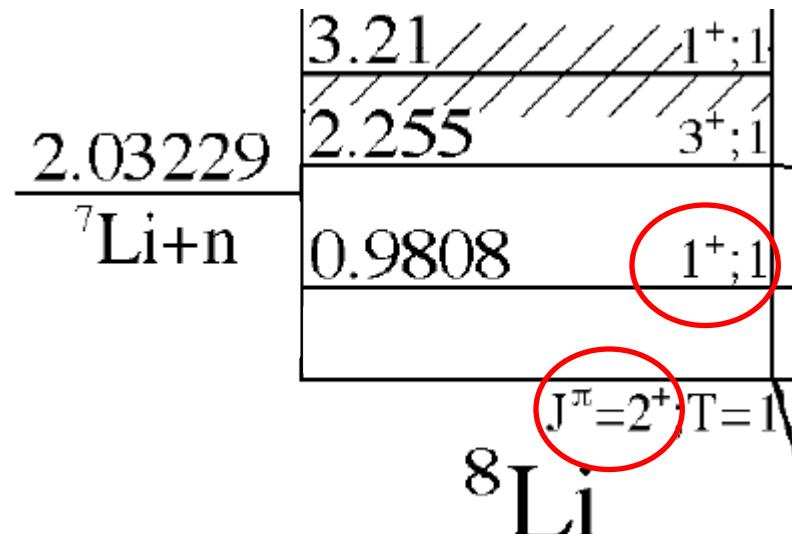
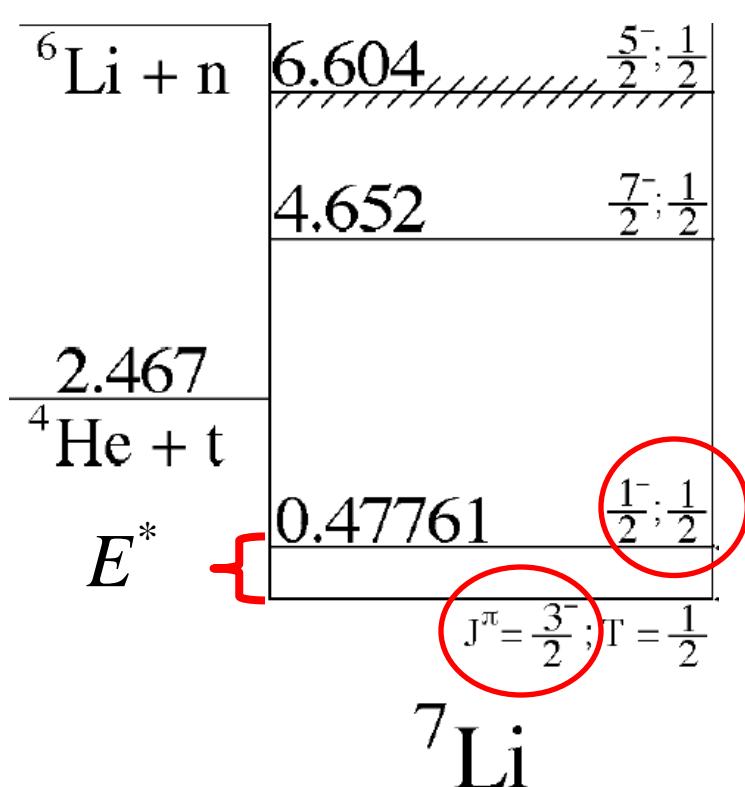


# Scales, spins, core excitations



IS  ${}^7\text{Li} + \text{n}$ :  ${}^3S_1, {}^5S_2, D$   
 IS  ${}^7\text{Li}^* + \text{n}$ :  ${}^1S_0^*, {}^3S_1^*$

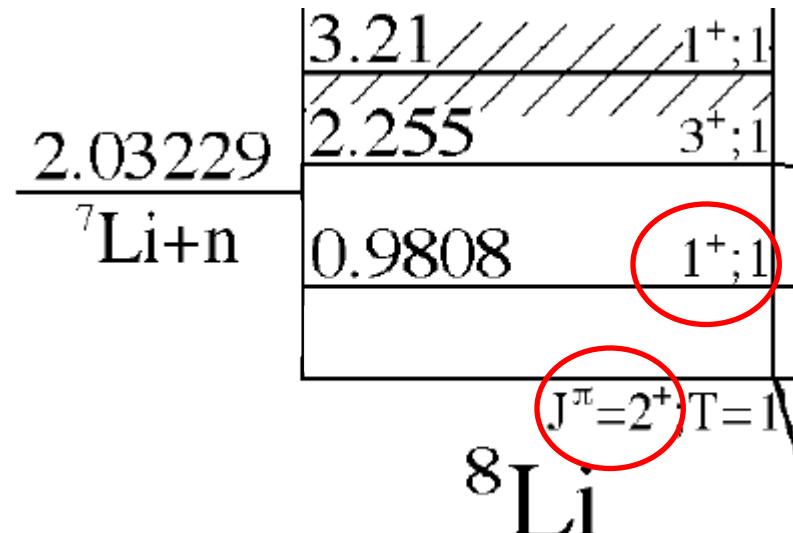
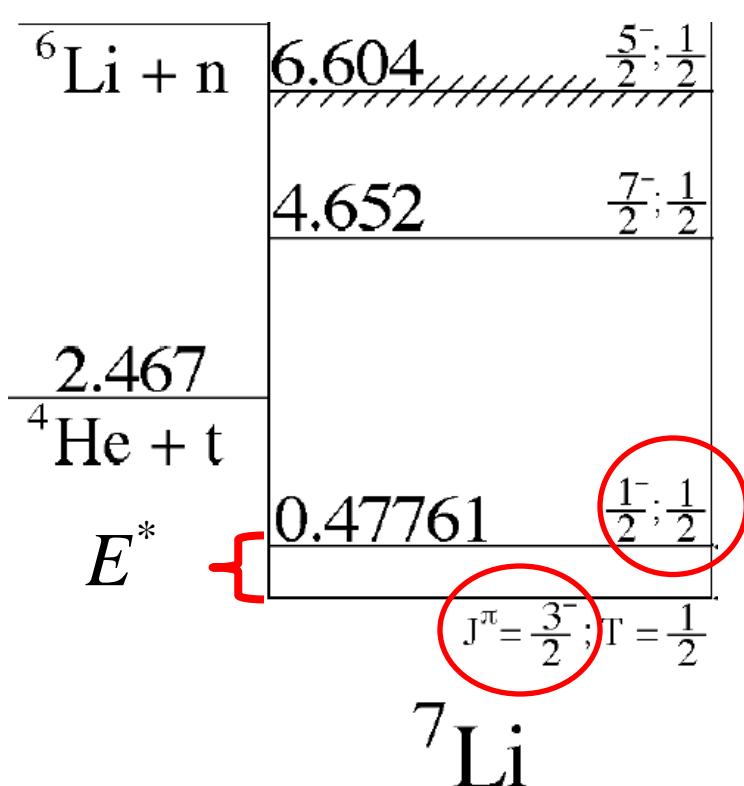
# Scales, spins, core excitations



IS  ${}^7\text{Li} + \text{n}$ :  ${}^3S_1, {}^5S_2, D$   
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FS( $2^+$ )  ${}^7\text{Li} + \text{n}$ :  ${}^3P_2, {}^5P_2$   
 FS( $2^+$ )  ${}^7\text{Li}^* + \text{n}$ :  ${}^3P_2^*$

# Scales, spins, core excitations



IS  ${}^7\text{Li} + \text{n}$ :  ${}^3S_1, {}^5S_2, D$   
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 FS( $2^+$ )  ${}^7\text{Li}^* + \text{n}$ :  ${}^3P_2^*$

FS( $1^+$ )  ${}^7\text{Li} + \text{n}$ :  ${}^3P_1, {}^5P_1$   
 FS( $1^+$ )  ${}^7\text{Li}^* + \text{n}$ :  ${}^1P_1^*, {}^3P_1^*$

# Scales, spins, core excitations

$$\Lambda \approx 100 - 300 \text{ MeV}$$

Momentum scale	Definition	Value
$\gamma$	$\sqrt{2M_R B_{^8\text{Li}}}$	57.8 MeV
$\gamma^*$	$\sqrt{2M_R(B_{^8\text{Li}} + E^*)}$	65.1 MeV
$\gamma_\Delta$	$\sqrt{2M_R E^*}$	30.0 MeV
$\tilde{\gamma}$	$\sqrt{2M_R B_{^8\text{Li}^*}}$	41.6 MeV
$\tilde{\gamma}^*$	$\sqrt{2M_R(B_{^8\text{Li}^*} + E^*)}$	51.3 MeV

Parameter	Channel	Value	Assigned scaling
$a_{(^5S_2)}$	$S$ -wave, $S = 2$	$-3.63(5)$ fm	$1/\gamma$
$a_{(^3S_1)}$	$S$ -wave, $S = 1$	$0.87(7)$ fm	$1/\Lambda$
$r$	$P$ -wave, $J = 2$	$-1.43(2)$ fm $^{-1}$	$\Lambda$
$\tilde{r}$	$P$ -wave, $J = 1$	$-1.86(6)$ fm $^{-1}$	$\Lambda$

# EFT

$$\begin{aligned}\mathcal{L}_0 = & n^{\dagger\sigma} \left( i\partial_t + \frac{\nabla^2}{2M_n} \right) n_\sigma + c^{\dagger a} \left( i\partial_t + \frac{\nabla^2}{2M_c} \right) c_a \\ & + d^{\dagger\delta} \left( i\partial_t + \frac{\nabla^2}{2M_c} \right) d_\delta + \pi^{\dagger\alpha} \left( i\partial_t + \frac{\nabla^2}{2M_{nc}} + \Delta \right) \pi_\alpha \\ & + \tilde{\pi}^{\dagger i} \left( i\partial_t + \frac{\nabla^2}{2M_{nc}} + \tilde{\Delta} \right) \tilde{\pi}_i ,\end{aligned}$$

# EFT

$$\begin{aligned}\mathcal{L}_0 = & n^{\dagger\sigma} \left( i\partial_t + \frac{\nabla^2}{2M_n} \right) n_\sigma + c^{\dagger a} \left( i\partial_t + \frac{\nabla^2}{2M_c} \right) c_a \\ & + d^{\dagger\delta} \left( i\partial_t + \frac{\nabla^2}{2M_c} \right) d_\delta + \pi^{\dagger\alpha} \left( i\partial_t + \frac{\nabla^2}{2M_{nc}} + \Delta \right) \pi_\alpha \\ & + \tilde{\pi}^{\dagger i} \left( i\partial_t + \frac{\nabla^2}{2M_{nc}} + \tilde{\Delta} \right) \tilde{\pi}_i ,\end{aligned}$$

$$\begin{aligned}\mathcal{L}_S = & g_{(3S_1)} c^{\dagger a'} n^{\dagger\sigma'} T_{a'\sigma'}^i T_i^{a\sigma} c_a n_\sigma \\ & + g_{(5S_2)} c^{\dagger a'} n^{\dagger\sigma'} T_{a'\sigma'}^\alpha T_\alpha^{a\sigma} c_a n_\sigma \\ & + g_{(3S_1^*)} d^{\dagger\delta} n^{\dagger\sigma'} T_{\delta\sigma'}^i T_i^{a\sigma} c_a n_\sigma + \text{C.C.} ,\end{aligned}$$

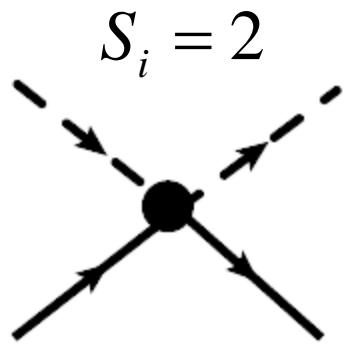
# EFT

$$\begin{aligned}\mathcal{L}_0 = & n^{\dagger\sigma} \left( i\partial_t + \frac{\nabla^2}{2M_n} \right) n_\sigma + c^{\dagger a} \left( i\partial_t + \frac{\nabla^2}{2M_c} \right) c_a \\ & + d^{\dagger\delta} \left( i\partial_t + \frac{\nabla^2}{2M_c} \right) d_\delta + \pi^{\dagger\alpha} \left( i\partial_t + \frac{\nabla^2}{2M_{nc}} + \Delta \right) \pi_\alpha \\ & + \tilde{\pi}^{\dagger i} \left( i\partial_t + \frac{\nabla^2}{2M_{nc}} + \tilde{\Delta} \right) \tilde{\pi}_i ,\end{aligned}$$

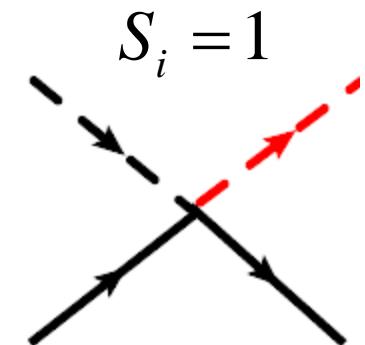
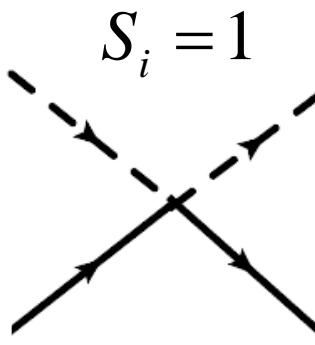
$$\begin{aligned}\mathcal{L}_S = & g_{(3S_1)} c^{\dagger a'} n^{\dagger\sigma'} T_{a'\sigma'}^i T_i^{a\sigma} c_a n_\sigma & \mathcal{L}_{P,gs} = h_{(3P_2)} \pi^{\dagger\alpha} T_\alpha^{ij} T_i^{\sigma a} n_\sigma i(V_n - V_c)_j c_a \\ & + g_{(5S_2)} c^{\dagger a'} n^{\dagger\sigma'} T_{a'\sigma'}^\alpha T_\alpha^{a\sigma} c_a n_\sigma & + h_{(5P_2)} \pi^{\dagger\alpha} T_\alpha^{\beta j} T_\beta^{\sigma a} n_\sigma i(V_n - V_c)_j c_a \\ & + g_{(3S_1^*)} d^{\dagger\delta} n^{\dagger\sigma'} T_{\delta\sigma'}^i T_i^{a\sigma} c_a n_\sigma + \text{C.C.} & + h_{(3P_2^*)} \pi^{\dagger\alpha} T_\alpha^{jk} T_k^{\delta\sigma} n_\sigma i(V_n - V_{c^*})_j d_\delta + \text{C.C.} ,\end{aligned}$$

$$\begin{aligned}\mathcal{L}_{P,es} = & \tilde{h}_{(3P_1)} \tilde{\pi}^{\dagger k} T_k^{ij} T_i^{\sigma a} n_\sigma i(V_n - V_c)_j c_a \\ & + \tilde{h}_{(5P_1)} \tilde{\pi}^{\dagger k} T_k^{\beta j} T_\beta^{\sigma a} n_\sigma i(V_n - V_c)_j c_a \\ & + \tilde{h}_{(1P_1^*)} \tilde{\pi}^{\dagger k} T_k^{0j} T_0^{\sigma\delta} n_\sigma i(V_n - V_{c^*})_j d_\delta \\ & + \tilde{h}_{(3P_1^*)} \tilde{\pi}^{\dagger k} T_k^{ij} T_i^{\sigma\delta} n_\sigma i(V_n - V_{c^*})_j d_\delta + \text{C.C.} .\end{aligned}$$

