The Equation of State of Neutron-Rich Matter and Neutron Star Observations

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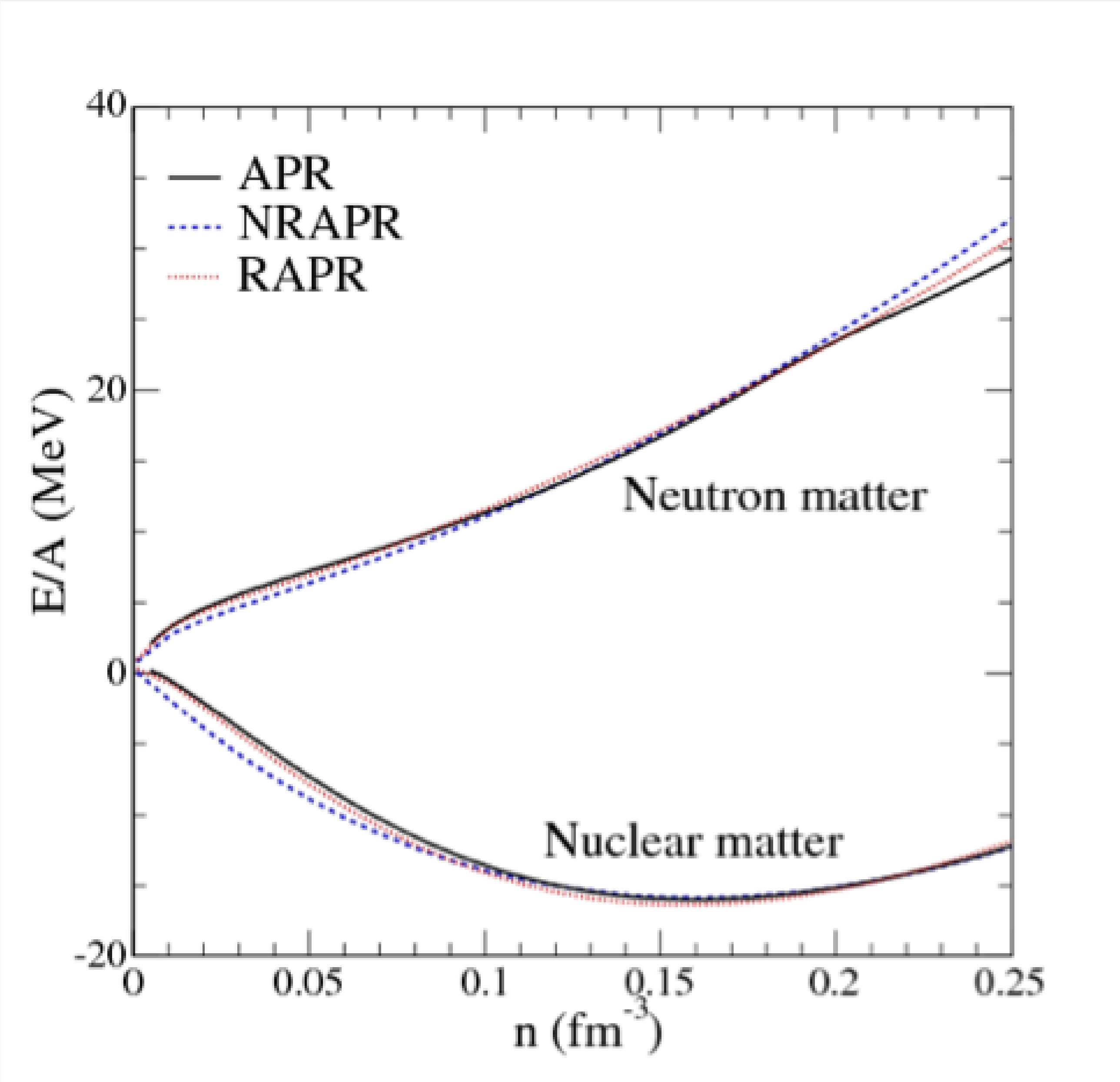
Outline

- Fundamental nuclear physics questions:
  - What is the nuclear symmetry energy?
  - What is the three-neutron force?
  - What is the nature of dense matter? Neutron-rich nuclei?
- Basic neutron star questions:
  - What is the (nearly) universal $M-R$ curve?
  - What is the radius of a 1.4 $M_\odot$ neutron star?
- How we can make these connections

Nucleonic matter



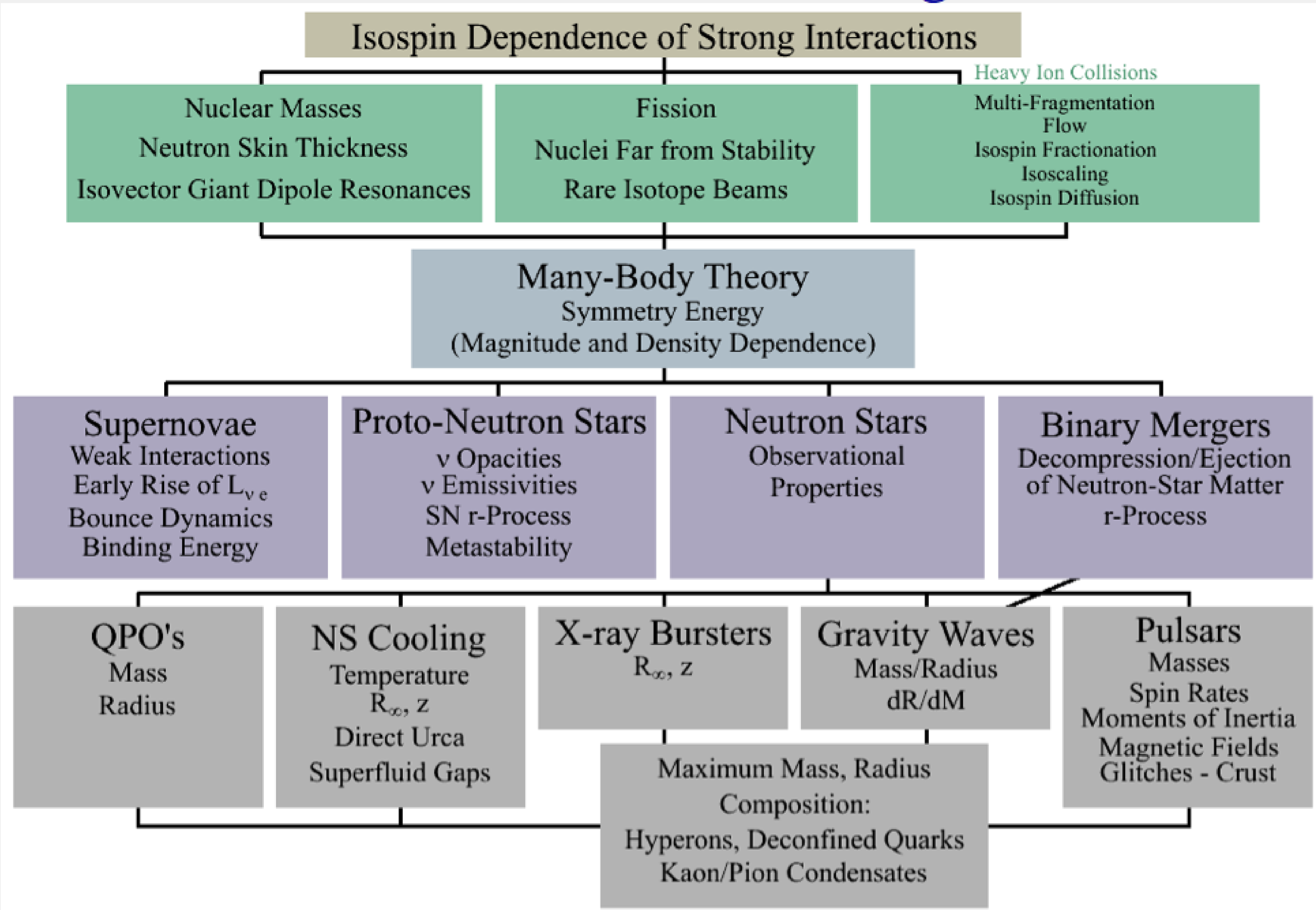
Taken from Steiner et al. (2005)

- baryon density $n_B = n_n + n_p$ (isospin) asymmetry $\alpha \equiv (n_n - n_p)/n_B$
- the nuclear saturation density $n_0 \approx 0.16\ \mathrm{fm}^{-3}$
- $\epsilon \equiv (n_B - n_0)/(3n_0)$
- Energy per baryon of nucleonic matter matter can be written as an expansion around $\epsilon = \alpha = 0$

$$E(n_B, \alpha)/A = -B + \frac{K}{2!}\epsilon^2 + \frac{Q}{3!}\epsilon^3 +$$

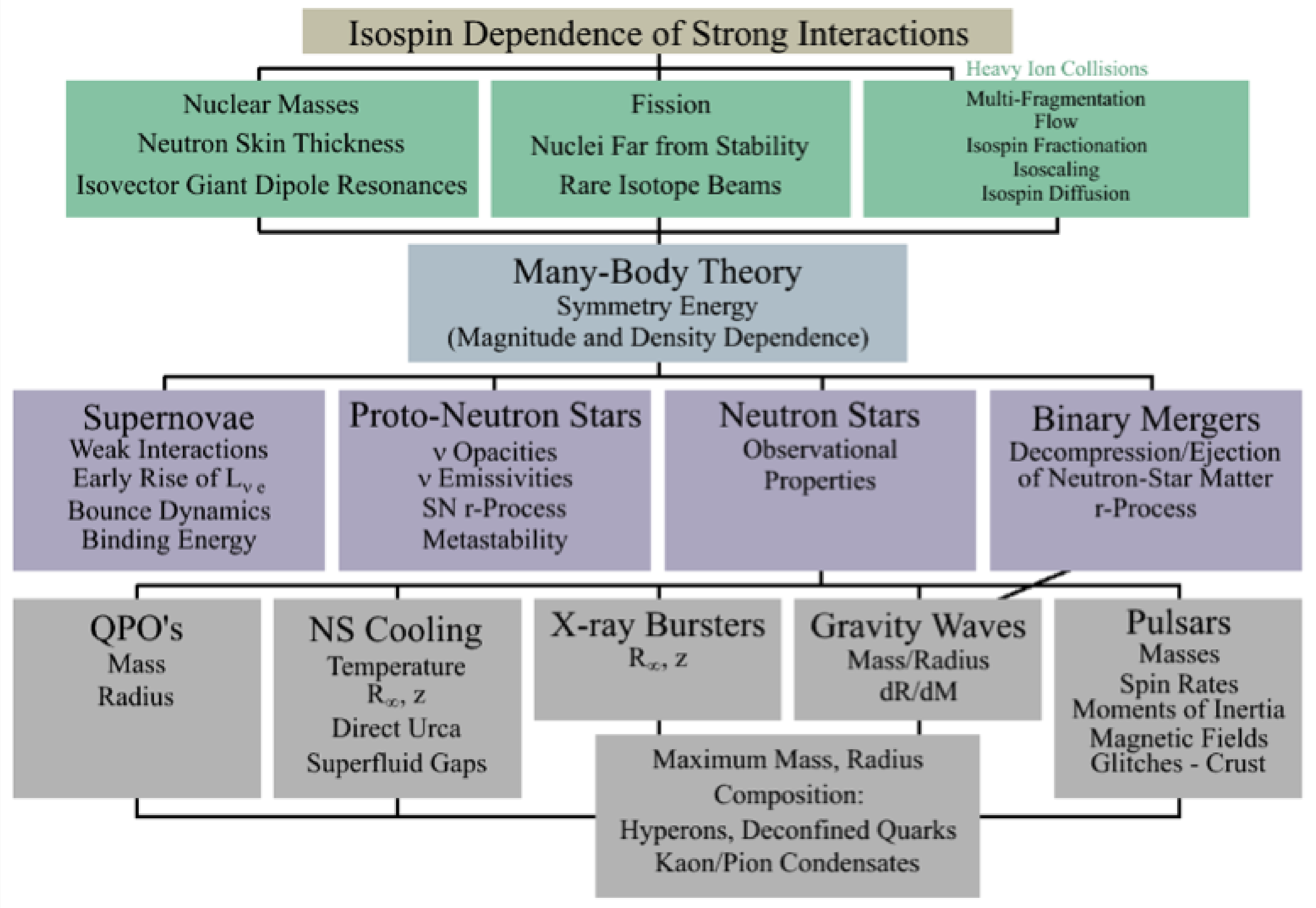
$$\alpha^2\left(S + L\epsilon + \frac{K_{\mathrm{sym}}}{2!}\epsilon^2 + \frac{Q_{\mathrm{sym}}}{3!}\epsilon^3\right) + \alpha^4(\ldots)$$

What is neutron-rich matter good for?