EFT

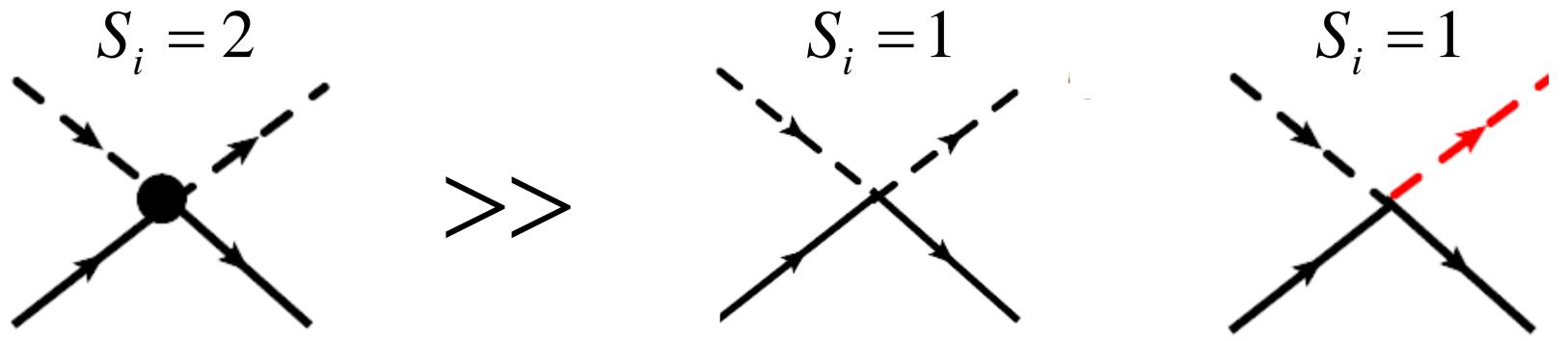


>>

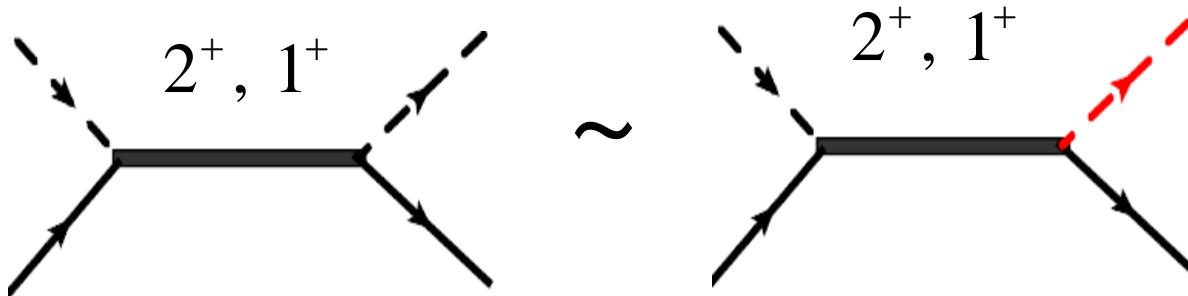


*One fine tuning in S wave*

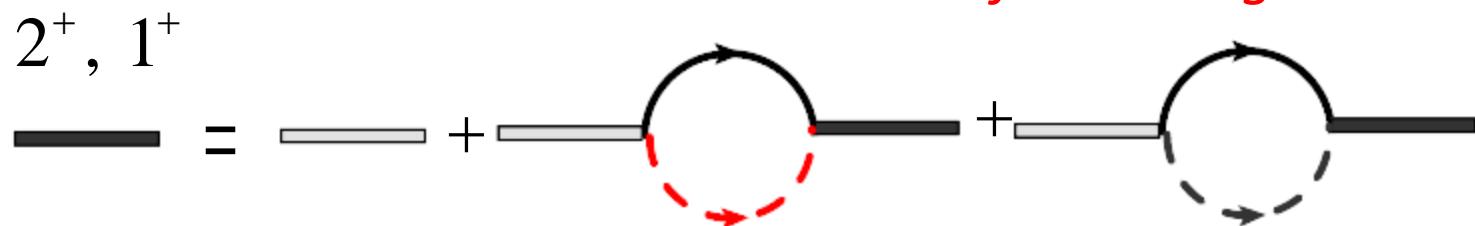
# EFT



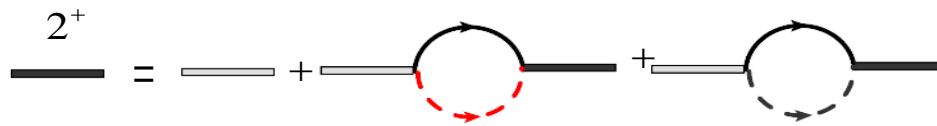
*One fine tuning in S wave*



*One fine tuning in P wave*



# P-wave



$$D^{-1} \frac{6\pi M_R}{h_t^2} = \frac{1}{a} - \frac{h_{(3P_2^*)}^2}{h_t^2} \gamma_\Delta^3 - \frac{1}{2} \left( r - 3 \frac{h_{(3P_2^*)}^2}{h_t^2} \gamma_\Delta \right) k^2 + i \left[ k^3 + \frac{h_{(3P_2^*)}^2}{h_t^2} (k^2 - \gamma_\Delta^2)^{\frac{3}{2}} \right]$$

$$Z = \frac{(-)6\pi}{h_t^2(r + 3\gamma) + 3h_{(3P_2^*)}^2(\gamma^* - \gamma_\Delta)}$$

$$\frac{C_{(3P_2)}^2}{h_{(3P_2)}^2 \gamma^2} = \frac{C_{(5P_2)}^2}{h_{(5P_2)}^2 \gamma^2} = \frac{C_{(3P_2^*)}^2}{h_{(3P_2^*)}^2 \gamma^{*2}} = \frac{Z}{3\pi}$$

	$C_{(3P_2)}$	$C_{(5P_2)}$	$C_{(3P_2^*)}$
Nollett	-0.283(12)	-0.591(12)	-0.384(6)
	-0.284(23)	-0.593(23)	



L. Trache, et.al., Phys. Rev. C 67, 062801(R) (2003)

# P-wave



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*4 parameters: 3 h + 1 Delta,  
or 3 C + gamma*

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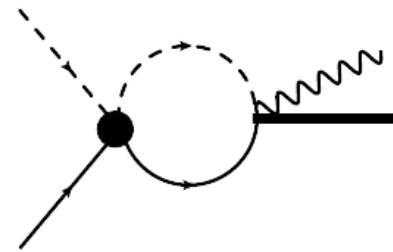
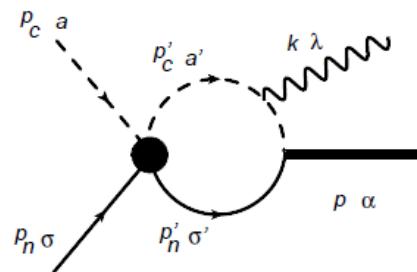
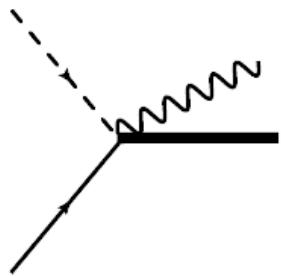
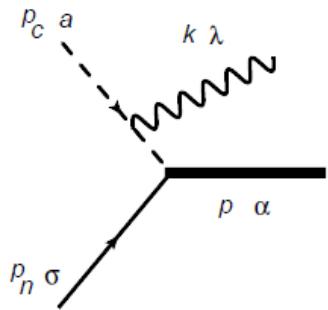
*5 parameters*

	$C_{(3P_2)}$	$C_{(5P_2)}$	$C_{(3P_2^*)}$	$\tilde{C}_{(3P_1)}$	$\tilde{C}_{(5P_1)}$	$\tilde{C}_{(1P_1^*)}$	$\tilde{C}_{(3P_1^*)}$
Nollett	-0.283(12)	-0.591(12)	-0.384(6)	0.220(6)	0.197(5)	-0.195(3)	-0.214(3)
	-0.284(23)	-0.593(23)		0.187(16)	0.217(13)		

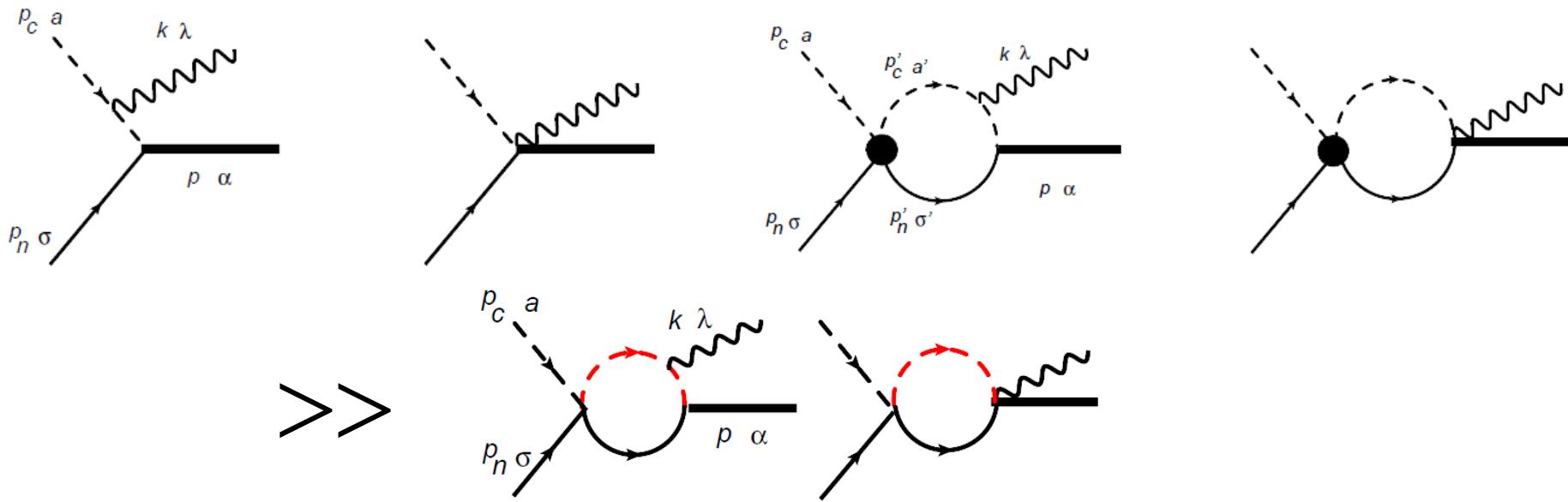


L. Trache, et.al., Phys. Rev. C 67, 062801(R) (2003)

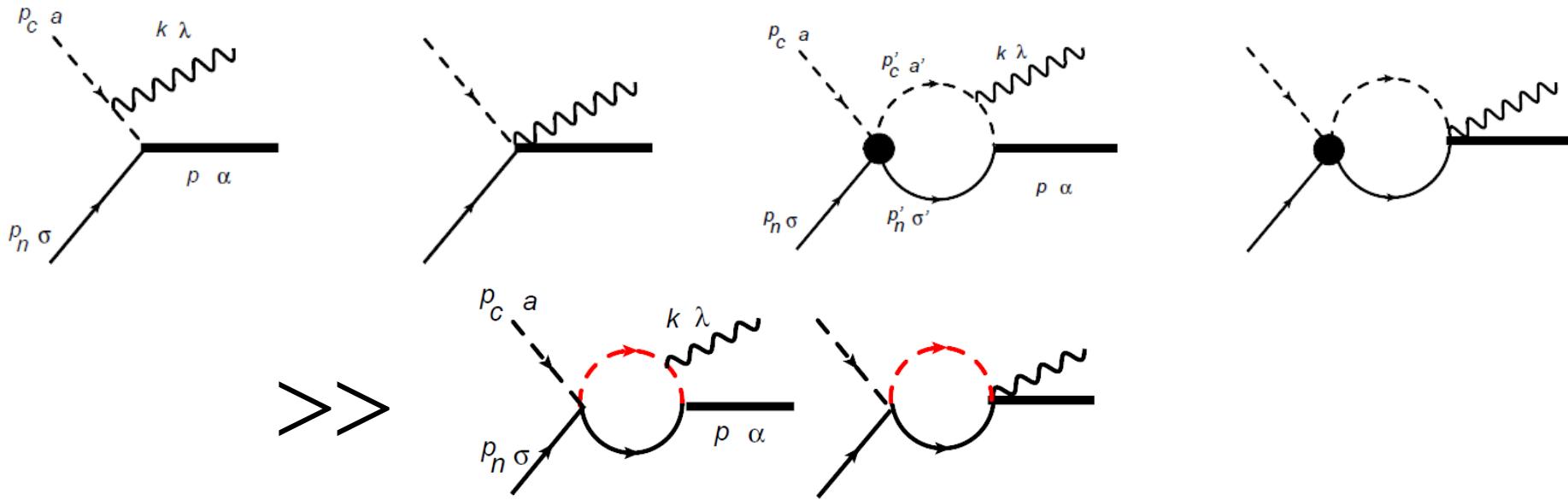
# Radiative captures: LO



# Radiative captures: LO



# Radiative captures: LO



*Initial total spin \$Si=2\$*

$$\mathcal{M} = ie_c h_{(5P_2)} \sqrt{8Z^{\text{LO}} M_n M_c M_{nc}} T_\beta^{\sigma a} T_\alpha^{\beta j} \left[ \frac{\epsilon^*(\lambda) \cdot V_c}{p_c^0 - \omega - \frac{(\mathbf{p}_c - \mathbf{k})^2}{2M_c} + i\epsilon} \left( \frac{p_c}{M_R} - \frac{\mathbf{k}}{M_c} \right)_j + (1 + X(p_c; \gamma, a_{(5S_2)})) \frac{\epsilon^*(\lambda)_j}{M_c} \right]$$