Steiner, Prakash, Lattimer, and Ellis (2005)

What is neutron-rich matter good for?



Steiner, Prakash, Lattimer, and Ellis (2005)

- "Data ⇒ Model ⇒ Data" and also, data-to-data correlations?

Nuclear Masses

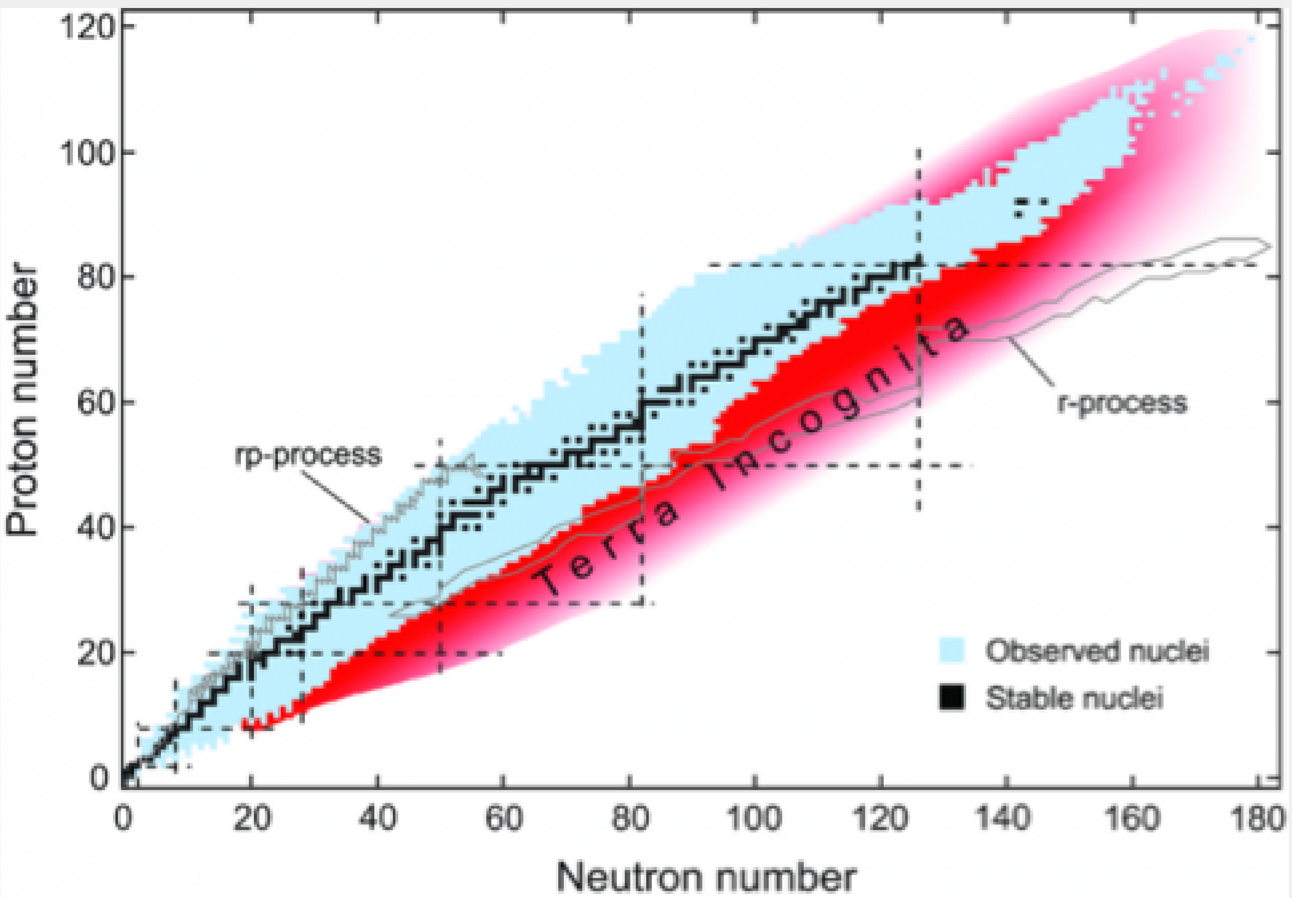


TABLE VIII. The same as Table VII, except for the UNEDF0.

k	Par.	$\hat{\mathbf{x}}$	95% CI	% of Int.	σ
1	ρ_c	0.160526	[0.160,0.161]	10	0.001
2	E^{NM}/A	−16.0559	[−16.146,−15.965]	45	0.055
3	K^{NM}	230	–	–	–
4	a_{sym}^{NM}	30.5429	[25.513,35.573]	126	3.058
5	L_{sym}^{NM}	45.0804	[−20.766,110.927]	219	40.037
6	$1/M_s^*$	0.9	–	–	–
7	$C_0^{\rho\Delta\rho}$	−55.2606	[−58.051,−52.470]	9	1.697
8	$C_1^{\rho\Delta\rho}$	−55.6226	[−149.309,38.064]	94	56.965
9	V_0^n	−170.374	[−173.836,−166.913]	3	2.105
10	V_0^p	−199.202	[−204.713,−193.692]	6	3.351
11	$C_0^{\rho\nabla J}$	−79.5308	[−85.160,−73.901]	16	3.423
12	$C_1^{\rho\nabla J}$	45.6302	[−2.821,94.081]	65	29.460

Taken from Kortelainen et al. (2010)

- Phenomenological Hamiltonian (or energy density functional) + Hartree-Fock-Bogoliubov
- Nuclear masses aren't great probes of S and L:
  - Mostly isospin-symmetric
  - Conflate bulk and surface effects
  - Result in correlation between S and L
- Nuclear masses near neutron drip line critical for r-process nucleosynthesis

Neutron Star Composition

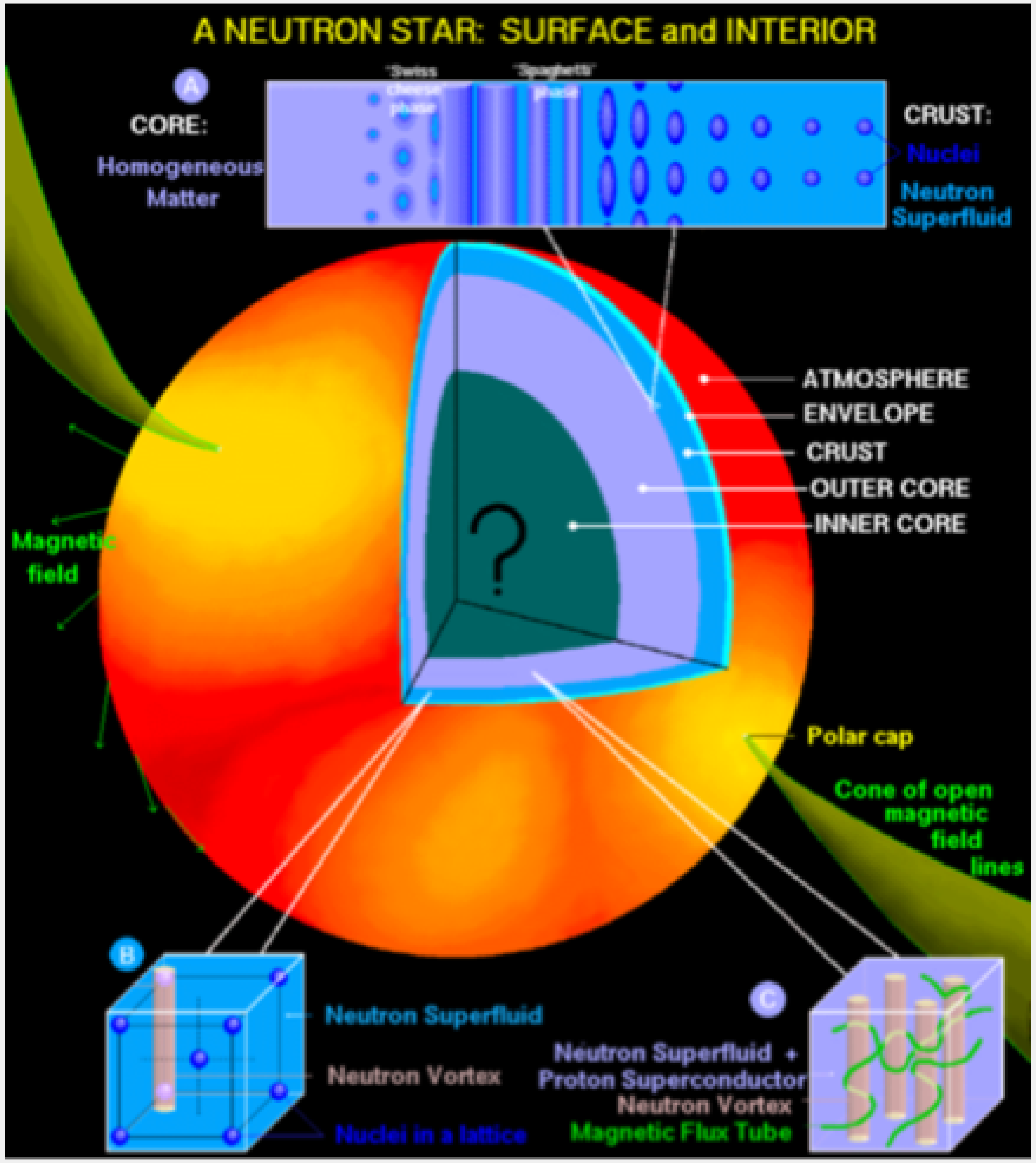


Figure by Dany Page

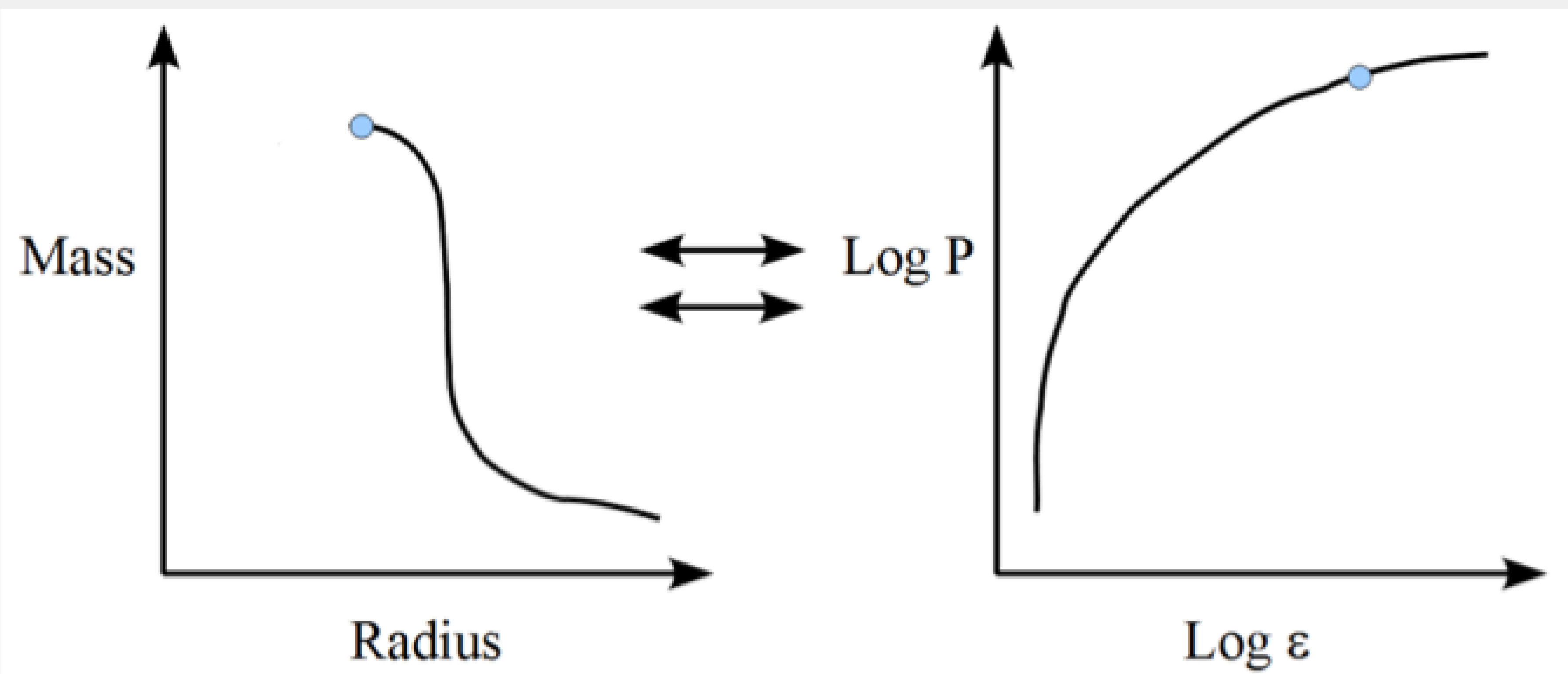
Neutron stars probe a unique region of the QCD phase diagram