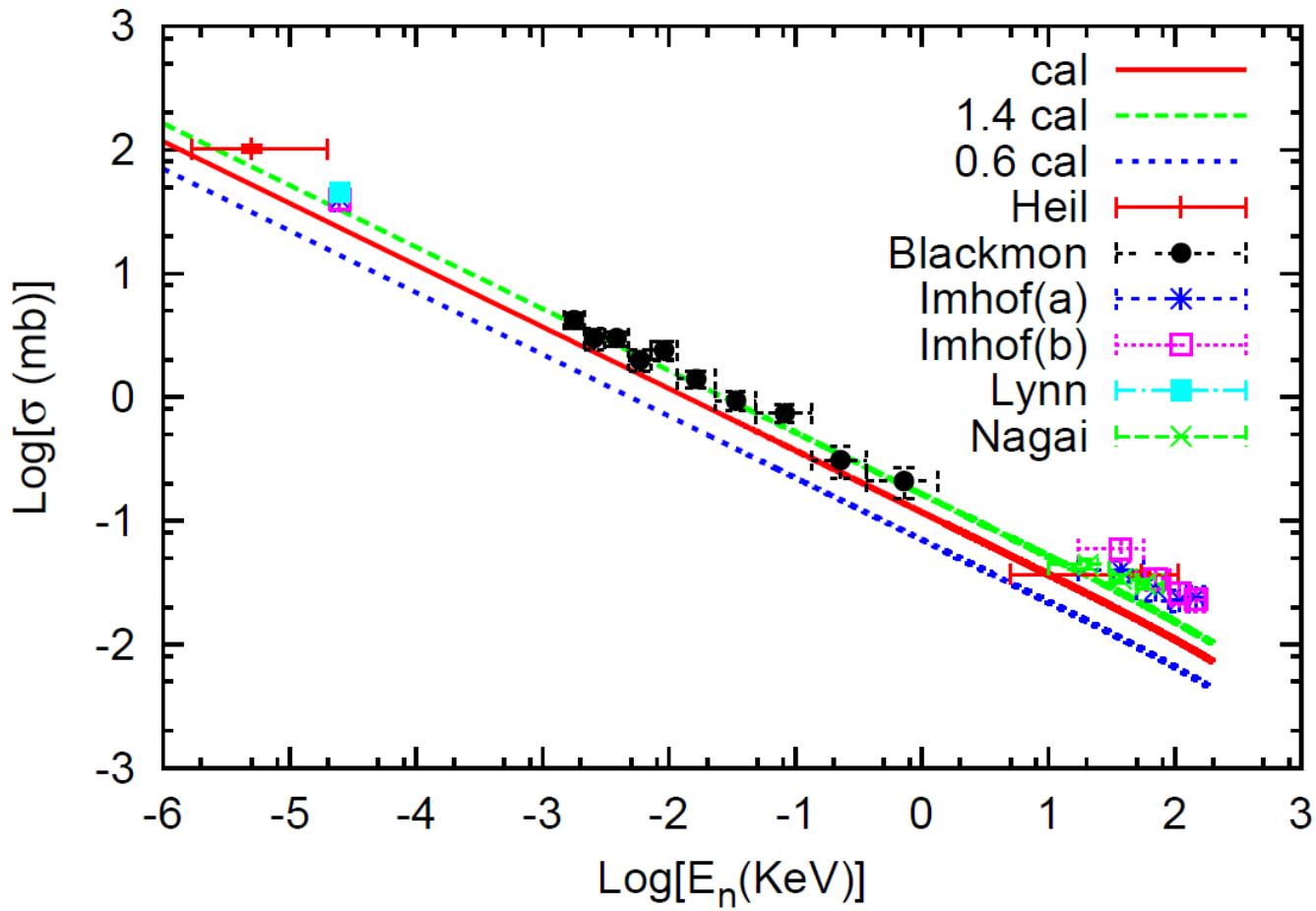
$$X(p_c; \gamma, a) \equiv \frac{(-)i}{a^{-1} + ip_c} \left[ p_c - \frac{2}{3}i \frac{\gamma^3 - ip_c^3}{\gamma^2 + p_c^2} \right] \quad a \sim \frac{1}{\gamma} \Rightarrow X \sim 1, \quad a \sim \frac{1}{\Lambda} \Rightarrow X \sim \frac{\gamma}{\Lambda}$$

$$\begin{aligned} \sum_{\sigma,a}^{\alpha,\lambda}|\mathcal{M}|^2 = & \frac{5}{3}64\pi\alpha Z_c^2\frac{3\pi}{\gamma^2}\frac{M_{\mathrm{n}}^2}{M_{\mathrm{R}}}\left(C_{(^5P_2)}^{\mathrm{LO}}\right)^2\left[\left|1+X(p_c;\gamma,a_{(^5S_2)})\right|^2-\frac{2p_c^2\sin^2\theta}{p_c^2+\gamma^2}\left(\frac{\gamma^2}{p_c^2+\gamma^2}+\mathrm{Re}\left\{X(p_c;\gamma,a_{(^5S_2)})\right\}\right)\right] \\ + & \frac{5}{3}64\pi\alpha Z_c^2\frac{3\pi}{\gamma^2}\frac{M_{\mathrm{n}}^2}{M_{\mathrm{R}}}\left(C_{(^3P_2)}^{\mathrm{LO}}\right)^2\left[1-\frac{p_c^2\sin^2\theta}{p_c^2+\gamma^2}\frac{2\gamma^2}{p_c^2+\gamma^2}\right] \end{aligned}$$

$$X(p_c;\gamma,a) \equiv \frac{(-)i}{a^{-1}+ip_c}\left[p_c-\frac{2}{3}i\frac{\gamma^3-ip_c^3}{\gamma^2+p_c^2}\right]$$

$$\begin{aligned} \sum_{\sigma,a}^{\alpha,\lambda}|\mathcal{M}|^2 &= \frac{5}{3}64\pi\alpha Z_c^2\frac{3\pi}{\gamma^2}\frac{M_{\text{n}}^2}{M_{\text{R}}}\left(C_{(^5P_2)}^{\text{LO}}\right)^2\left[|1+X(p_c;\gamma,a_{(^5S_2)})|^2-\frac{2p_c^2\sin^2\theta}{p_c^2+\gamma^2}\left(\frac{\gamma^2}{p_c^2+\gamma^2}+\text{Re}\left\{X(p_c;\gamma,a_{(^5S_2)})\right\}\right)\right] \\ &+ \frac{5}{3}64\pi\alpha Z_c^2\frac{3\pi}{\gamma^2}\frac{M_{\text{n}}^2}{M_{\text{R}}}\left(C_{(^3P_2)}^{\text{LO}}\right)^2\left[1-\frac{p_c^2\sin^2\theta}{p_c^2+\gamma^2}\frac{2\gamma^2}{p_c^2+\gamma^2}\right] \\ X(p_c;\gamma,a) &\equiv \frac{(-)i}{a^{-1}+ip_c}\left[p_c-\frac{2}{3}i\frac{\gamma^3-ip_c^3}{\gamma^2+p_c^2}\right] \\ \sum_{i,f}|\mathcal{M}|^2 &= 64\pi\alpha Z_c^2\frac{3\pi}{\tilde{\gamma}^2}\frac{M_{\text{n}}^2}{M_{\text{R}}}\Bigg\{\left(\tilde{C}_{(^3P_1)}^{\text{LO}}\right)^2\left[1-\frac{p_c^2\sin^2\theta}{p_c^2+\tilde{\gamma}^2}\left(\frac{2\tilde{\gamma}^2}{p_c^2+\tilde{\gamma}^2}\right)\right] \\ &+ \left(\tilde{C}_{(^5P_1)}^{\text{LO}}\right)^2\left[|1+X(p_c;\tilde{\gamma},a_{(^5S_2)})|^2-\frac{2p_c^2\sin^2\theta}{p_c^2+\tilde{\gamma}^2}\left(\frac{\tilde{\gamma}^2}{p_c^2+\tilde{\gamma}^2}+\text{Re}\left\{X(p_c;\tilde{\gamma},a_{(^5S_2)})\right\}\right)\right]\Bigg\} \end{aligned}$$

# LO results on Li7(n,gamma)Li8(Li8\*)



N. K. Timofeyuk *et.al.*, PRL 91, 232501 (2003); D. Howell *et.al.*, PRC 88, 025804 (2013);  
D. Gul'ko *et.al.*, SJNP 6, 477 (1968); E. Lynn *et.al.*, PRC 44, 764 (1991);  
Y. Nagai *et. al.*, PRC 71, 055803 (2005); J. C. Blackmon *et. al.*, PRC 54, 383 (1996); J. E. Lynn *et. al.*, PRC 44, 764 (1991); M. Heil *et.al.*, Astro. J. 507, 997 (1998); W. L. Imhof *et.al.*, PR 114, 1037 (1959).

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$$\frac{\sigma[(S_i = 1) \rightarrow 2^+]}{\sigma[(S_i = 2) \rightarrow 2^+]} = \frac{\left(C_{(3P_2)}^{\text{LO}}\right)^2}{\left(C_{(5P_2)}^{\text{LO}}\right)^2 (1 - \frac{2}{3}\gamma a_{(5S_2)})^2}$$

→  $\frac{\sigma[(S_i = 2) \rightarrow 2^+]}{\sigma(\rightarrow 2^+)} = 0.93(2) \quad [> 0.86]$

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A. D. Gul'ko, S. S. Trostin, and A. Hudoklin, *Sov. J. Nucl. Phys.* 6, 477 (1968);  
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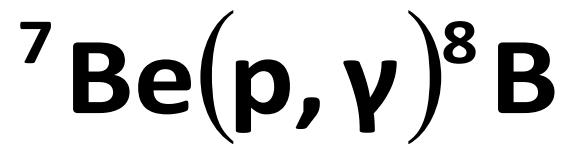
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---

$$\frac{\sigma(\rightarrow 1^+)}{\sigma(\rightarrow 2^+)} = \frac{3}{5} \frac{\left(\tilde{C}_{(3P_1)}^{\text{LO}}\right)^2 + \left(\tilde{C}_{(5P_1)}^{\text{LO}}\right)^2 |1 - \frac{2}{3}a_{(5S_2)}\tilde{\gamma}|^2}{\left(C_{(3P_2)}^{\text{LO}}\right)^2 + \left(C_{(5P_2)}^{\text{LO}}\right)^2 |1 - \frac{2}{3}a_{(5S_2)}\gamma|^2}$$

→  $\frac{\sigma(\rightarrow 2^+)}{\sigma} = 0.88(4) \quad [0.89(1)]$

A. D. Gul'ko, S. S. Trostin, and A. Hudoklin, *Sov. J. Nucl. Phys.* 6, 477 (1968);  
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 Y. Nagai et. al., *Phys. Rev. C* 71, 055803 (2005).



- It is considered as isospin mirror of Li7 capture on the nucleon level
- From EFT/core+proton picture, they are quite different due to strong Coulomb effect

# Nonperturbative Coulomb effect

$$k_C \equiv Q_c Q_n \alpha_{EM} M_R \quad \eta \equiv \frac{k_C}{k} \sim 1$$

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$$\chi_{\mathbf{k}}^{(\pm)}(r) = e^{-\frac{\pi}{2}\eta} e^{i\mathbf{k}r} \Gamma(1 \pm i\eta) M(\mp i\eta, 1; \pm ikr - ikr)$$

*Kummer function*

$$\chi_{\mathbf{k}}^{(\pm)*}(r=0) \chi_{\mathbf{k}}^{(\pm)}(r=0) = \frac{2\pi\eta}{e^{2\pi\eta} - 1} = C_{\eta,0}^2$$

$$\chi_{\mathbf{k}}^{(\mp)*}(r=0) \chi_{\mathbf{k}}^{(\pm)}(r=0) = C_{\eta,0}^2 e^{\pm 2i\sigma_0}$$

*Coulomb barrier, and phase*

$$C_{\eta,l} = \frac{2^l e^{-\frac{\pi}{2}\eta} |\Gamma(l+1+i\eta)|}{\Gamma(2l+2)} \quad e^{2i\sigma_l} \equiv \frac{\Gamma(l+1+i\eta)}{\Gamma(l+1-i\eta)}$$

# ERE in EFT

$$\begin{aligned}\langle \chi_{\mathbf{p}'}^{(-)} | T_{cs}(E) | \chi_{\mathbf{p}}^{(+)} \rangle &= (-) \frac{2\pi}{M_R} \frac{\chi_{\mathbf{p}'}^{(-)*}(0) \chi_{\mathbf{p}}^{(+)}(0)}{-a_0^{-1} - 2k_C H(\eta)} \\ &\rightarrow (-) \frac{2\pi}{M_R} \frac{C_{\eta,0}^2 e^{2i\sigma_0}}{-a_0^{-1} - 2k_C H(\eta)}\end{aligned}$$

$$C_{\eta,0}^2 k (\cot \delta_0 - i) = -\frac{1}{a_0} + \dots - 2k_C H(\eta) \quad H(\eta) = \psi(i\eta) + 1/(2i\eta) - \ln(i\eta)$$

---

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---

$$\begin{aligned} \langle \chi_{\mathbf{p}'}^{(-)} | T_{cs}(E) | \chi_{\mathbf{p}}^{(+)} \rangle &= (-) \frac{6\pi}{M_R} \frac{\partial \chi_{\mathbf{p}'}^{(-)*}(0) \partial \chi_{\mathbf{p}}^{(+)}(0)}{-\frac{1}{a_1} + \frac{r_1}{2} k^2 - k^2(1+\eta^2) 2k_C H(\eta)} \\ &\rightarrow (-) \frac{6\pi}{M_R} \frac{k^2 C_{\eta,1}^2 e^{2i\sigma_1}}{-\frac{1}{a_1} + \frac{r_1}{2} k^2 - k^2(1+\eta^2) 2k_C H(\eta)} \end{aligned}$$

$$C_{\eta,1}^2 k^3 (\cot \delta_1 - i) = -\frac{1}{a_1} + \frac{r_1}{2} k^2 + \dots - k^2(1+\eta^2) 2k_C H(\eta)$$

# ERE in EFT

$$\begin{aligned} \langle \chi_{\mathbf{p}'}^{(-)} | T_{cs}(E) | \chi_{\mathbf{p}}^{(+)} \rangle &= (-) \frac{2\pi}{M_R} \frac{\chi_{\mathbf{p}'}^{(-)*}(0) \chi_{\mathbf{p}}^{(+)}(0)}{-a_0^{-1} - 2k_C H(\eta)} \\ &\rightarrow (-) \frac{2\pi}{M_R} \frac{C_{\eta,0}^2 e^{2i\sigma_0}}{-a_0^{-1} - 2k_C H(\eta)} \end{aligned}$$

*One parameter:  
g (or a0)*

$$C_{\eta,0}^2 k (\cot \delta_0 - i) = -\frac{1}{a_0} + \dots - 2k_C H(\eta) \quad H(\eta) = \psi(i\eta) + 1/(2i\eta) - \ln(i\eta)$$

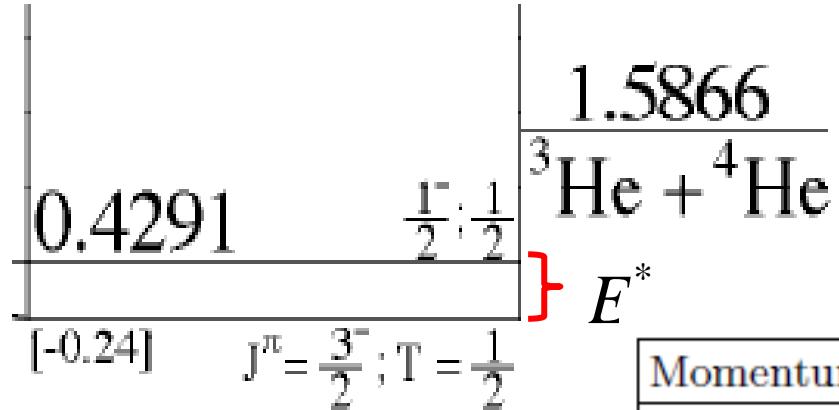

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*Two parameters: Delta and h (or a1 and r1)*

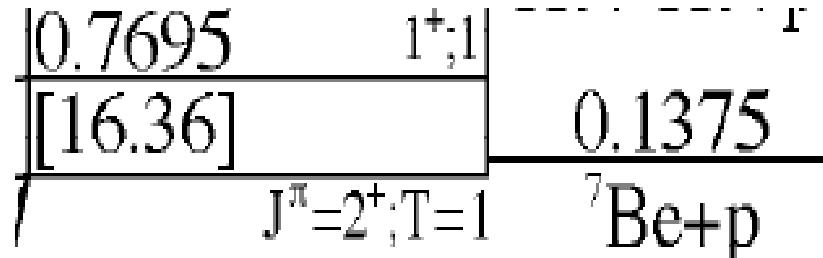
# Scales, spins, core excitations



${}^7\text{Be}$

$$\frac{p_c}{\Lambda}, \frac{\gamma}{\Lambda}, \frac{\tilde{\gamma}}{\Lambda} \sim 0.2$$

$$\eta = \frac{k_c}{k} \sim 1$$



*Shallow bound state*

Momentum scale	Definition	Value
$k_C \sim \gamma$	$Q_c Q_n \alpha_{EM} M_R$	24.02 MeV
$\gamma$	$\sqrt{2M_R B_{8\text{B}}}$	15.04 MeV
$\Lambda$	$\sqrt{2M'_R B_{7\text{Be}}}$	70 MeV
$\gamma^* \sim \gamma$	$\sqrt{2M_R (B_{8\text{B}} + E^*)}$	30.53 MeV
$\gamma_\Delta \sim \gamma$	$\sqrt{2M_R E^*}$	26.57 MeV
$a_{3S_1}, a_{5S_2} \sim 1/\gamma$	scattering lengths	Varies
$r_0 \sim 1/\Lambda$	$l = 0$ effective ranges	Varies
$a_1 \sim \gamma^{-2} \Lambda^{-1}$	scattering volume	$1054.1 \text{ fm}^3$
$r_1 \sim \Lambda$	$l = 1$ effective “range”	$-0.34 \text{ fm}^{-1}$

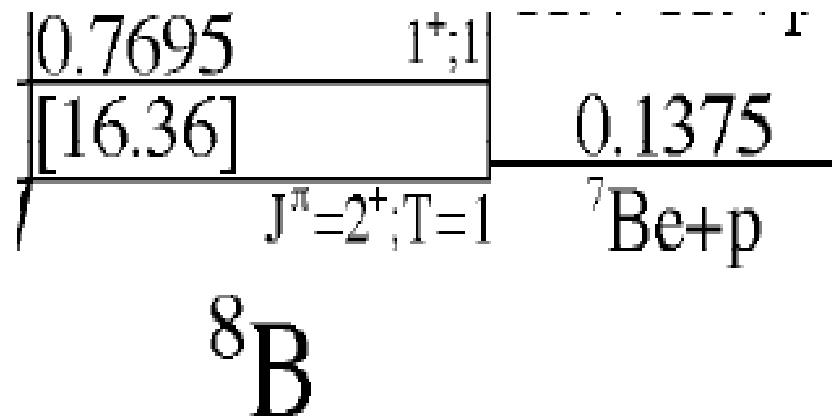
# Repeat

		$\frac{1}{2}^-, \frac{1}{2}^+$
0.4291		
[-0.24]		$J^\pi = \frac{3}{2}^-, T = \frac{1}{2}$
	1.5866	${}^3\text{He} + {}^4\text{He}$

${}^7\text{Be}$

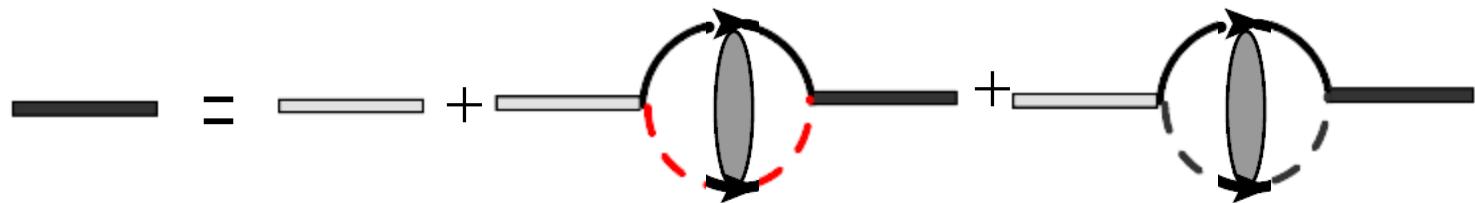
FS( $2^+$ ) Be7 + p:  ${}^3P_2$ ,  ${}^5P_2$

FS( $2^+$ ) Be7\* + p:  ${}^3P_2^*$



IS Be7 + p:  ${}^3S_1$ ,  ${}^5S_2$ , D  
 IS Be7\* + p:  ${}^1S_0$ ,  ${}^3S_1^*$

# P-wave

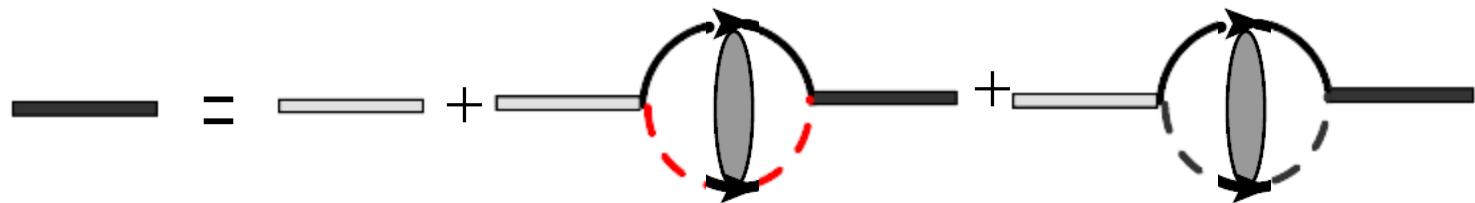


$$\frac{-6\pi M_R}{h_t^2 D} = -\frac{1}{a_1} + \frac{r_1}{2} k^2 - 2k_C(k^2 + k_C^2)H(k_C/k) - 2k_C \frac{h_{(3P_2^*)}^2}{h_t^2} (k_*^2 + k_C^2)H(k_C/k_*)$$

$$\begin{aligned} \frac{6\pi}{Z} + h_t^2 r_1 &= 2 \frac{k_C}{\gamma} \left\{ \frac{h_t^2}{\gamma^2} \left[ 2\gamma^3 \tilde{H} \left( \frac{k_C}{\gamma} \right) + (k_C^3 - k_C \gamma^2) \tilde{H}' \left( \frac{k_C}{\gamma} \right) \right] \right. \\ &\quad \left. + \frac{h_{(3P_2^*)}^2}{\gamma^{*2}} \left[ 2\gamma^{*3} \tilde{H} \left( \frac{k_C}{\gamma^*} \right) + (k_C^3 - k_C \gamma^{*2}) \tilde{H}' \left( \frac{k_C}{\gamma^*} \right) \right] \right\} \end{aligned}$$

$$\frac{C_Y^2}{h_Y^2 \gamma^2 \Gamma^2 (2 + k_C/\gamma)} = \frac{C_{(3P_2^*)}^2}{h_{(3P_2^*)}^2 \gamma^{*2} \Gamma^2 (2 + k_C/\gamma^*)} = \frac{Z}{3\pi} \quad Y = {}^3P_2 \text{ and } {}^5P_2$$

# P-wave



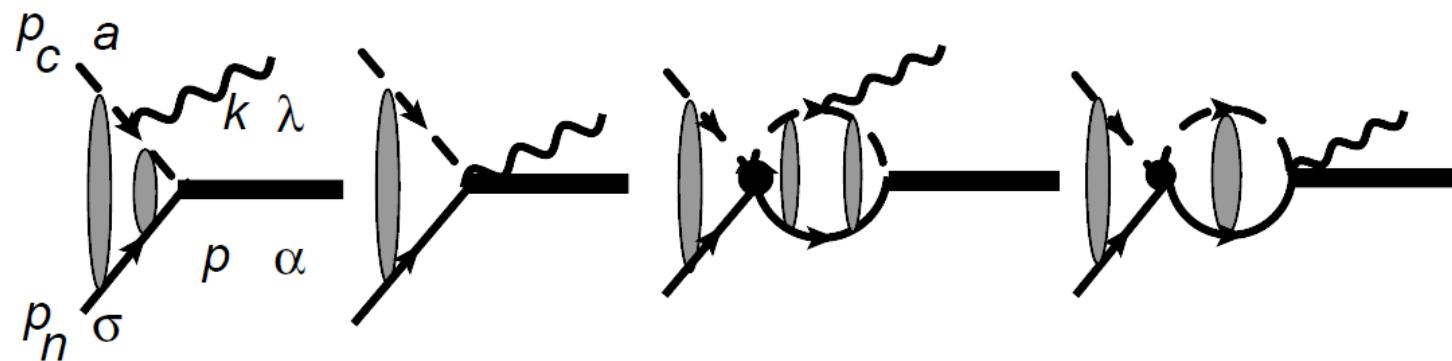
$$\frac{-6\pi M_R}{h_t^2 D} = -\frac{1}{a_1} + \frac{r_1}{2} k^2 - 2k_C(k^2 + k_C^2)H(k_C/k) - 2k_C \frac{h_{(3P_2^*)}^2}{h_t^2}(k_*^2 + k_C^2)H(k_C/k_*)$$

$$\begin{aligned} \frac{6\pi}{Z} + h_t^2 r_1 &= 2 \frac{k_C}{\gamma} \left\{ \frac{h_t^2}{\gamma^2} \left[ 2\gamma^3 \tilde{H} \left( \frac{k_C}{\gamma} \right) + (k_C^3 - k_C \gamma^2) \tilde{H}' \left( \frac{k_C}{\gamma} \right) \right] \right. \\ &\quad \left. + \frac{h_{(3P_2^*)}^2}{\gamma^{*2}} \left[ 2\gamma^{*3} \tilde{H} \left( \frac{k_C}{\gamma^*} \right) + (k_C^3 - k_C \gamma^{*2}) \tilde{H}' \left( \frac{k_C}{\gamma^*} \right) \right] \right\} \end{aligned}$$

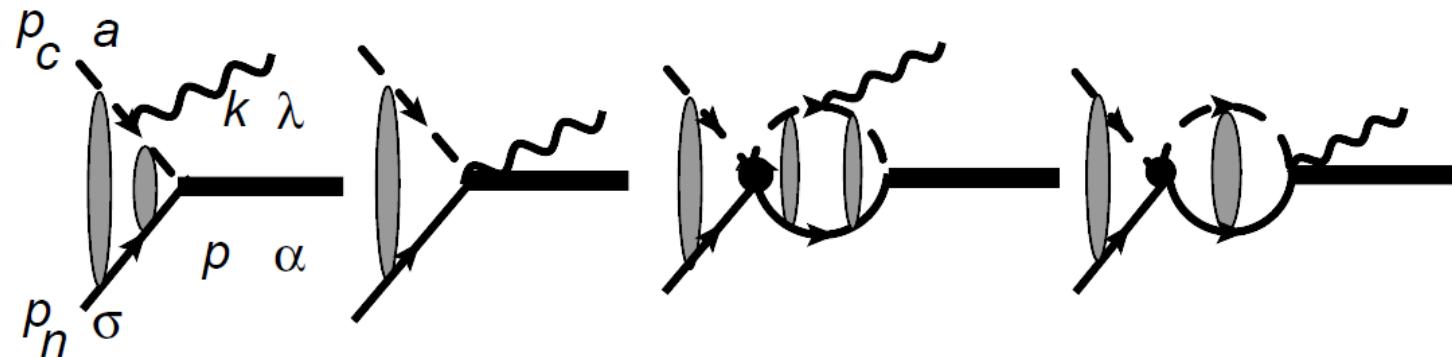
$$\frac{C_Y^2}{h_Y^2 \gamma^2 \Gamma^2(2 + k_C/\gamma)} = \frac{C_{(3P_2^*)}^2}{h_{(3P_2^*)}^2 \gamma^{*2} \Gamma^2(2 + k_C/\gamma^*)} = \frac{Z}{3\pi} \quad Y = {}^3P_2 \text{ and } {}^5P_2$$