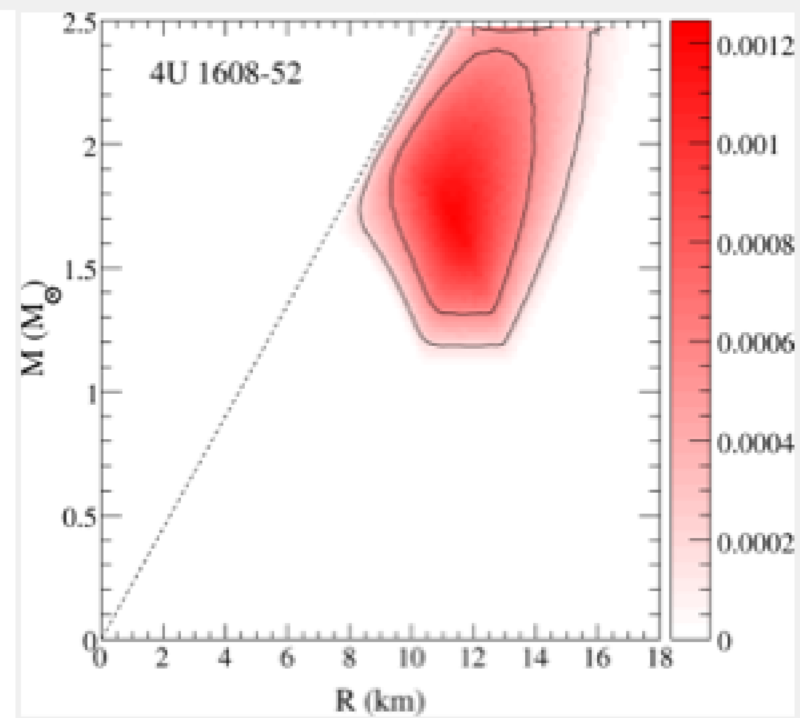
- In outer crust, μ_e increases faster than $\mu_{n,p}$, higher densities more neutron-rich
- In inner crust, S determines EOS of neutron matter as well as properties of nuclei
- As one proceeds into the core $\mu_{n,p}$ increase faster, tend to restore isospin symmetry
- High μ_e can favor phase transitions, i.e. $\mu_{\pi^-} = \mu_e$
- Relationship with hyperons more complicated
- When strange quarks appear, there is a hypercharge asymmetry energy

Neutron Star Masses and Radii and the EOS

- Unlike planets, neutron stars (to better than 10%) all lie on one universal mass-radius curve
- Except for "strange quark stars"
- Rotation is a <10% effect
- A strong enough magnetic field can also deform the star

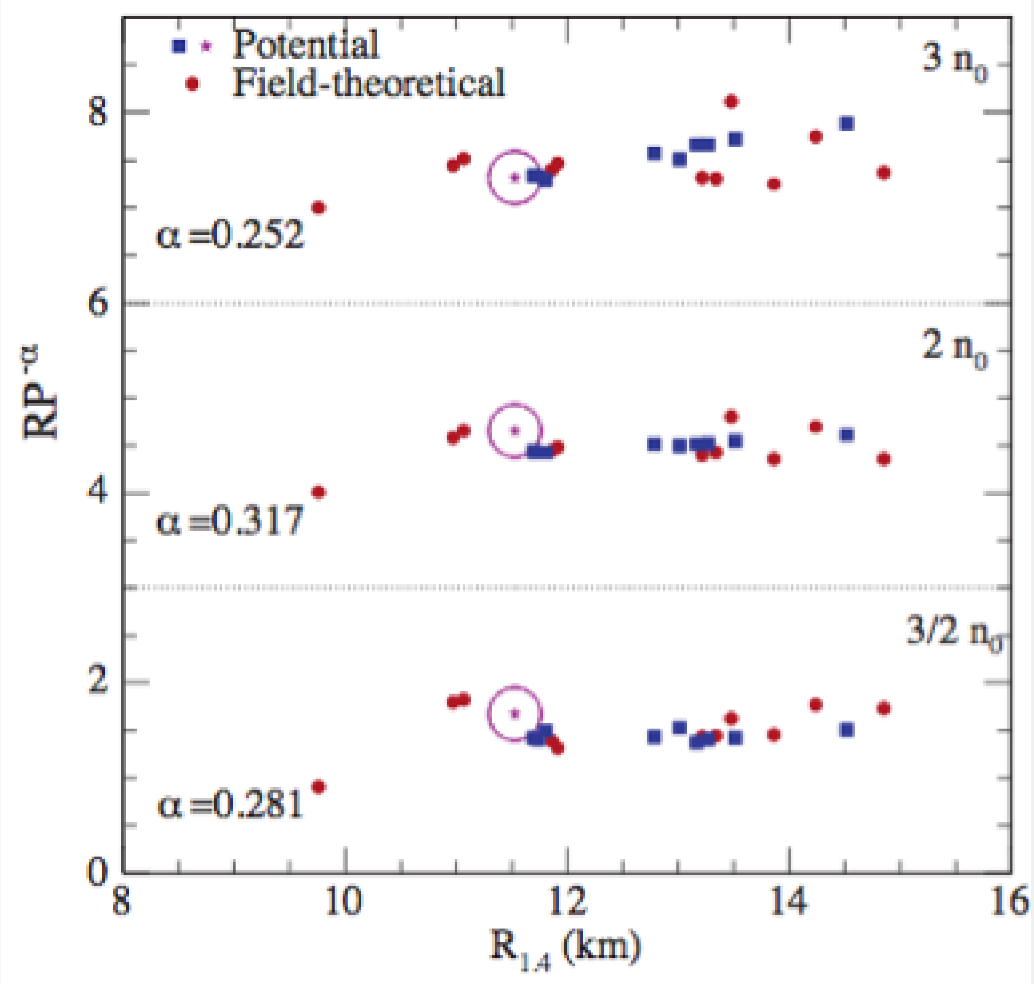
- Until recently, neutron star radii constrained to 8-15 km
  Lattimer and Prakash (2007)
- Recent measurement of two $2\ M_\odot$ neutron stars
  Demorest et al. (2010), Antoniadis et al. (2013)
- Convert X-ray photons into $\mathcal{P}(R, M)$

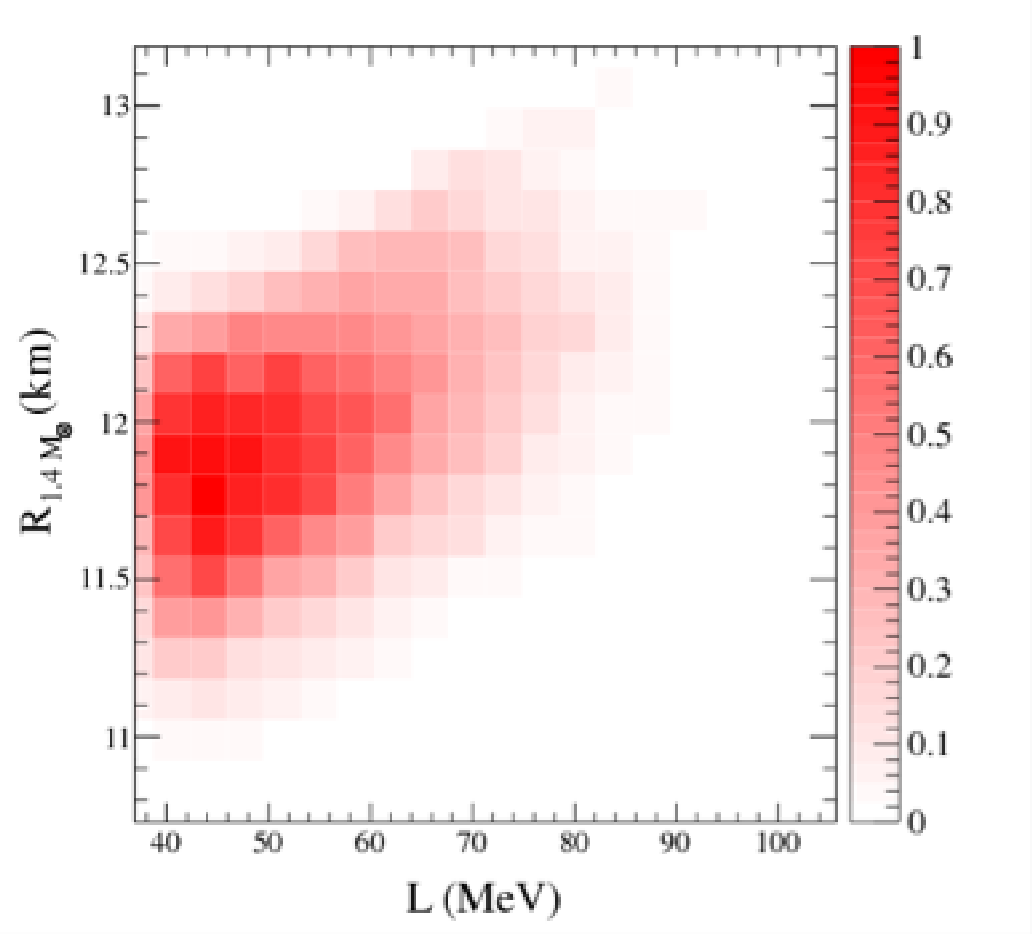




Steiner, Lattimer, and Brown (2010)

Neutron Star Radii and the Symmetry Energy





- $R_{1.4}$ correlated with pressure of neutron matter at a fixed density
- Pressure of neutron-star matter tightly connected to L
- These correlations are characterizations of a model space,
  $\rightarrow$ somewhat model-dependent
- depend on your parameterization of your model space
- Or: "sensitive to Bayesian prior distributions over the model space"

Heavy-Ion Collisions and the Symmetry Energy

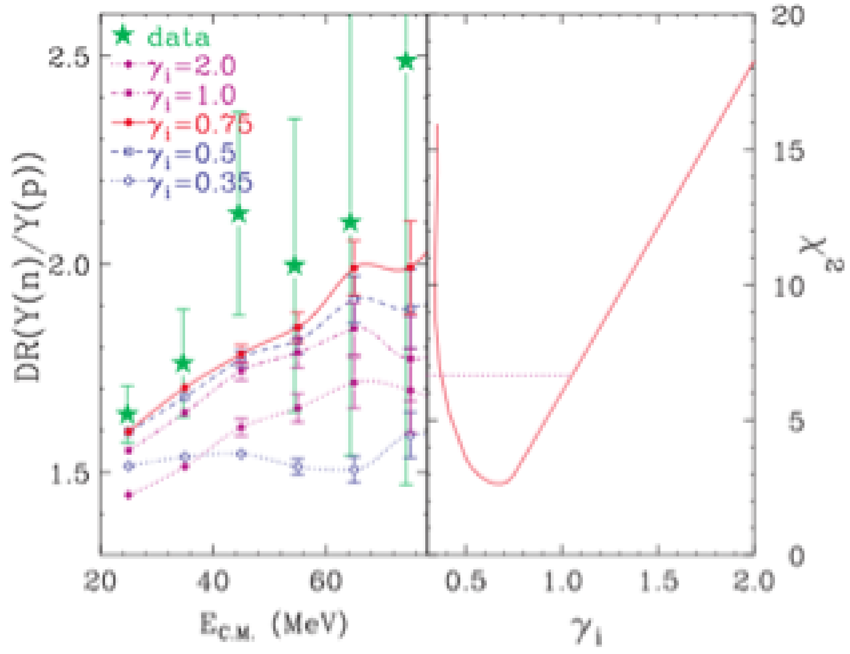


FIG. 1 (color online). Left panel: Comparison of experimental double neutron-proton ratios [18] (star symbols), as a function of nucleon center-of-mass energy, to ImQMD calculations (lines) with different density dependencies of the symmetry energy parameterized by γ_i in Eq. (1). Right panel: A plot of χ^2 as a function of γ_i.