*4 parameters: 3 h + 1 Delta, or 3 C + gamma*

# Radiative captures: LO



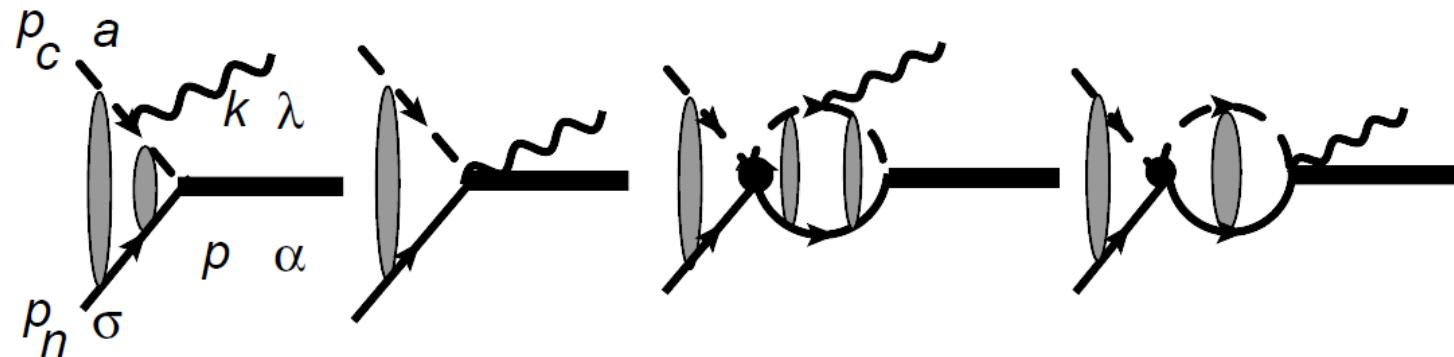
# Radiative captures: LO



$$\langle \pi^\alpha | L_{EM} | \chi_{\mathbf{p}}^{(+)}, \delta, a \rangle \equiv T_i^{\delta a} T_\alpha^{ij} \mathcal{M}_j \quad \text{Initial total spin } Si=1$$

$$\mathcal{M}_j = (-i) C_{\eta,0} C_{(3P_2)}^{\text{LO}} \frac{Z_{eff}}{M_R} \frac{2\pi}{\sqrt{3}} (\gamma^2 + k^2) \left[ e^{i\sigma_0} \epsilon_j^* Y_{00}(\hat{p}) S(^3S_1) + e^{i\sigma_2} \epsilon_k^* \sqrt{2} T_j^{ka} Y_{2a}(\hat{p}) D \right]$$

# Radiative captures: LO



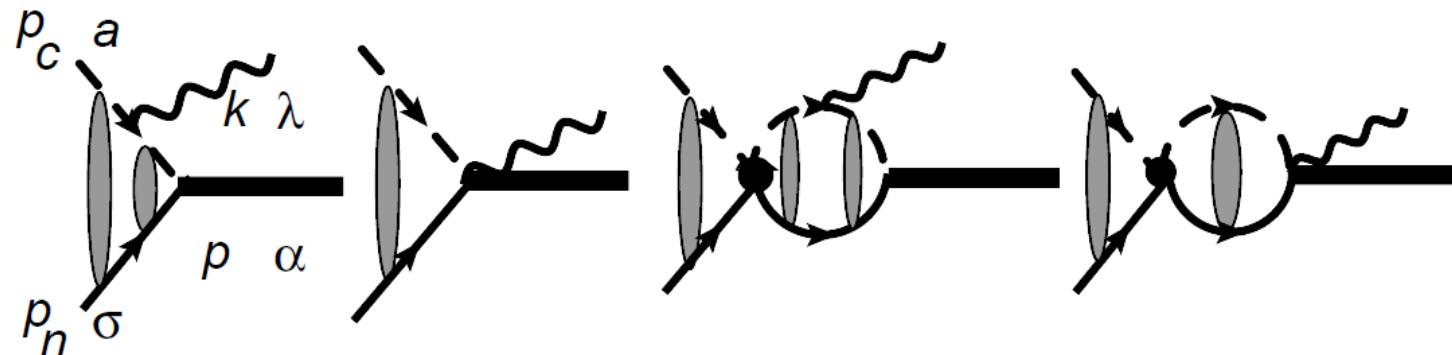
$$\langle \pi^\alpha | L_{EM} | \chi_{\mathbf{p}}^{(+)}, \delta, a \rangle \equiv T_i^{\delta a} T_\alpha^{ij} \mathcal{M}_j \quad \text{Initial total spin } Si=1$$

$$\mathcal{M}_j = (-i) C_{\eta,0} C_{(3P_2)}^{\text{LO}} \frac{Z_{eff}}{M_R} \frac{2\pi}{\sqrt{3}} (\gamma^2 + k^2) \left[ e^{i\sigma_0} \epsilon_j^* Y_{00}(\hat{p}) S(^3S_1) + e^{i\sigma_2} \epsilon_k^* \sqrt{2} T_j^{ka} Y_{2a}(\hat{p}) D \right]$$

$$S(X) \equiv \int_0^{+\infty} dr W_{-\eta_B, \frac{3}{2}}(2\gamma r) r \left[ \frac{C_{\eta,0} G_0(k,r)}{-a_{(X)}^{-1} - 2k_C H(\eta)} + \frac{F_0(k,r)}{C_{\eta,0} k} \frac{-a_{(X)}^{-1} - 2k_C \text{Re}[H(\eta)]}{-a_{(X)}^{-1} - 2k_C H(\eta)} \right]$$

$$\mathcal{D} \equiv \int_0^{+\infty} dr W_{-\eta_B, \frac{3}{2}}(2\gamma r) r \frac{F_2(k,r)}{C_{\eta,0} k}$$

# Radiative captures: LO



$$\langle \pi^\alpha | L_{EM} | \chi_{\mathbf{p}}^{(+)}, \delta, a \rangle \equiv T_i^{\delta a} T_\alpha^{ij} \mathcal{M}_j \quad \text{Initial total spin } Si=1$$

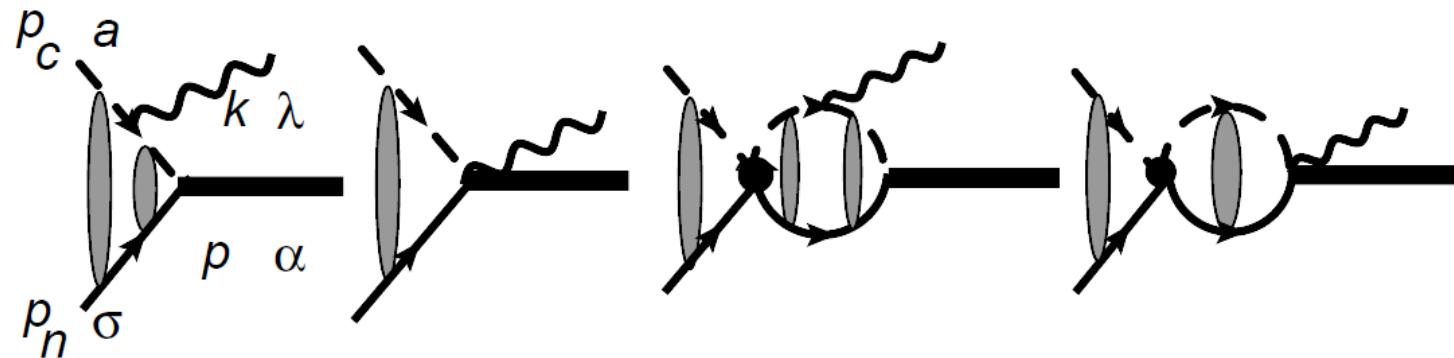
$$\mathcal{M}_j = (-i) C_{\eta,0} C_{(3P_2)}^{\text{LO}} \frac{Z_{eff}}{M_R} \frac{2\pi}{\sqrt{3}} (\gamma^2 + k^2) \left[ e^{i\sigma_0} \epsilon_j^* Y_{00}(\hat{p}) S(^3S_1) + e^{i\sigma_2} \epsilon_k^* \sqrt{2} T_j^{ka} Y_{2a}(\hat{p}) D \right]$$

$$\mathcal{S}(X) \equiv \int_0^{+\infty} dr W_{-\eta_B, \frac{3}{2}}(2\gamma r) r \left[ \frac{C_{\eta,0} G_0(k,r)}{-a_{(X)}^{-1} - 2k_C H(\eta)} + \frac{F_0(k,r)}{C_{\eta,0} k} \frac{-a_{(X)}^{-1} - 2k_C \text{Re}[H(\eta)]}{-a_{(X)}^{-1} - 2k_C H(\eta)} \right]$$

$$\mathcal{D} \equiv \int_0^{+\infty} dr W_{-\eta_B, \frac{3}{2}}(2\gamma r) r \frac{F_2(k,r)}{C_{\eta,0} k}$$

$$\begin{aligned} F &\rightarrow j \\ G &\rightarrow n \\ W &\rightarrow h \end{aligned}$$

# Radiative captures: LO



$$S(E) = \frac{e^{2\pi\eta}}{e^{2\pi\eta} - 1} \frac{Z_{eff}^2}{M_R^2} \frac{\pi}{24} \omega k_C (\gamma^2 + k^2)^2 \frac{5}{3} \times \\ \left[ C_{(3P_2)}^{\text{LO}} {}^2 (| \mathcal{S}(^3S_1) |^2 + 2 |\mathcal{D}|^2) + C_{(5P_2)}^{\text{LO}} {}^2 (| \mathcal{S}(^5S_2) |^2 + 2 |\mathcal{D}|^2) \right]$$

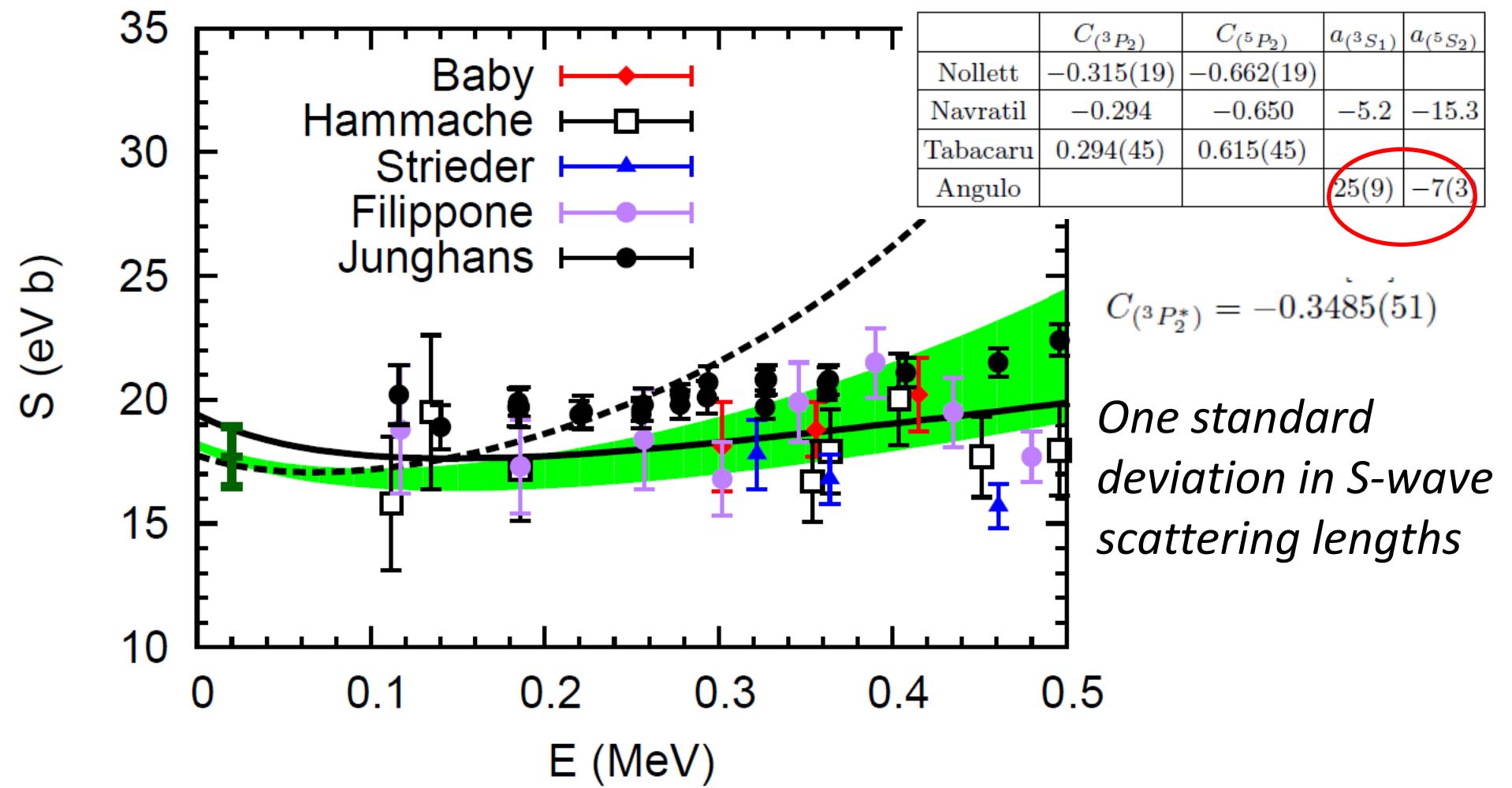
# LO results on Be7(p,gamma)B8

	$C_{(3P_2)}$	$C_{(5P_2)}$	$a_{(3S_1)}$	$a_{(5S_2)}$
Nollett	-0.315(19)	-0.662(19)		
Navratil	-0.294	-0.650	-5.2	-15.3
Tabacaru	0.294(45)	0.615(45)		
Angulo			25(9)	-7(3)

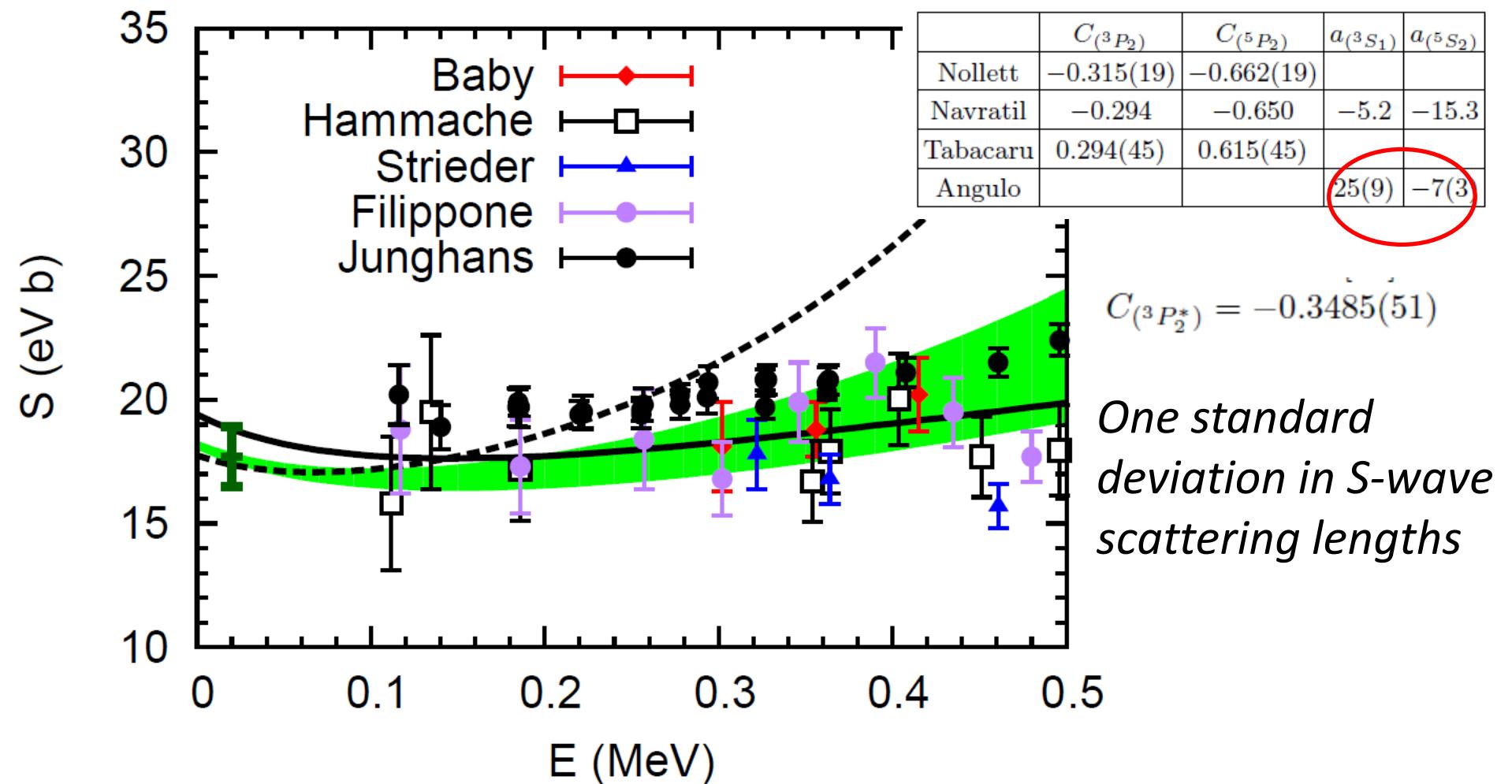
$$C_{(3P_2^*)} = -0.3485(51)$$

- P. Navratil, R. Roth and S. Quaglioni, *Phys. Lett. B* 704, 379 (2011);  
C. Angulo *et. al.*, *Nucl. Phys. A* 716, 211 (2003);  
G. Tabacaru, *et. al.*, *Phys. Rev. C* 73, 025808 (2006)

# LO results on Be7(p,gamma)B8



# LO results on Be7(p,gamma)B8



**Need better measurement of S-wave scattering lengths  
and/or effective ranges to extrapolate data to zero energy**

# LO results on Be7(p,gamma)B8

$$S(E) = S(0)(1 + d_1 E + d_2 E^2) \quad \text{Fit to } 0 < E < 50 \text{ keV}$$

	$S(0)$ (eV b)	$S_{(^3S_1)}(0)$	$d_1(\text{MeV}^{-1})$	$d_2 (\text{MeV}^{-2})$
No+A	18.2(12)	3.1(4)	-1.62	10.3
Na	17.8	3.0	-1.26	10.8
T+A	15.7(27)	2.7(8)	-1.62	10.3
	20.8(16)		-1.5(1)	6.5(2.0)



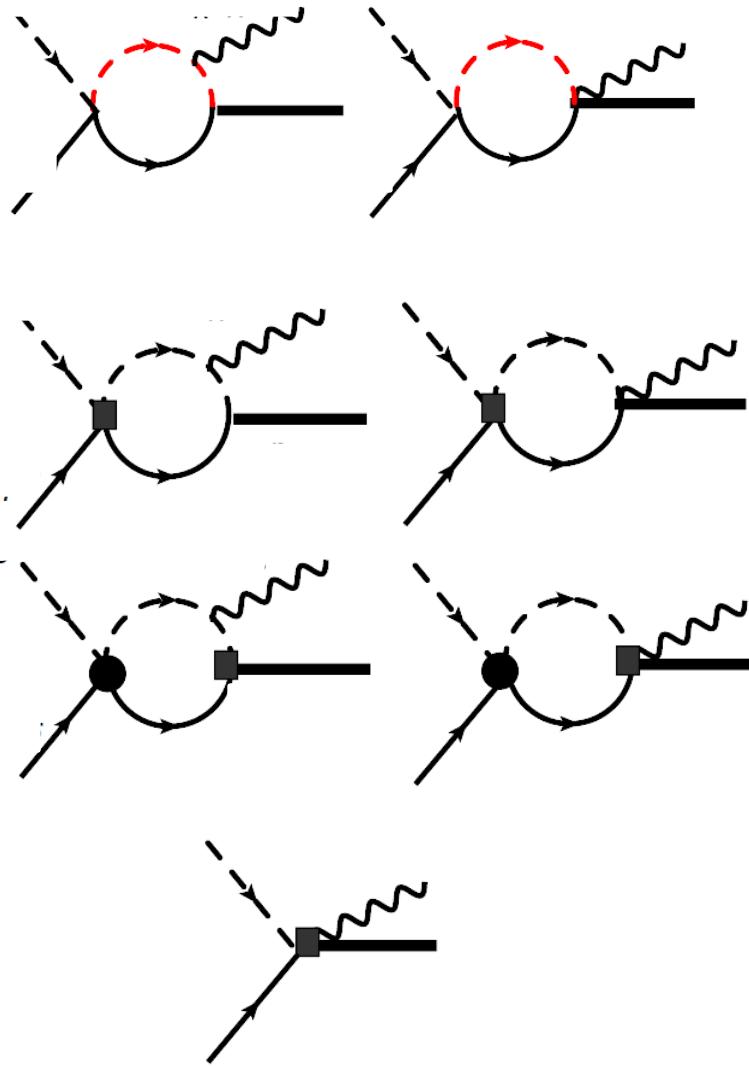
E. G. Adelberger, et al., *Rev. Mod. Phys.* 83, 195 (2011)

- L. T. Baby, et. al., [ISOLDE Collaboration], *Phys. Rev. Lett.* 90, 022501 (2003);  
F. Hammache, et. al., *Phys. Rev. Lett.* 86, 3985 (2001);  
F. Strieder, et. al., *Nucl. Phys. A* 696, 219 (2001);  
B. W. Filippone, et. al., *Phys. Rev. C* 28, 2222 (1983);  
A. R. Junghans, et. al., *Phys. Rev. C* 68, 065803 (2003);  
A. R. Junghans, et. al., *Phys. Rev. C* 81, 012801 (2010).

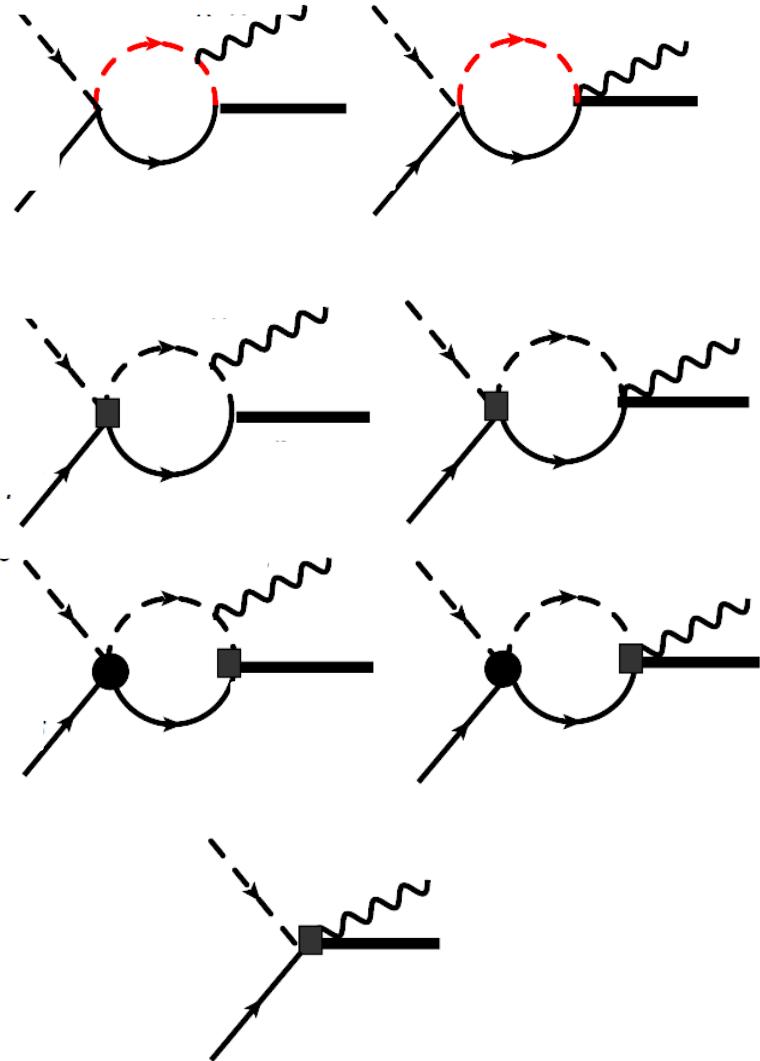
# Summary

- EFT (power counting)+ab initio works as expected at LO
- LO need s-wave scattering length, p-wave ANC<sub>s</sub>, and binding momentum
- The p-wave is a coupled-channel problem
- For Be7 capture, improving s-wave measurement is important for extrapolating data to stellar energies.

# Outlook: NLO

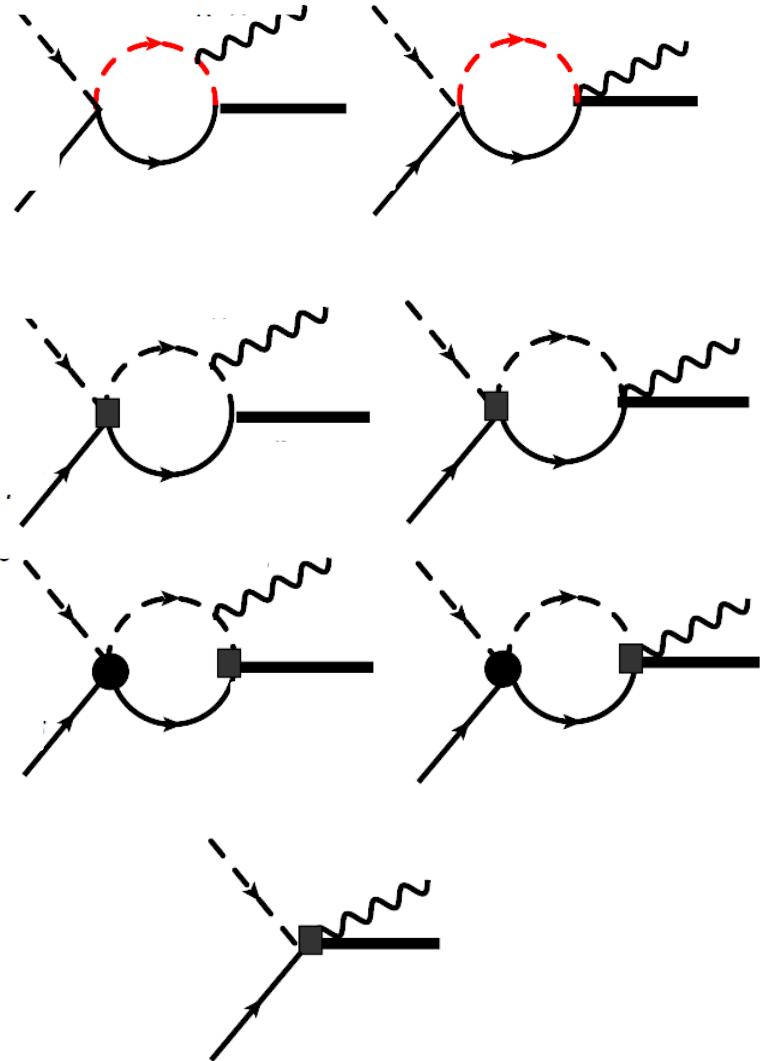


# Outlook: NLO



- Need to fix higher order couplings, i.e., need more “observables”.
- Extract from ab initio calculations?
- Change the boundary conditions?
- Change the background fields?