Tsang et al. (2009)

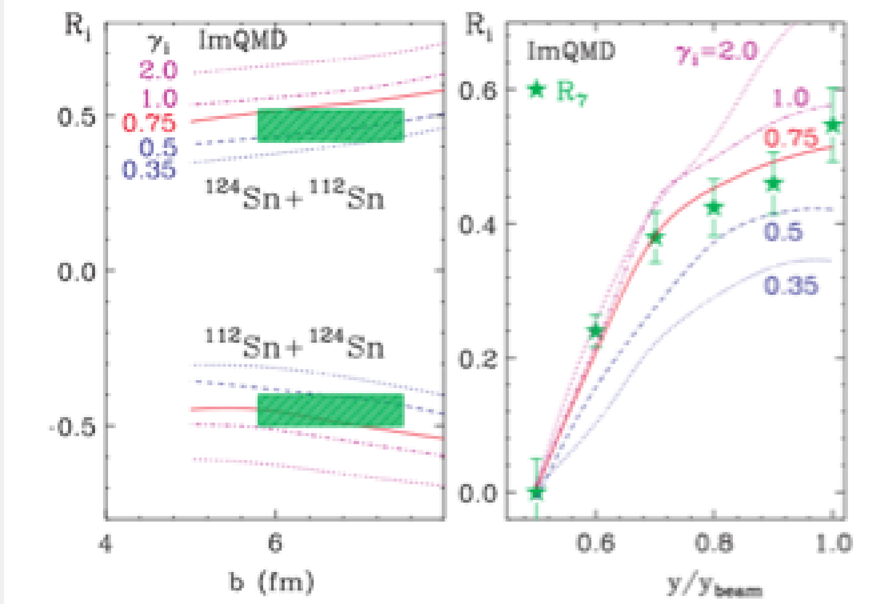


FIG. 2 (color online). Left panel: Comparison of experimental isospin transport ratios [16] (shaded regions) to ImQMD results (lines), as a function of impact parameter for different values of γ_i. Right panel: Comparison of experimental isospin transport ratios obtained from the yield ratios of $A = 7$ isotopes [17] (star symbols), as a function of the rapidity to ImQMD calculations (lines) at $b = 6$ fm.

Tsang et al. (2009)

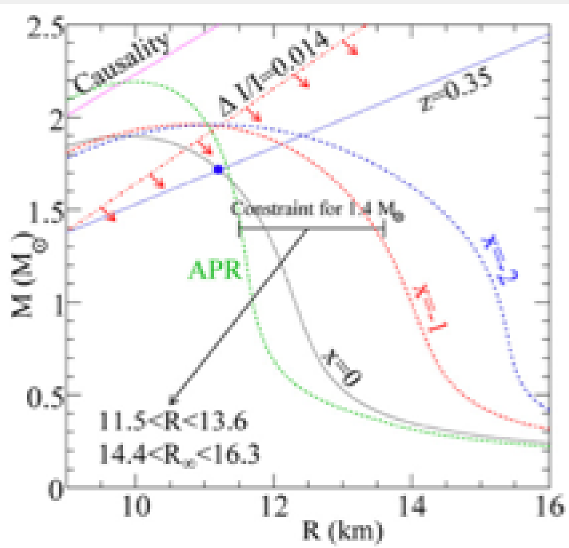
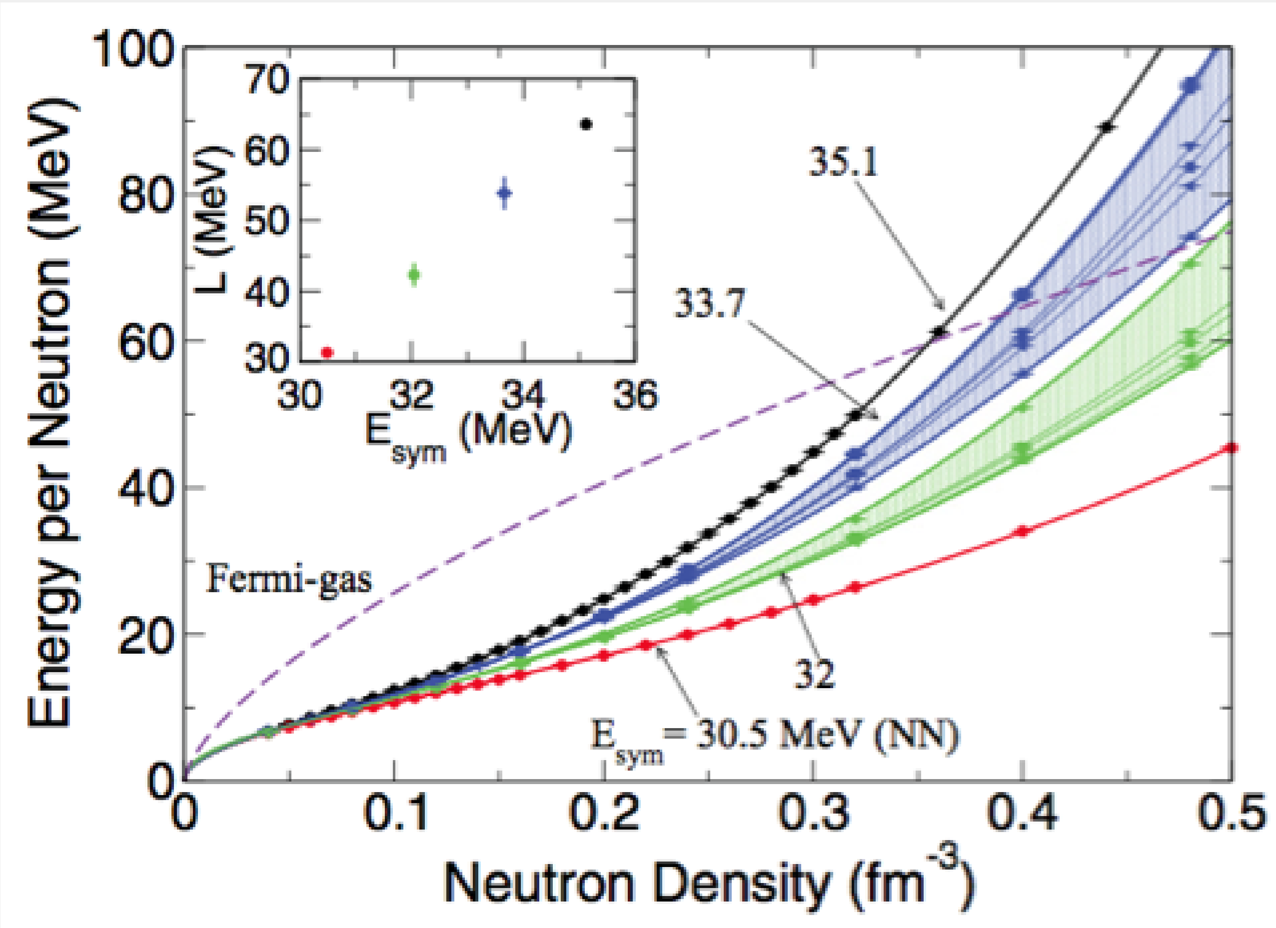


Fig. 3. The mass-radius curves for $x = 0, -1$, and -2 and the APR EOS. The limit from causality, the Vela pulsar, and the redshift of EXO0748 are all indicated. The inferred radius of a 1.4 solar mass neutron star and the inferred value of R_∞ are given.

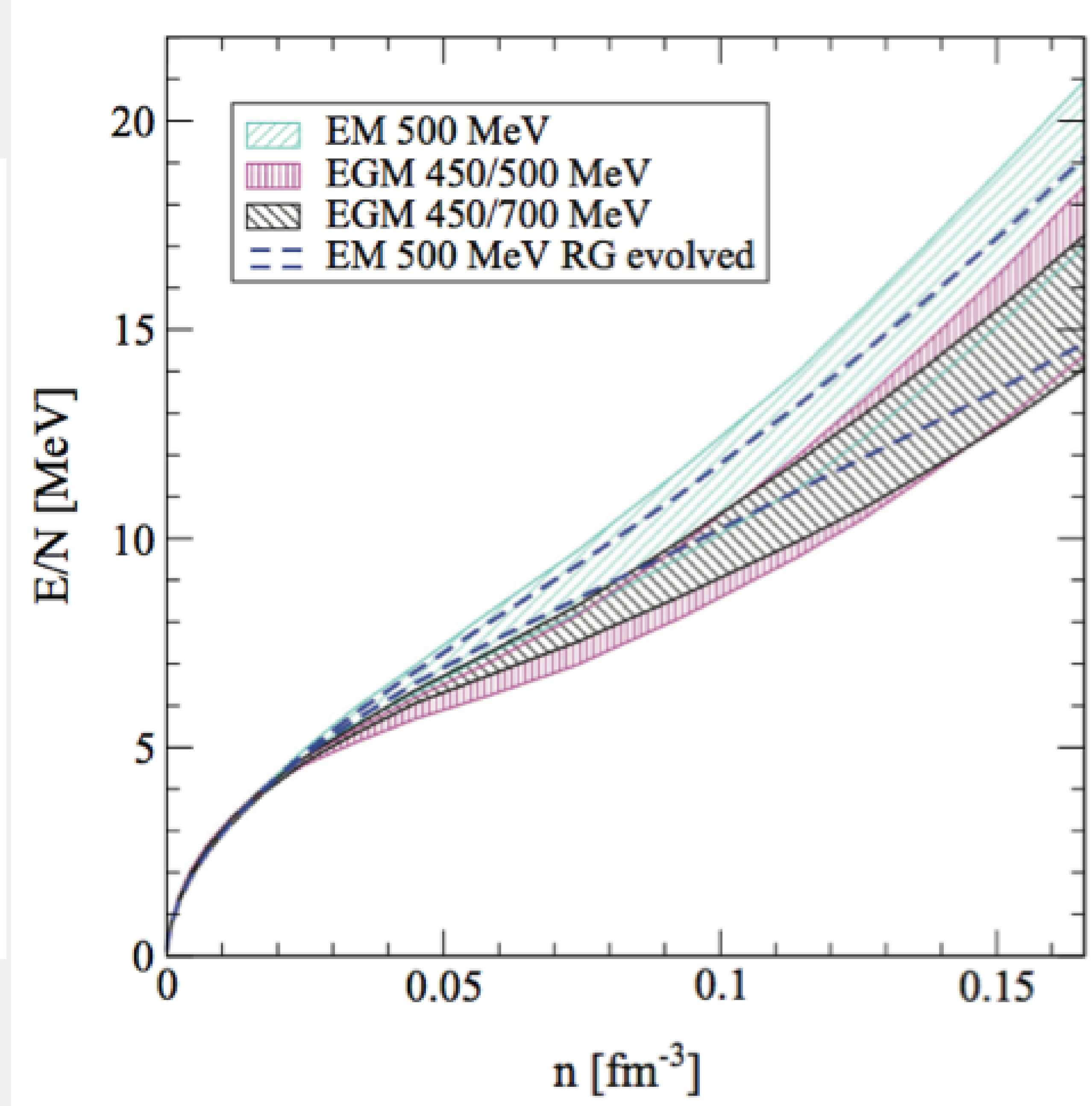
Li and Steiner (2006)

- Particle ratios and composition of the projectile-like fragment "isospin-diffiusion"
- Sensitive to L and to neutron star radii
- 11.5 km < $R_{1.4}$ < 13.6 km