# Outlook: NLO

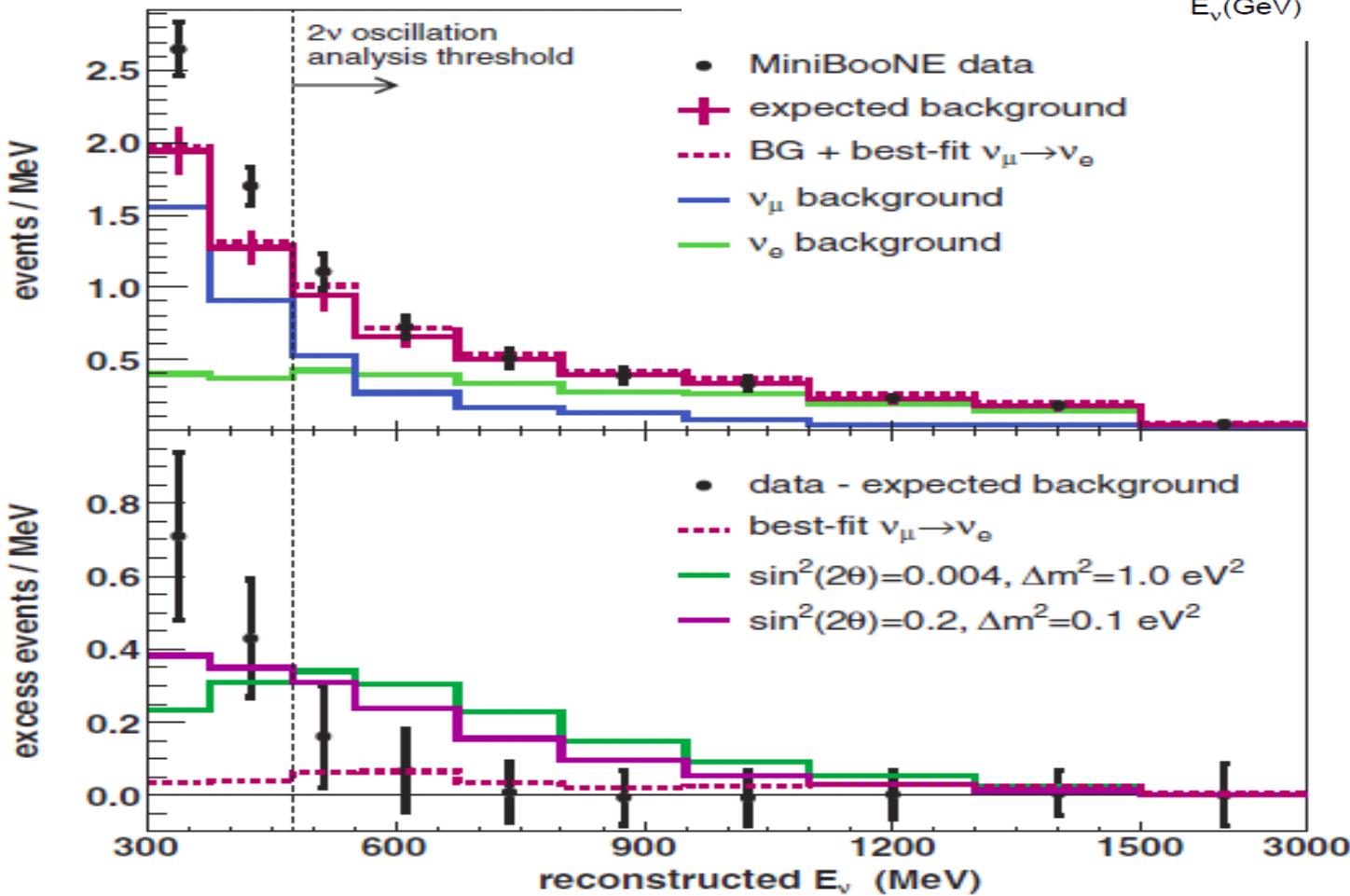
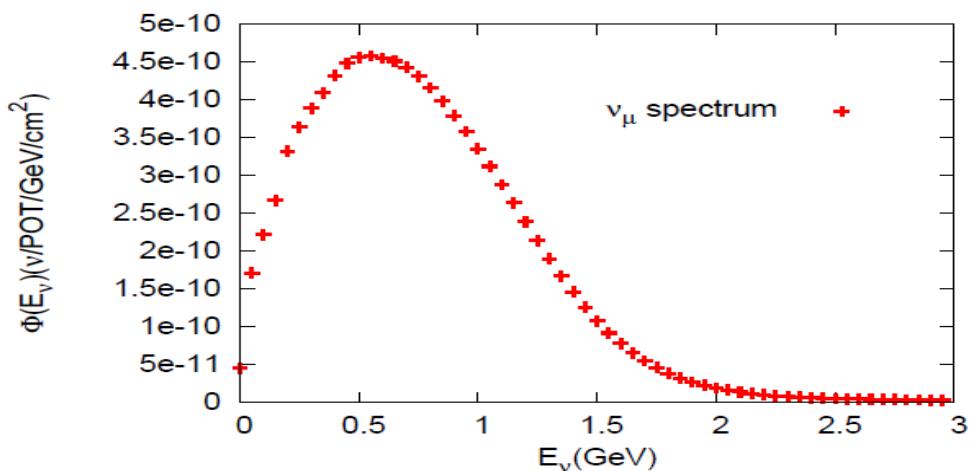


- Need to fix higher order couplings, i.e., need more “observables”.
- Extract from ab initio calculations?
- Change the boundary conditions?
- Change the background fields?
- Another approach by using data?

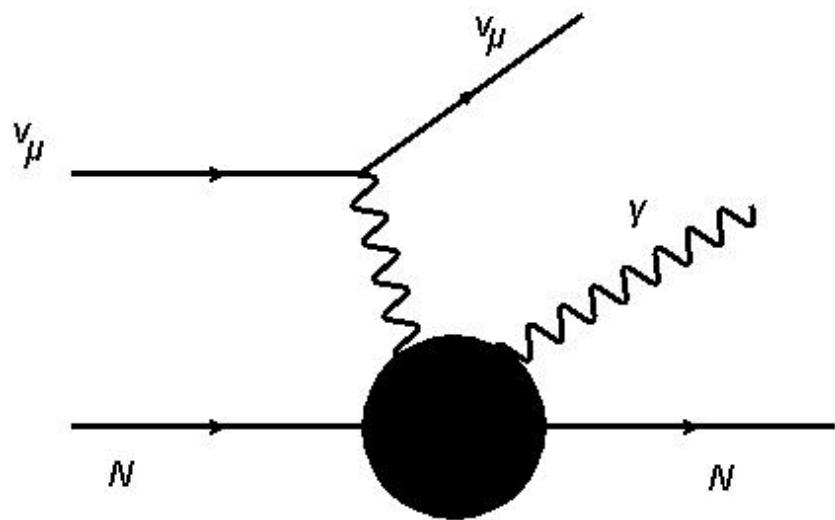
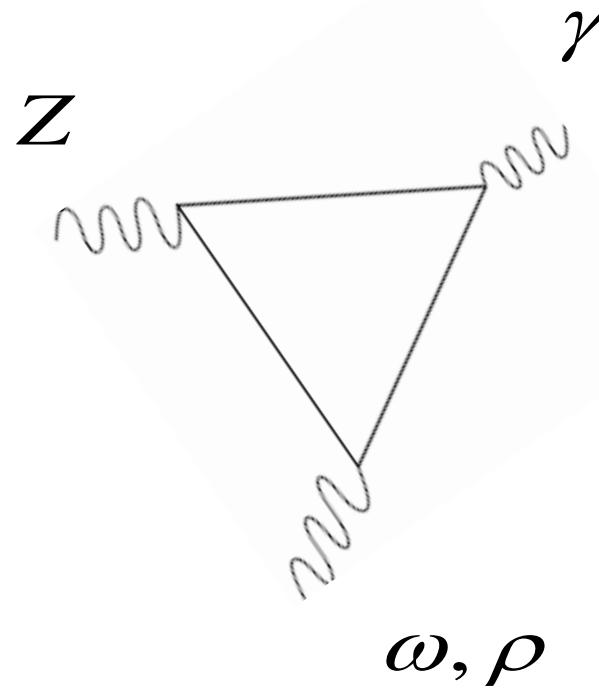
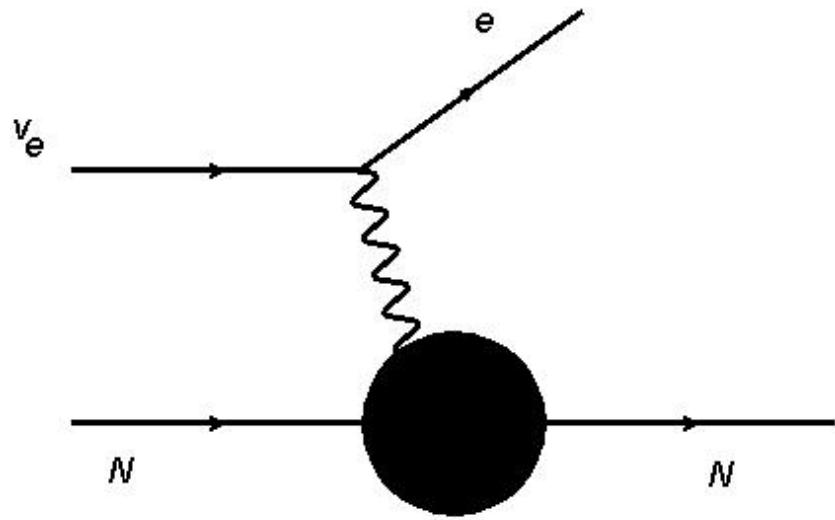
# Other works

- Neutrino-nucleus interactions (GeV): neutral-current induced photon (motivated by MiniBooNE low energy excess), pion productions, nuclear effects [with Brian Serot]
- Jet quenching in heavy ion collisions: initial state fluctuation, different phenomenological jet energy loss models, possible near-Tc enhancement [with Jinfeng Liao]
- Two-loop contributions: nuclear matter, neutron matter, finite temperature [With Madappa Prakash]  
<https://sites.google.com/site/xilinzhangphysics/>

# MiniBooNE



# New hadronic interactions?



J.A. Harvey, C.T. Hill, R.J. Hill, Phys. Rev. Lett. **99**, 261601 (2007), Phys. Rev. D **77**, 085017(2008).  
R.J. Hill, Phys. Rev. D **81**, 013008 (2010), **84** 017501(2011).

# Production of single photons in the exclusive neutrino process $\nu N \rightarrow \nu \gamma N$

S. S. Gershtein, Yu. Ya. Komachenko, and M. Yu. Khlopov

Institute of High Energy Physics, Serpukhov

(Submitted 16 January 1981)

Yad. Fiz. 33, 1597–1604 (June 1981)

It is shown that the experimentally observed production of single photons in neutrino interactions involving neutral currents without visible accompaniment of other particles can be explained by the scattering of the neutrino by a virtual  $\omega$  meson with small momentum transfer to a nucleon and subsequent coherent enhancement of the process in the nucleus.

PACS numbers: 13.15. + g, 14.80.Kx

## 1. INTRODUCTION

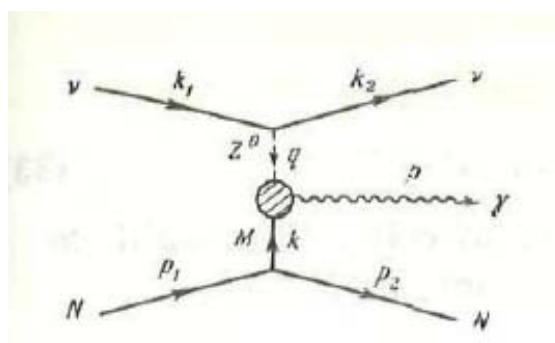
In neutrino experiments performed at CERN using the chamber Gargamelle, more than ten events were detected in which it was observed that single photons with energy 1–10 GeV were produced without visible tracks of any other particles.<sup>1</sup> It can be assumed that the observed events correspond to the weak-electromagnetic process of single-photon production in the reaction

$$\nu N \rightarrow \nu \gamma N, \quad (1)$$

$$H_{\mu\nu} = \sum_M T^{(M)} P^{(M)} J_{\mu\nu}^{(M)}, \quad (3)$$

in which  $T^{(M)}$  is the vertex for emission of a virtual meson ( $M$ ) by the target nucleon,  $P^{(M)}$  is the meson propagator, and  $J_{\mu\nu}^{(M)} = \langle 0 | T(J_\mu^W(x), J_\nu^{EM}(y)) | M \rangle e^{iqx + ipy} d^4x d^4y$  is the weak-electromagnetic  $Z^0 M \gamma$  vertex. The notation for the particle momenta is given in Fig. 2.

In accordance with the estimates of Ref. 3, we shall take into account the contributions to the diagram of Fig.

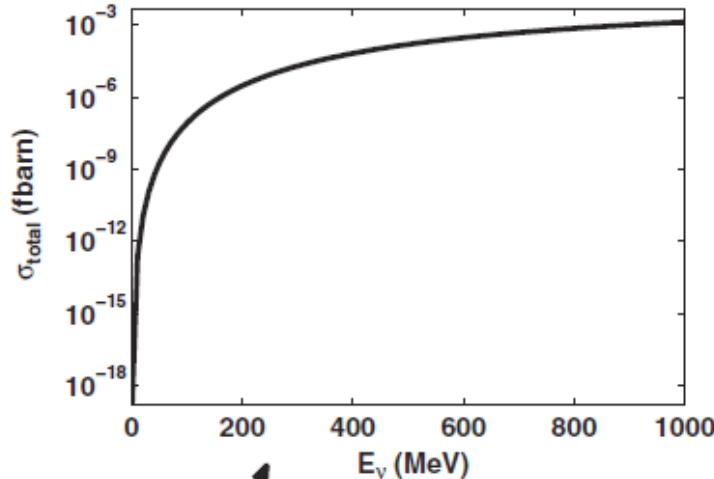
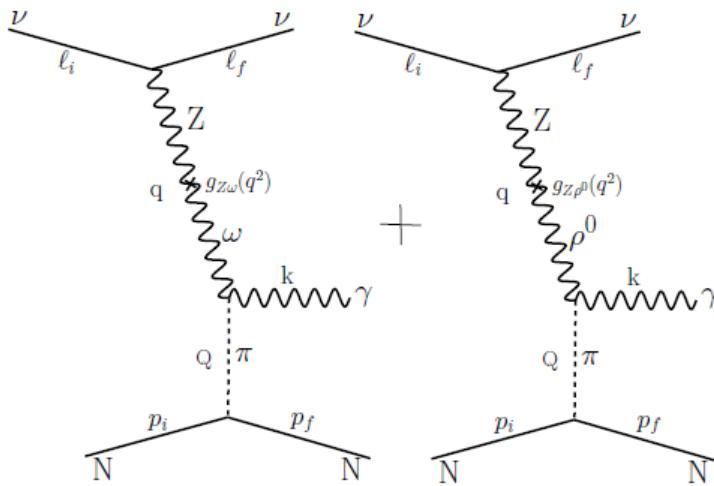


# A phenomenological study of photon production in low energy neutrino nucleon scattering

James Jenkins and T. Goldman

Theoretical Division, Los Alamos National Laboratory, Los Alamos, NM 87545

Low energy photon production is an important background to many current and future precision neutrino experiments. We present a phenomenological study of  $t$ -channel radiative corrections to neutral current neutrino nucleus scattering. After introducing the relevant processes and phenomenological coupling constants, we will explore the derived energy and angular distributions as well as total cross-section predictions along their estimated uncertainties. This is supplemented throughout with comments on possible experimental signatures and implications. We conclude with a general discussion of the analysis in the context of complimentary methodologies.