The EOS of Neutron Matter



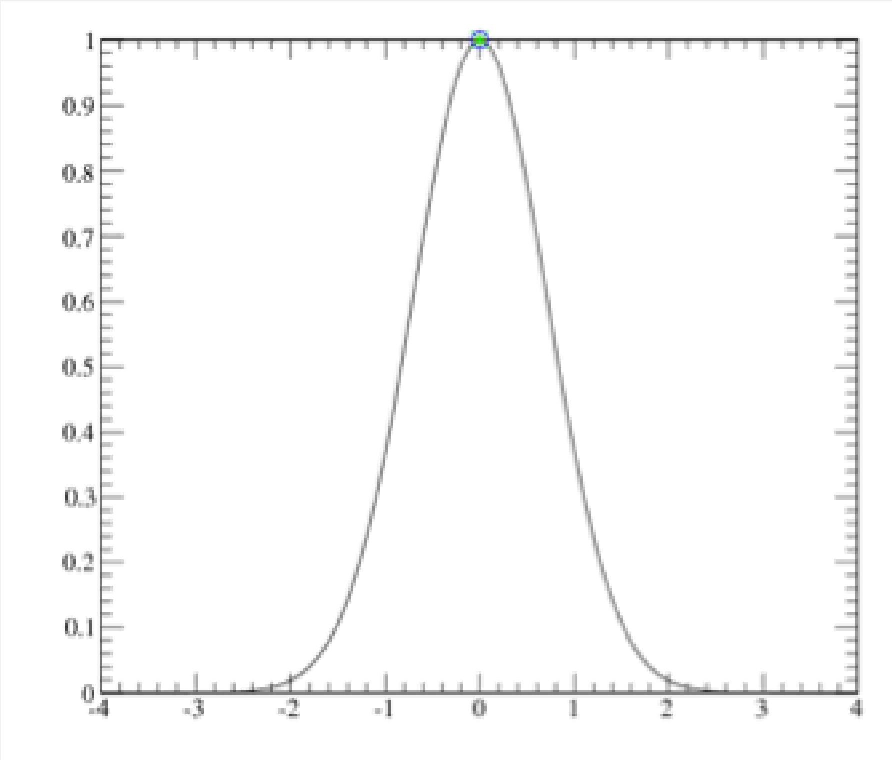
Gandolfi, Carlson, and Reddy (2012)
Describes scattering up to higher momenta

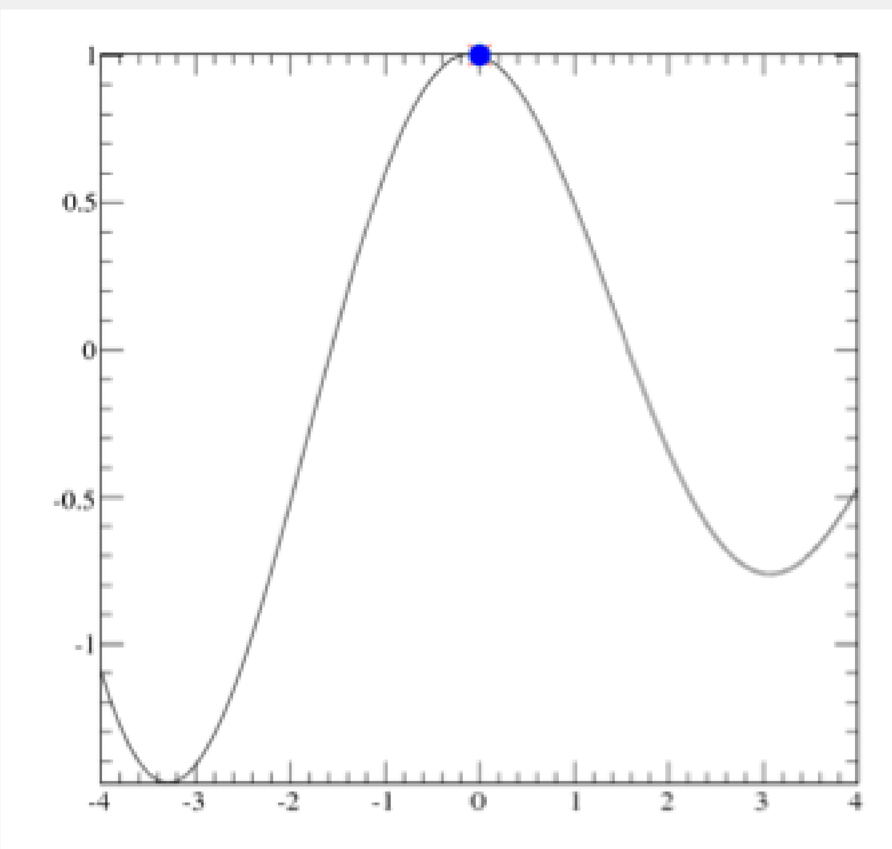


Krüger et al. (2013)
Easier to describe asymmetric matter

- EOS of pure neutron matter up to saturation density (maybe beyond)

Likelihood Functions





$$\chi^2 = \sum_i \left(\frac{O_i - M_i}{E_i} \right)^2$$

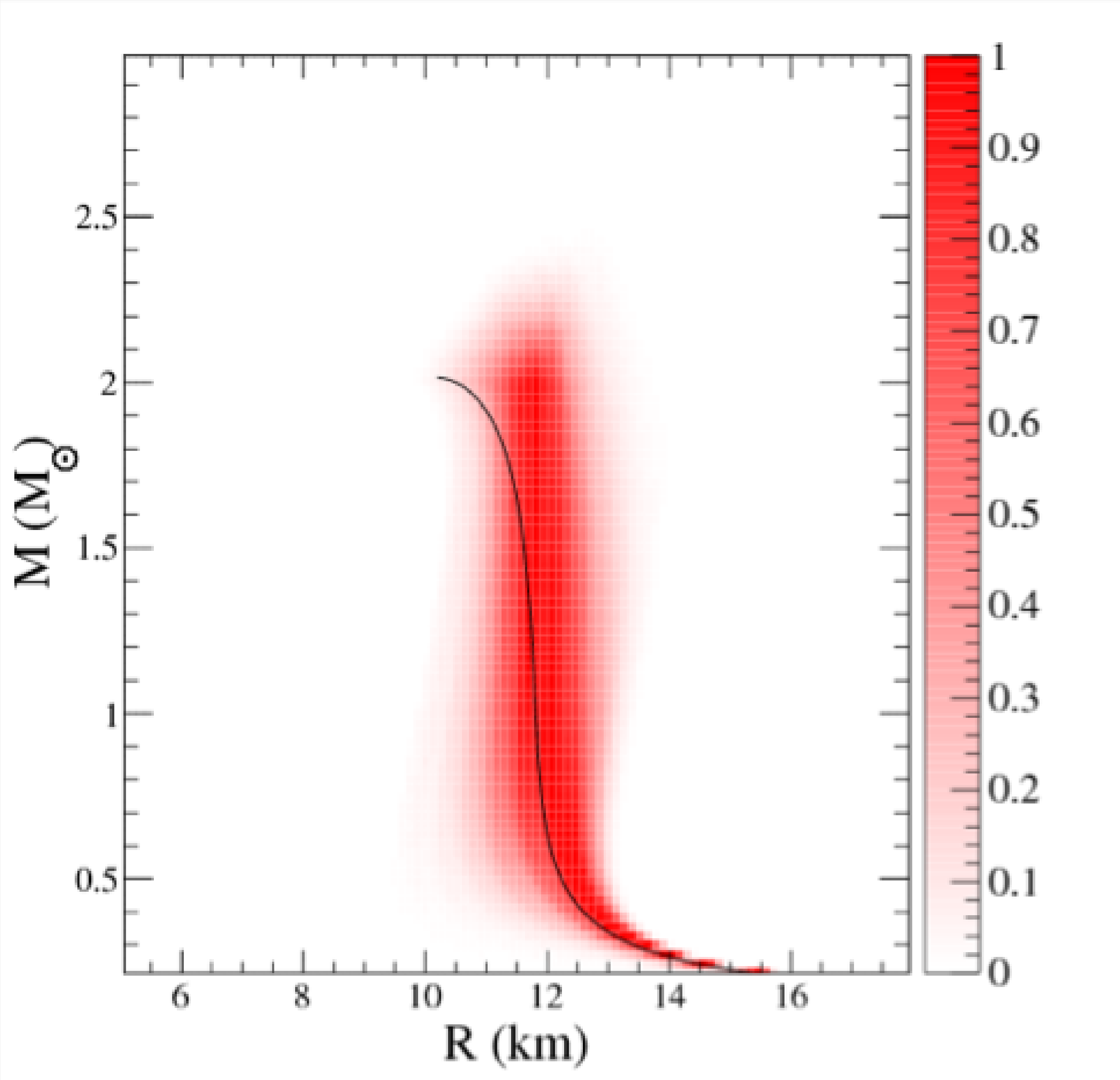
$$\mathcal{L} = \exp\left(-\chi^2/2\right)$$

- Many 'classical' methods assume something about the shape of the likelihood function near the maximum

$$\mathcal{L} \sim \exp\left[-\frac{1}{2}(p_i - p_{0,i})\Sigma_{ij}^{-1}(p_j - p_{0,j})\right]$$

- In this case, only need the neighborhood near the best fit
- Can be difficult to assign E_i when dominated by systematics

The Geometry of M-R curves

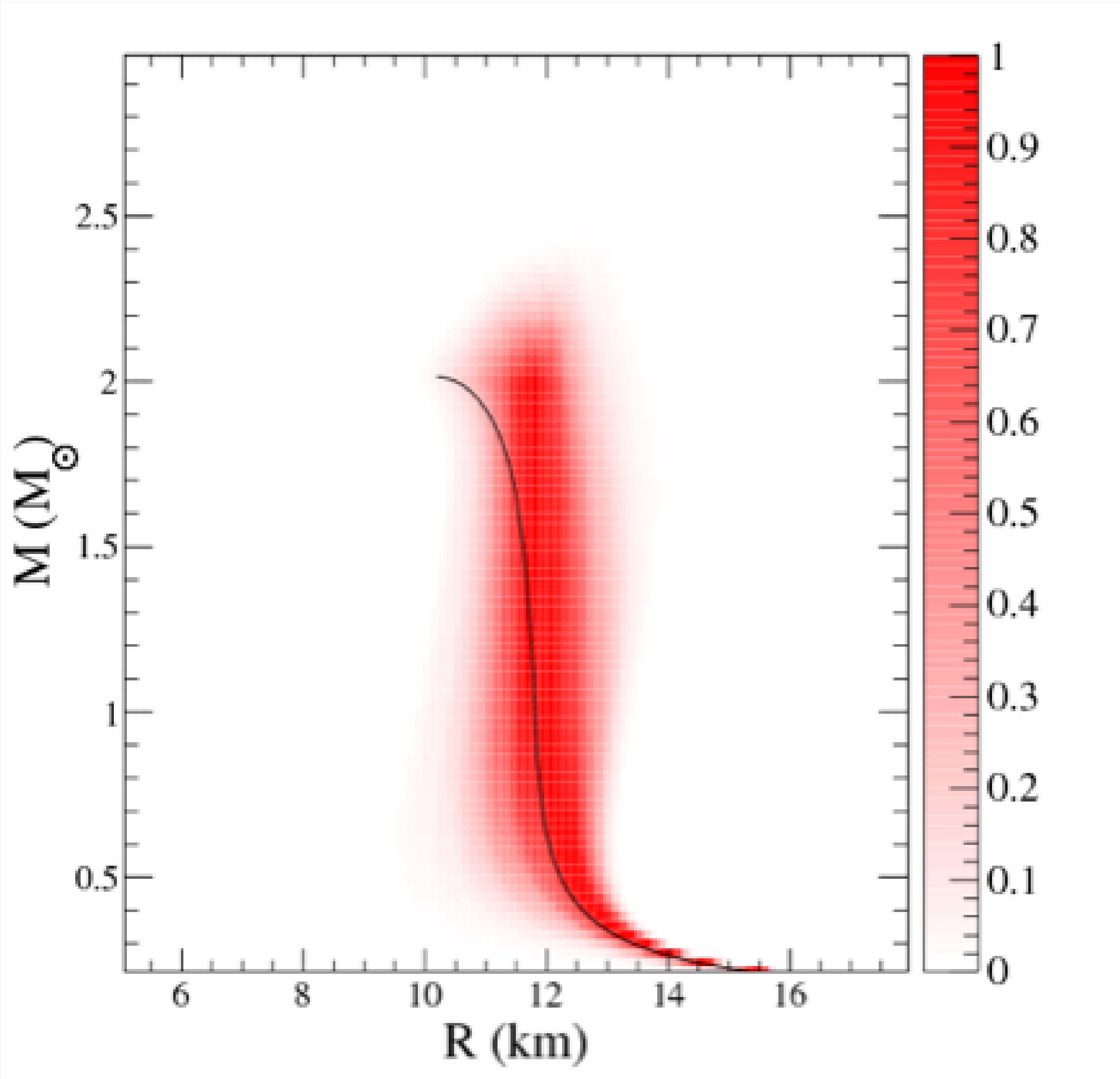


- Neither $M(R)$ nor $R(M)$ need to be functions (but $M(P_c)$ and $R(P_c)$ are)
  even though R(M) is continuous and differentiable
- In the language of χ^2 fitting: c.f. Deming or orthogonal regression and total least squares
  no unique solution in the general case
- Minimize distance between data and the curve (instead of vertical displacement)
  defining a distance is nontrivial

- Formally an underconstrained problem
  cannot divide chi-squared by the number of degrees of freedom
- Unless one performs a parameterization, $M - R$ or the EOS
- However: (R, M) space is difficult to translate to (ε, P) space
  not even a homeomorphism

Bayesian Analysis

How do we get the EOS from several $\mathcal{P}(R, M)$'s?



- Over/under-constrained subspaces (Low vs. high densities)

- Bayesian analysis proven successful
  Lepage et al. (2002) and Schindler and Phillips (2009)

- Many standard frequentist methods assume something about the shape of the likelihood function near the maximum

- This fails in this case: the best fit not same as "typical" M-R curve
  Posterior maximum mass distribution is strongly skewed

- Naive covariance analysis unrelated to typical M-R curve for high masses
  Just an example of how that method can fail

Analysis Details

$$\varepsilon = m_n n_n + m_p n_p + B + \frac{K}{18 n_0^2}(n - n_0)^2 + \frac{K'}{162 n_0^3}(n - n_0)^3$$

$$+ (1 - 2x)^2 \left[S_k \left(\frac{n}{n_0} \right)^{2/3} + S_p \left(\frac{n}{n_0} \right)^{\gamma} \right]$$

$$K, K', S_p + S_k, \text{ and } \gamma$$

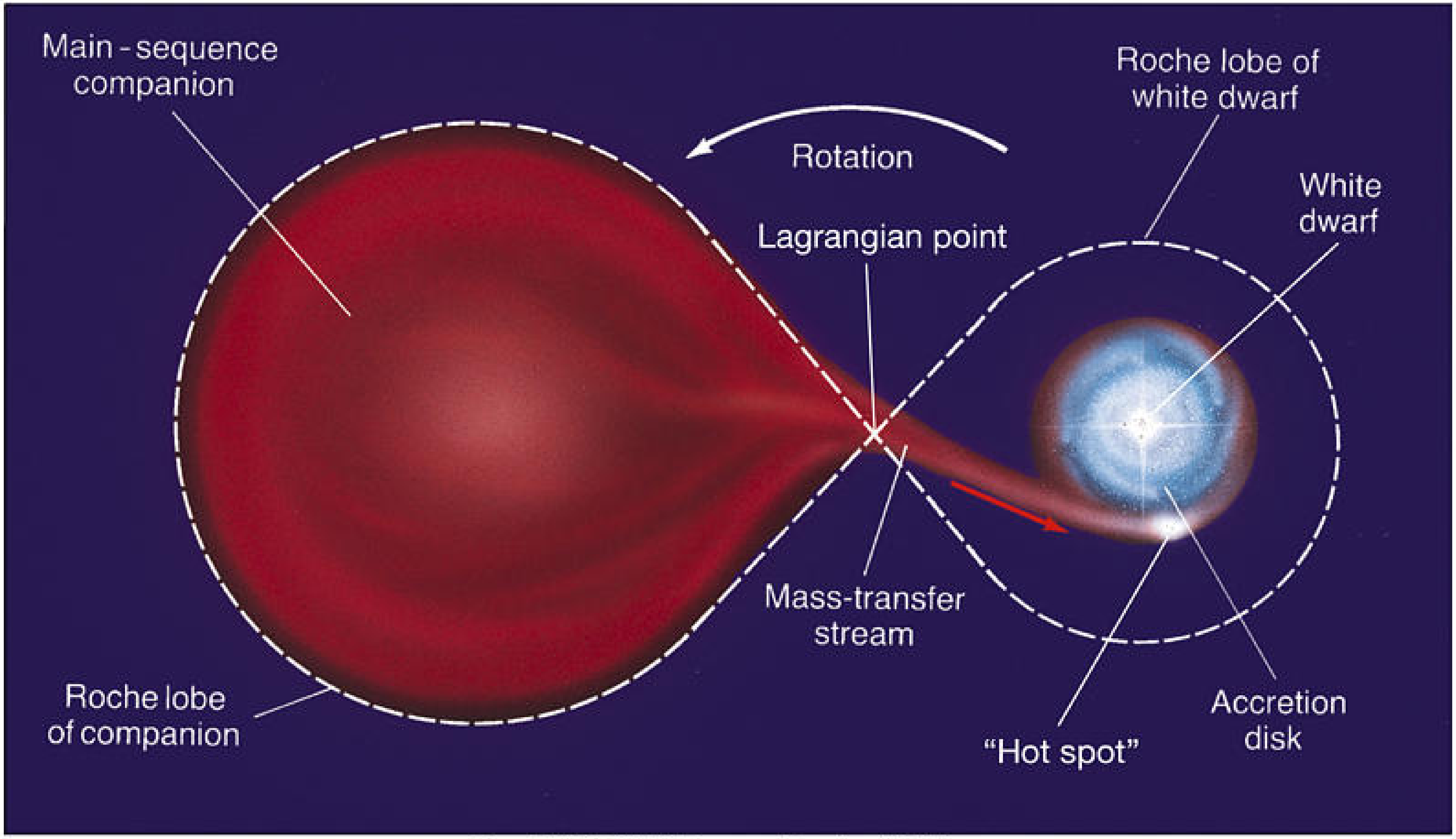
$$P(\varepsilon) = K \varepsilon^{\Gamma} \text{ with } \Gamma \equiv 1 + \frac{1}{n} : \quad \Gamma_i \text{ and } \varepsilon_i$$

$$\text{crust} \mid \varepsilon_{\text{trans}} \mid \text{schematic} \mid \varepsilon_1 \mid \text{Polytrope 1} \mid \varepsilon_2 \mid \text{Polytrope 2}$$

- Bayes theorem: $P[\mathcal{M}_i | D] \propto P[D | \mathcal{M}_i] P[\mathcal{M}_i]$
- Prior $\Leftrightarrow$ EOS parameterization
- Determine parameters through marginalization, i.e.

$$P(\mathcal{M}_i^0) = \int \delta(\mathcal{M}_i - \mathcal{M}_i^0) P[\mathcal{M}_i | D] \, d\mathcal{M}$$

Accreting Neutron Stars: LMXBs



- Most stars have companions: neutron stars can have main-sequence companions
- Accretion heats the crust and is episodic
- At high enough density, H and He are unstable to thermonuclear explosions

Radius Measurements in qLMXBs

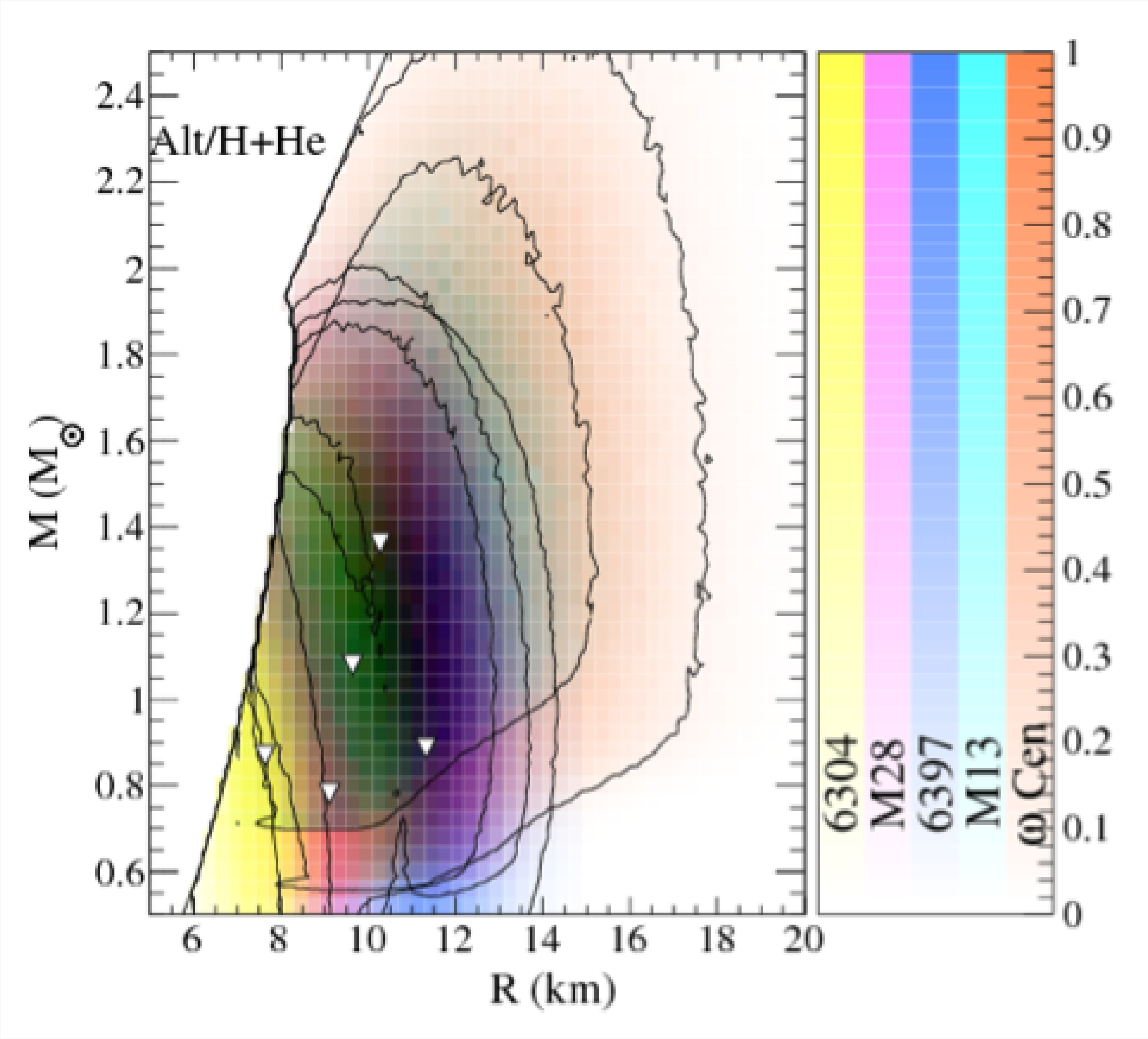
Quiescent LMXBs

- Measure flux of photons and their energy distribution
- Know distance if in a globular cluster
- Implies radius measurement

$$F \propto T_{\rm eff}^{4} \left(\frac{R_{\infty}}{D} \right)^{2}$$

i.e. Rutledge et al. (1999)

- Need information about the atmosphere, including composition
- Also need X-ray absorption and absolute flux calibration
- Inevitably give small radii for some low-mass stars



Lattimer and Steiner (2013)

- Rotation, anisotropy, and magnetic fields may also be important

Photospheric Radius Expansion X-ray Bursts

- X-ray bursts sufficiently strong to blow off the outer layers - radiate at the Eddington limit
- Flux peaks, then temperature reaches a maximum, "touchdown"