## A search for single photon events in neutrino interactions

**NOMAD exp.**

C.T. Kullenberg<sup>s</sup>, S.R. Mishra<sup>s,\*</sup>, D. Dimmery<sup>s</sup>, X.C. Tian<sup>s</sup>, D. Autiero<sup>h</sup>, S. Gninenco<sup>h,l</sup>, A. Rubbia<sup>h,x</sup>, S. Alekhin<sup>y</sup>, P. Astier<sup>n</sup>, A. Baldisseri<sup>r</sup>, M. Baldo-Ceolin<sup>m</sup>, M. Banner<sup>n</sup>, G. Bassompierre<sup>a</sup>, K. Benslama<sup>i</sup>, N. Besson<sup>r</sup>, I. Bird<sup>h,i</sup>, B. Blumenfeld<sup>b</sup>, F. Bobisut<sup>m</sup>, J. Bouchez<sup>r</sup>, S. Boyd<sup>t,1</sup>, A. Bueno<sup>c,x</sup>, S. Bunyatov<sup>f</sup>, L. Camilleri<sup>h</sup>, A. Cardini<sup>j</sup>, P.W. Cattaneo<sup>o</sup>, V. Cavasinni<sup>p</sup>, A. Cervera-Villanueva<sup>h,v</sup>, R. Challis<sup>k</sup>,

Letter of Intent: A new investigation of  $\nu_\mu \rightarrow \nu_e$  oscillations with improved sensitivity in an enhanced MiniBooNE experiment

### MiniBooNE Collaboration

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Z. Djurcic  
*Argonne National Laboratory, Argonne, IL 60439*

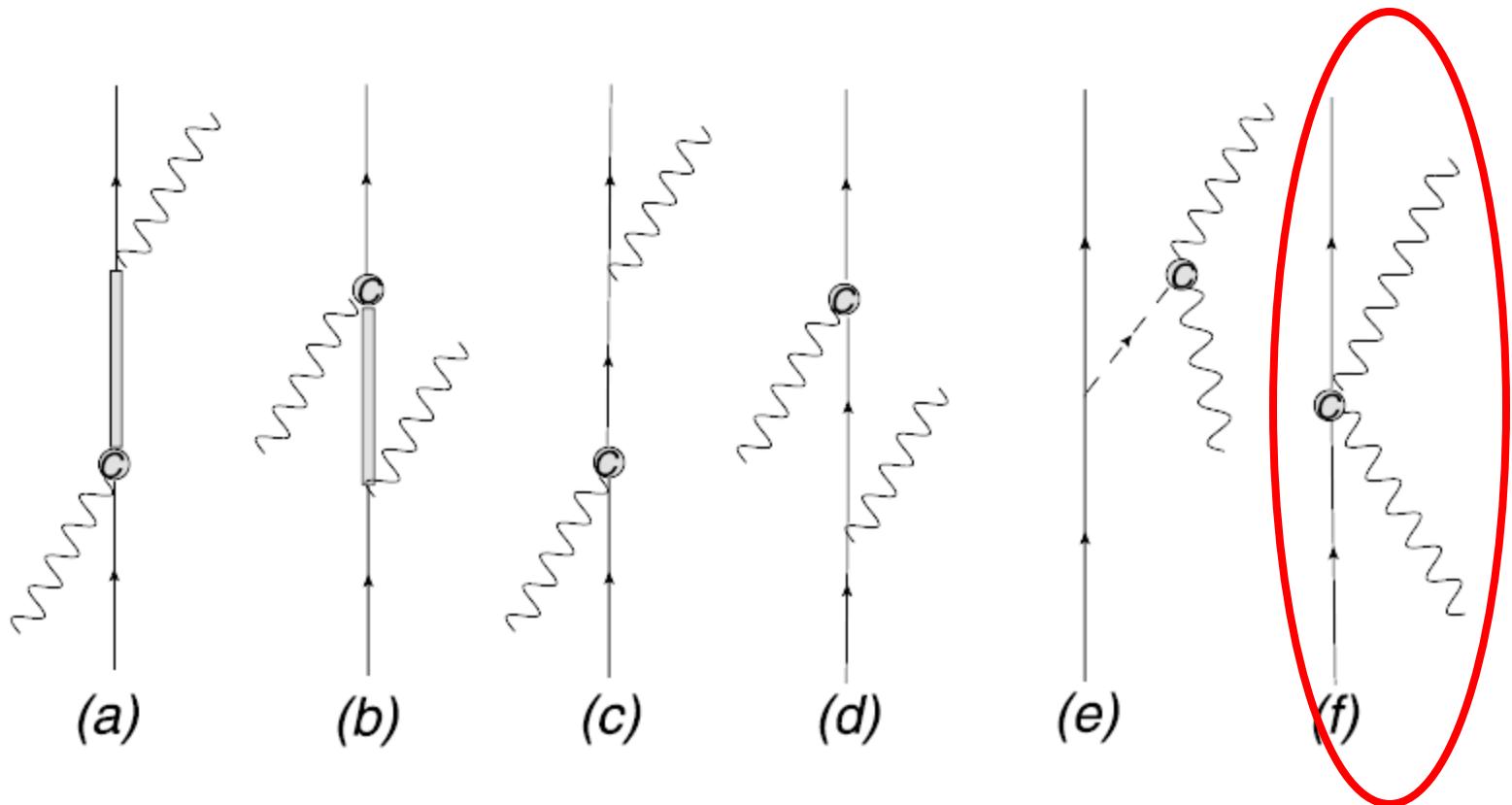
R. A. Johnson & A. Wickremasinghe  
*University of Cincinnati, Cincinnati, OH 45221*

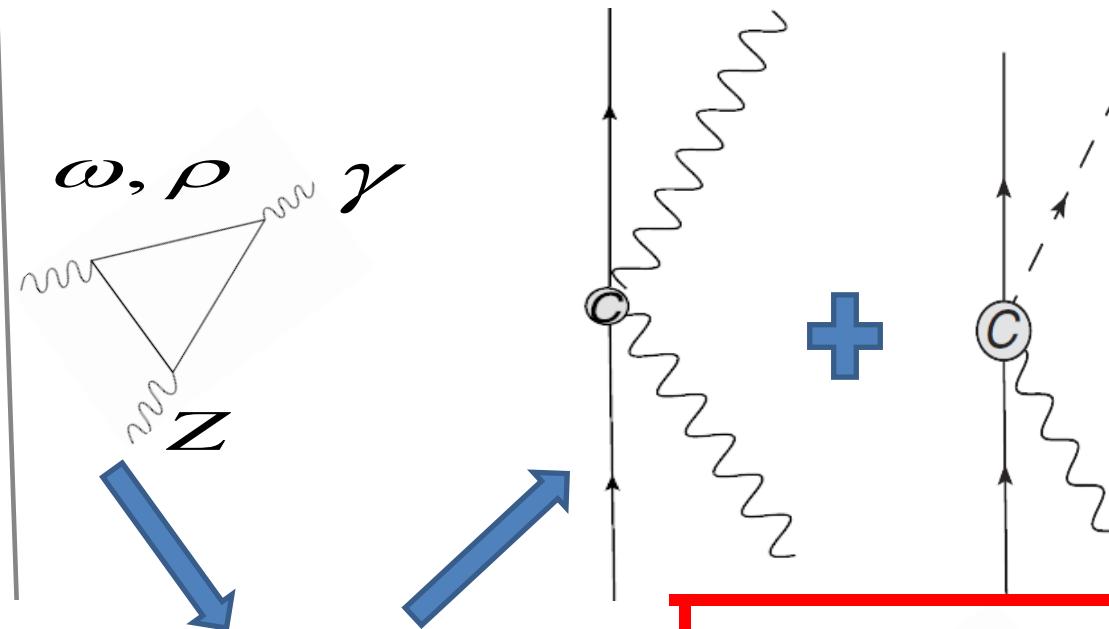
G. Karagiorgi & M. H. Shaevitz  
*Columbia University, New York, NY 10027*

B. C. Brown, F.G. Garcia, R. Ford, W. Marsh, C. D. Moore,  
D. Perevalov, & C. C. Polly  
*Fermi National Accelerator Laboratory, Batavia, IL 60510*

J. Grange, J. Mousseau, B. Osmanov, & H. Ray

# NC photon production off the nucleon





$$\frac{c_1}{M^2} \bar{N} \gamma^\mu N \text{Tr}(\tilde{a}^\nu \bar{F}_{\mu\nu}^{(+)}) , \quad \frac{e_1}{M^2} \bar{N} \gamma^\mu \tilde{a}^\nu N \bar{f}_{s\mu\nu}$$

$$F_{\mu\nu}^{(+)} \approx 2\partial_{[\mu} v_{\nu]} + 2\epsilon^{ijk} \frac{\pi_j}{f_\pi} \frac{\tau_k}{2} \partial_{[\mu} a_{iv]}$$

$$\tilde{a}_\mu \approx \frac{1}{f_\pi} \partial_\mu \pi^i \frac{\tau_i}{2} + a_{i\mu} \frac{\tau^i}{2} + \epsilon^{ijk} \frac{\pi_j}{f_\pi} \frac{\tau_k}{2} v_{i\mu}$$

R. J. Hill, Phys. Rev. D **81**, 013008 (2010)  
 W. Peters I, H. Lenske, U. Mosel,  
 Nucl. Phys. A640, 89 (1998)

$c_1 = 1.5$

$e_1 = 0.8$

$\omega, \rho \gamma \pi$

$\langle J_{NC}^\mu \rangle_\gamma = \delta_B^A \frac{-iec_1}{M^2} \epsilon^{\mu\nu\alpha\beta} \bar{u}_f \gamma_\nu k_\alpha \epsilon_\beta^*(k) u_i$

$+ \delta_B^A \frac{-iec_1 q^\mu}{M^2 (q^2 - m_\pi^2)} \epsilon^{\lambda\nu\alpha\beta} \bar{u}_f \gamma_\lambda q_\nu k_\alpha \epsilon_\beta^*(k) u_i$

N intermediate state  $\sim \frac{1}{M}$

*These terms are small*

# MiniBooNE NC photon events

$E_{QE}(\text{GeV})$	[0.2, 0.3]	[0.3, 0.475]	[0.475, 1.25]
coh	1.5 (2.9)	6.0 (9.2)	2.1 (8.0)
inc	12.0 (14.1)	25.5 (31.1)	12.6 (23.2)
H	4.1 (4.4)	10.6 (11.6)	4.6 (6.3)
Total	17.6 (21.4)	42.1 (51.9)	19.3 (37.5)
MiniBN	19.5	47.3	19.4
Excess	$42.6 \pm 25.3$	$82.2 \pm 23.3$	$21.5 \pm 34.9$

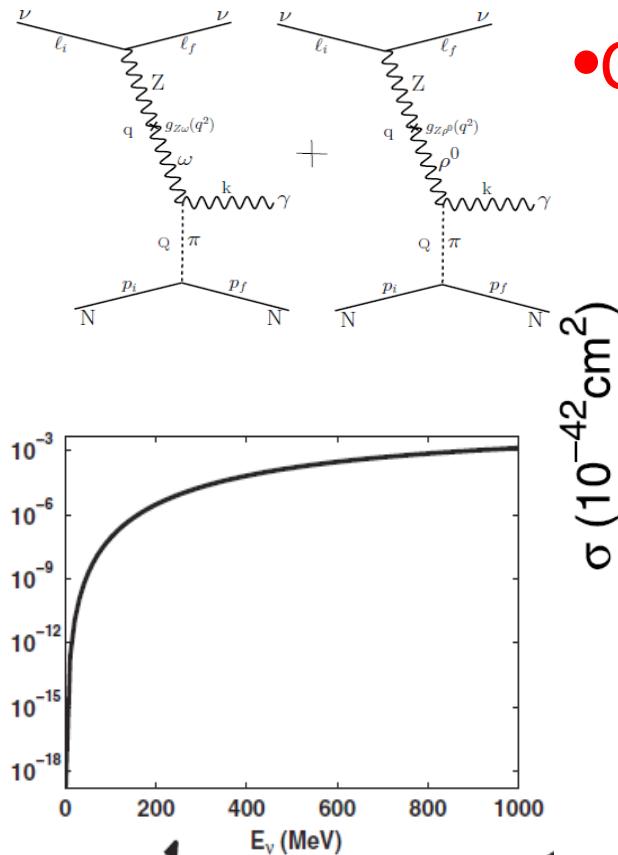
Xection  
needs to be  
doubled at  
least.

$E_{QE}(\text{GeV})$	[0.2, 0.3]	[0.3, 0.475]	[0.475, 1.25]
coh	1.0 (2.2)	3.1 (5.5)	0.87 (5.4)
inc	4.5 (5.3)	10.0 (12.2)	4.0 (10.2)
H	1.3 (1.6)	3.6 (4.3)	1.1 (2.4)
Total	6.8 (9.1)	16.7 (22.0)	6.0 (18.0)
MiniBN	8.8	16.9	6.8
Excess	$34.6 \pm 13.6$	$23.5 \pm 13.4$	$20.2 \pm 22.8$

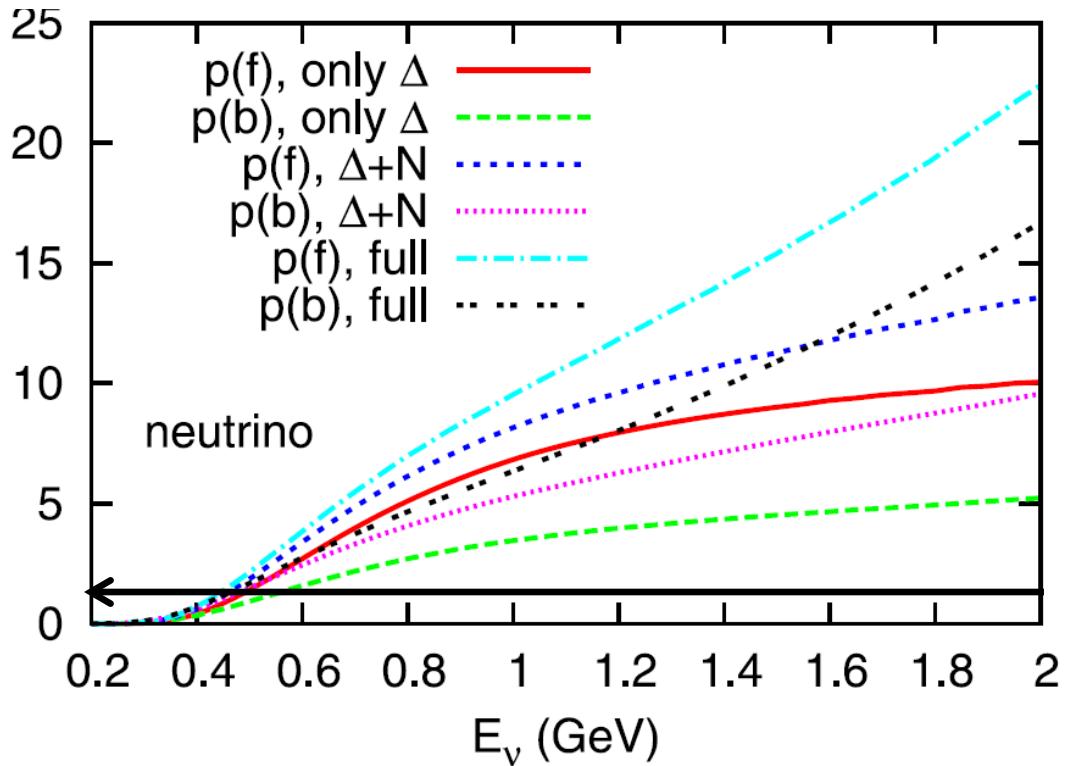
# A phenomenological study of photon production in low energy neutrino nucleon scattering

James Jenkins and T. Goldman

Theoretical Division, Los Alamos National Laboratory, Los Alamos, NM 87545



- Incoherent one is small at  $E_{\nu} \sim 1 \text{ GeV}$
- Coherent one is zero



# backup

- Capture cross section
- 20 keV ~ fb
- 1 MeV ~ mb