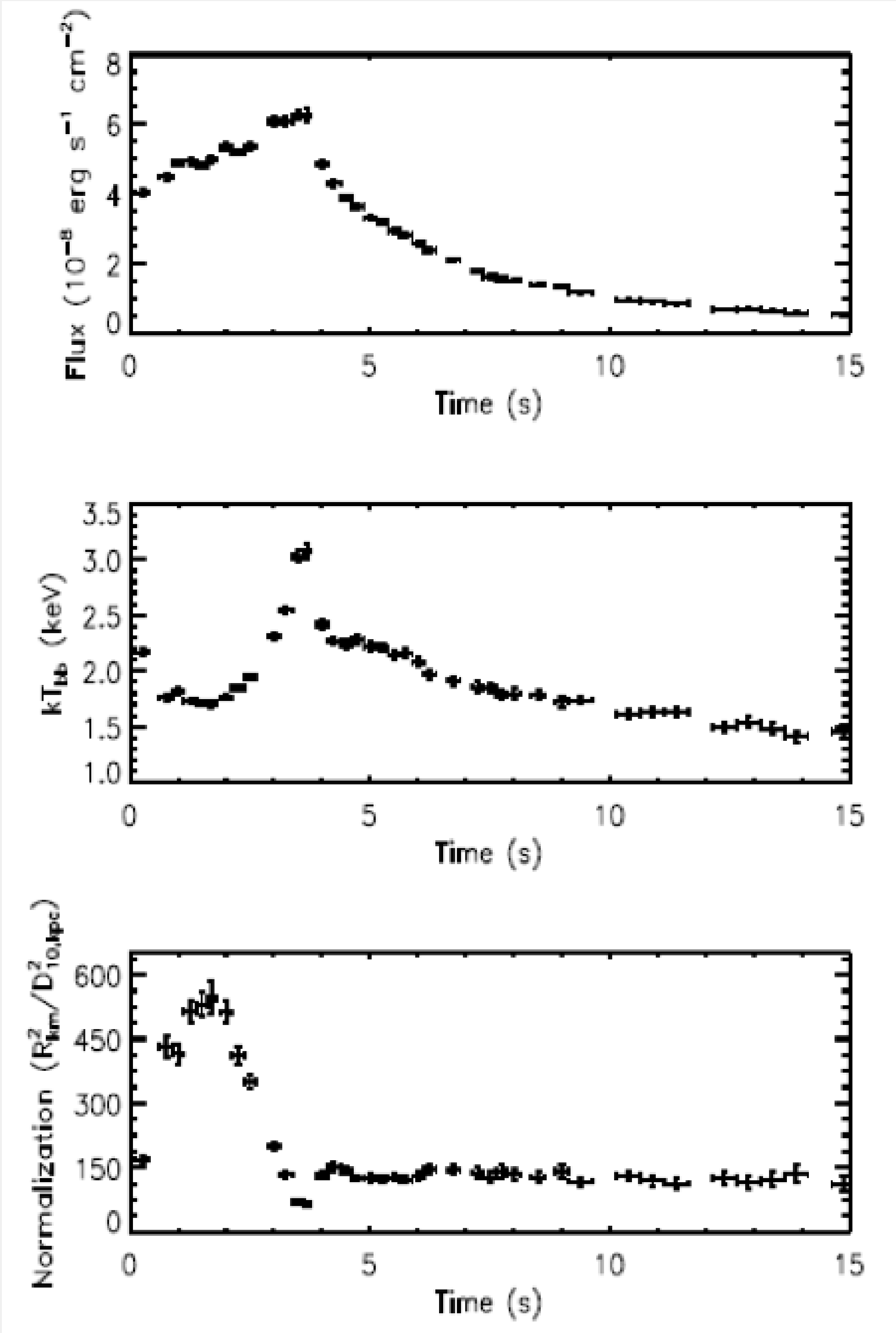
$$F_{TD} = \frac{GMc}{\kappa D^2}\sqrt{1-2\beta(r_{ph})}$$

- Normalization during the tail of the burst:

$$\frac{F_\infty}{\sigma T^4_{bb,\infty}} = f_c^{-4}\left(\frac{R}{D}\right)^2(1-2\beta)^{-1}$$

- If we have the distance, two constraints for mass and radius
- Dimensionless parameter

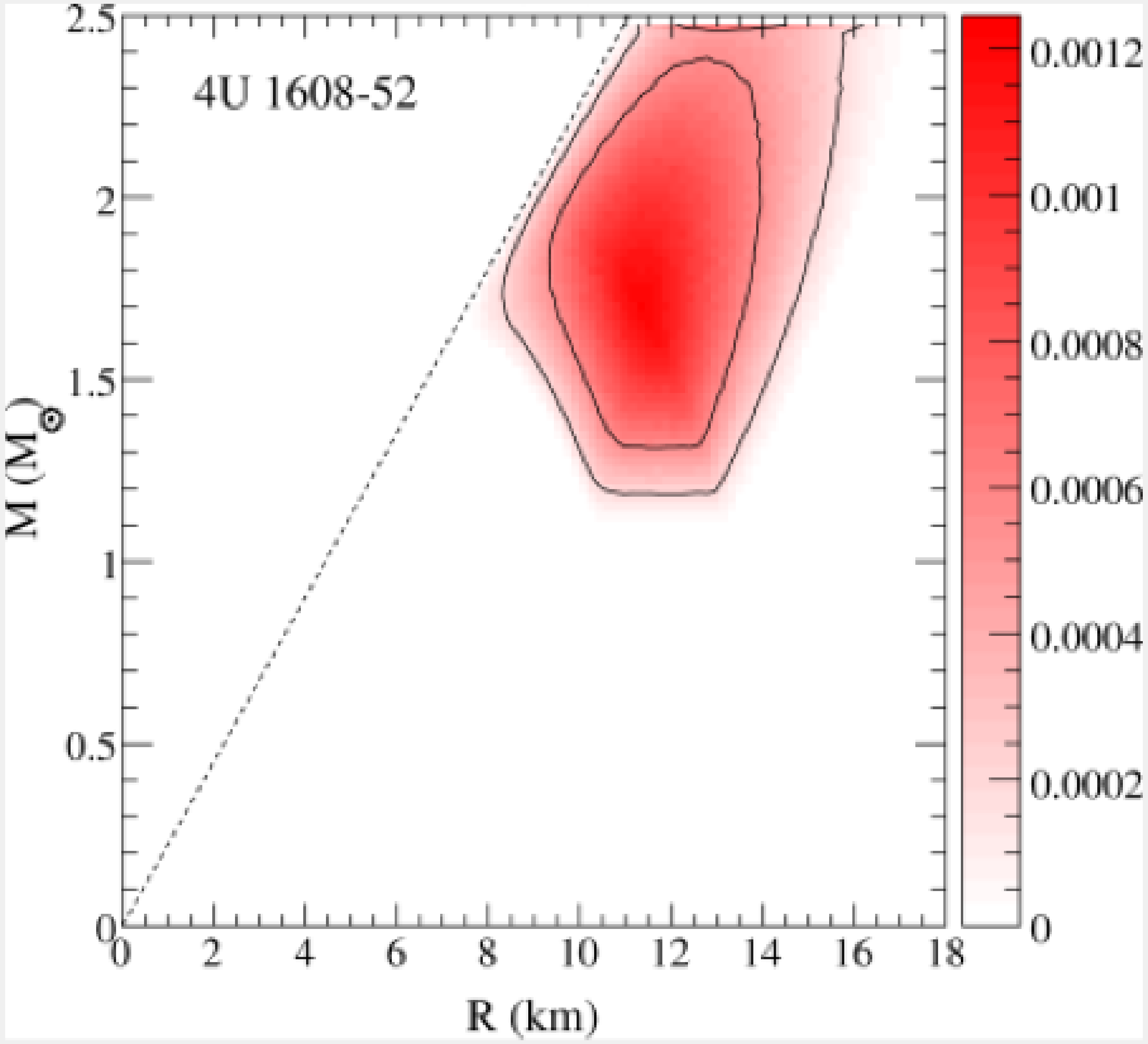
$$\alpha \equiv \frac{F_{TD}\kappa D}{\sqrt{A}c^3 f_c^2}$$



Ozel et al. (2010)

Photospheric Radius Expansion X-ray Bursts

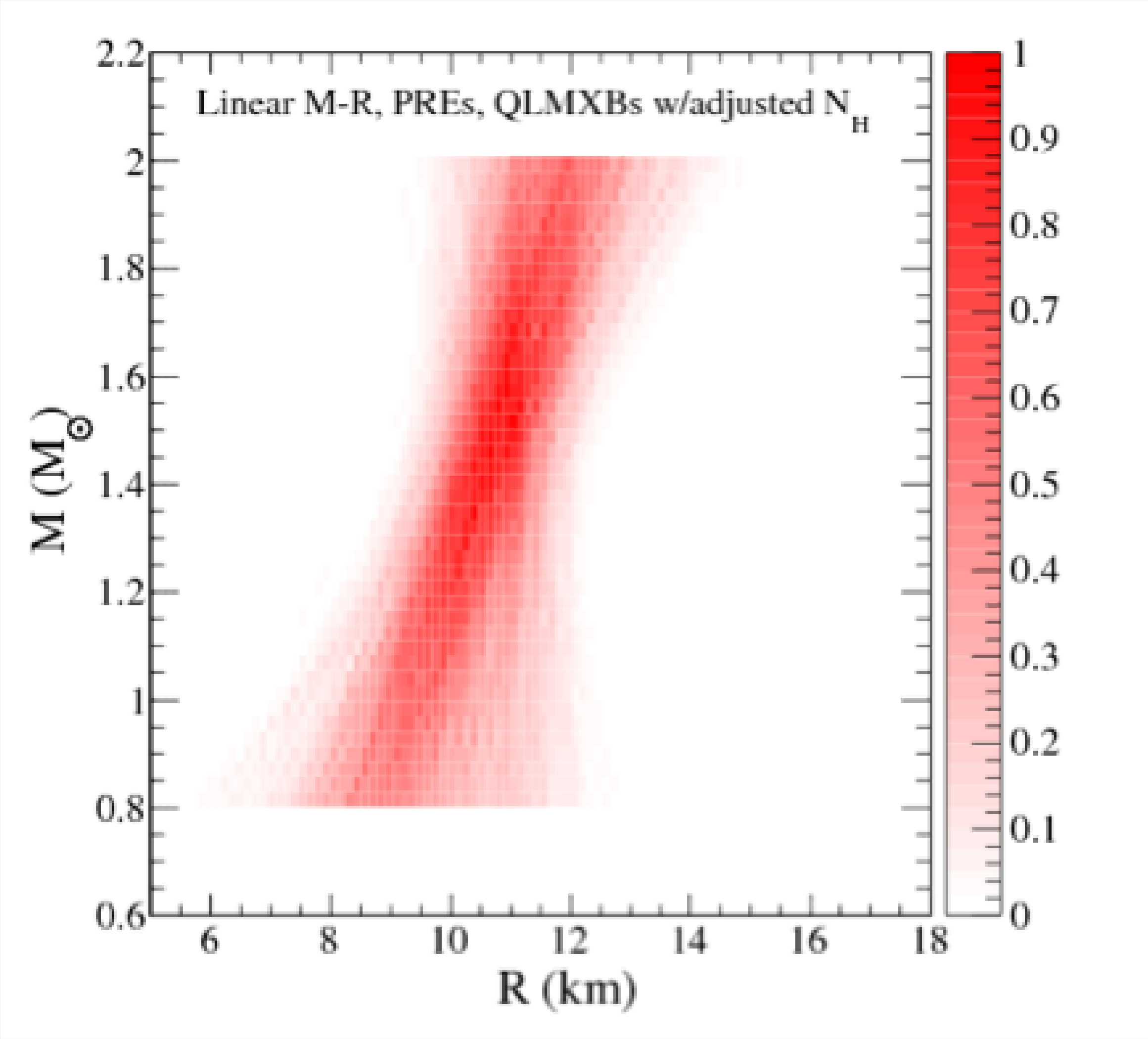
- Several potential systematic uncertainties
- All the complications of qLMXBs
- plus requires assumptions about time-dependence



Steiner et al. (2010)

Minimal Nuclear Physics Models

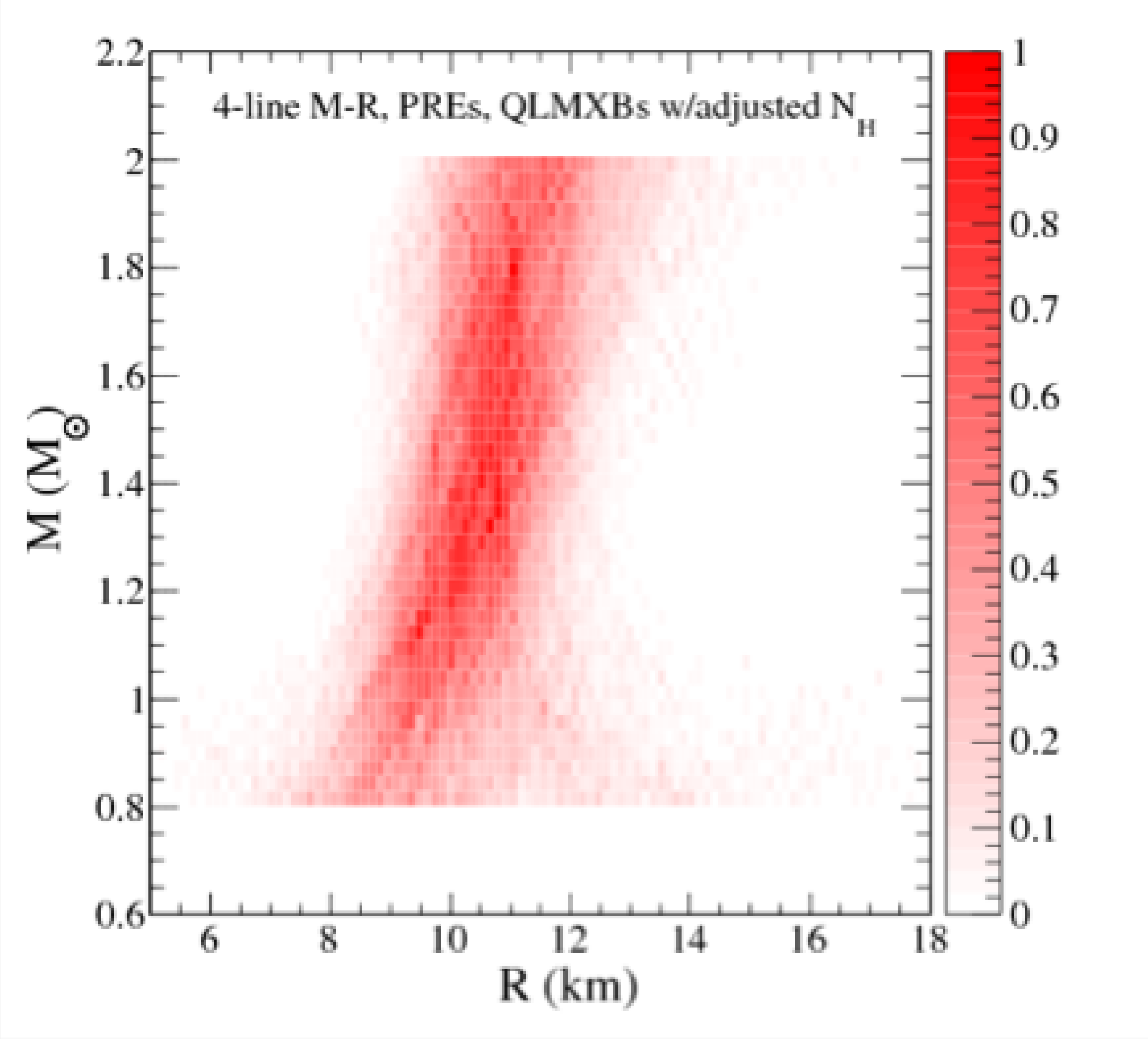
What if we directly parameterize the $M - R$ curve?



Linear model

Lattimer and Steiner (2013)

- Maybe the closest thing to a "model-independent" result
- Consistent with a vertical $M - R$ line at the 2 σ level



Four-line segments (8 parameters)

Lattimer and Steiner (2013)

- Some of these $M - R$ curves may be unphysical
- Tension between nuclear physics and observations

The M-R curve and the EOS of dense matter

EOS Model	Data modifications	$R_{95\%>}$	$R_{68\%>}$ (km)	$R_{68\%<}$	$R_{95\%<}$
Variations in the EOS model					
A	-	11.18	11.49	12.07	12.33
B	-	11.23	11.53	12.17	12.45
C	-	10.63	10.88	11.45	11.83
D	-	11.44	11.69	12.27	12.54
Variations in the data interpretation					
A	I	11.82	12.07	12.62	12.89
A	II	10.42	10.58	11.09	11.61
A	III	10.74	10.93	11.46	11.72
A	IV	10.87	11.19	11.81	12.13
A	V	10.94	11.25	11.88	12.22
A	VI	11.23	11.56	12.23	12.49
Global limits		10.42	10.58	12.62	12.89

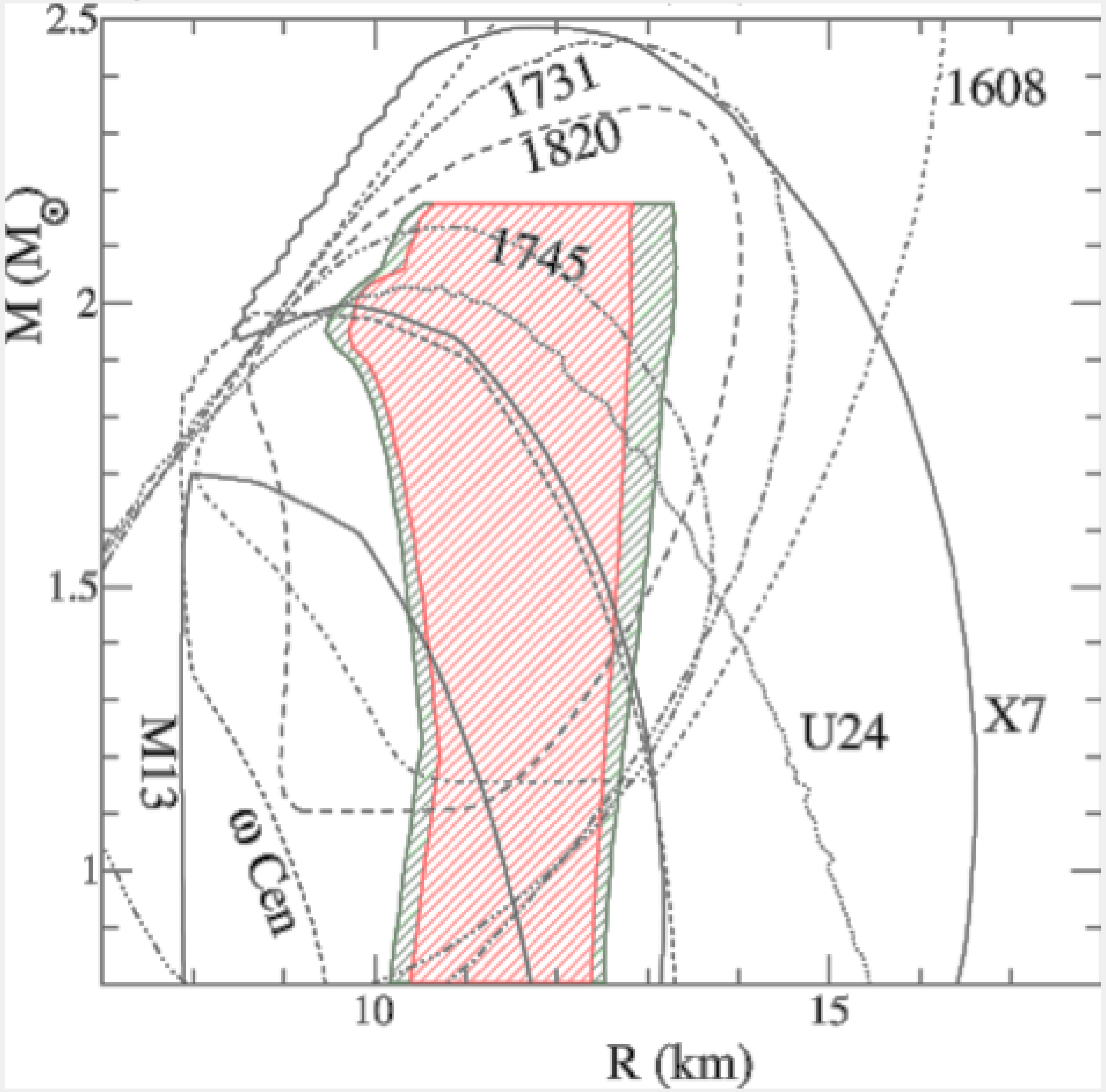
Steiner, Lattimer, and Brown (2013)

- Critical component: trying different EOS parameterizations and different interpretations of the data

- Modest attempt to address systematic uncertainties

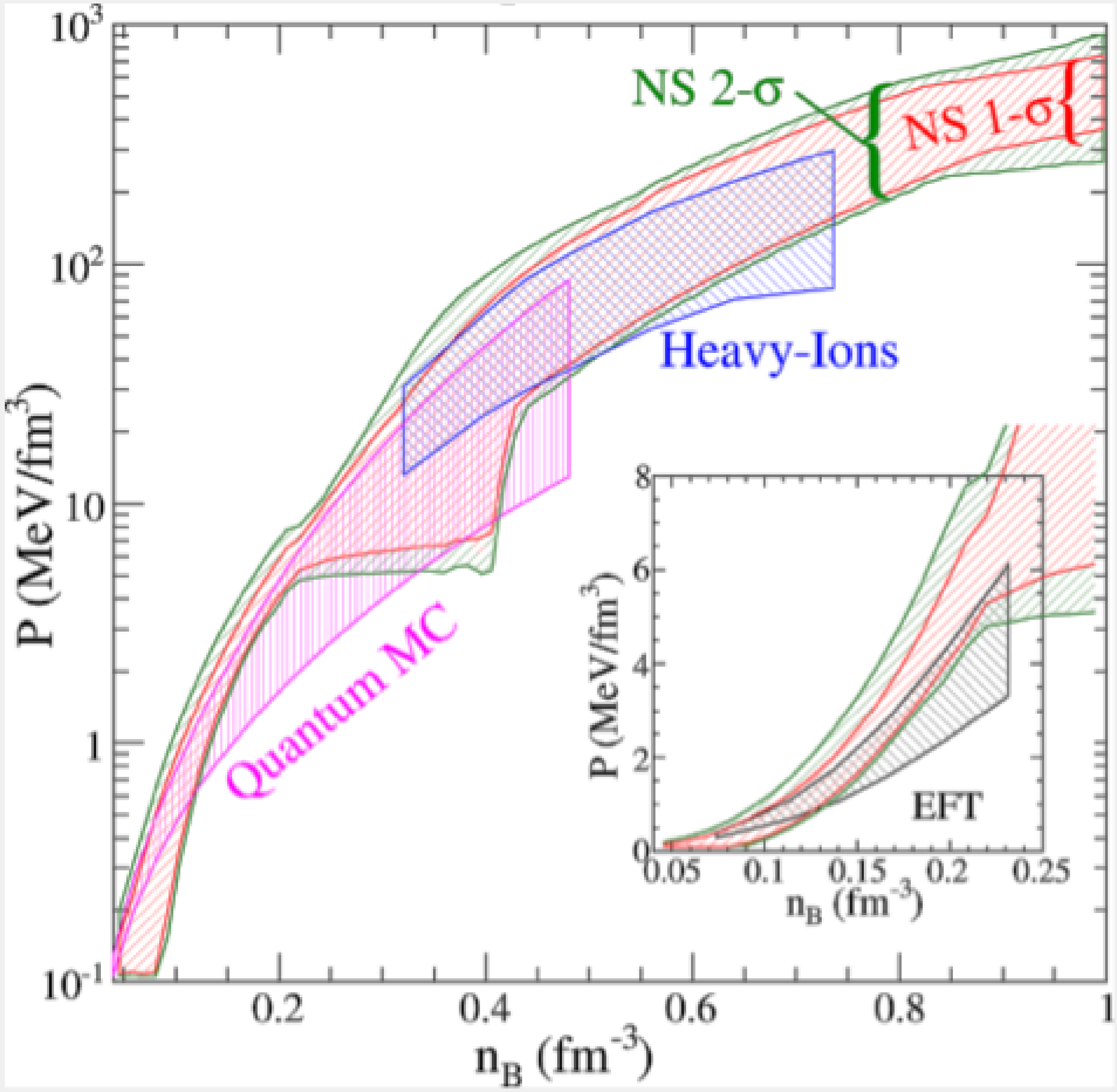
The M-R curve and the EOS of dense matter

Now parameterize the EOS:



Steiner, Lattimer, and Brown (2013)

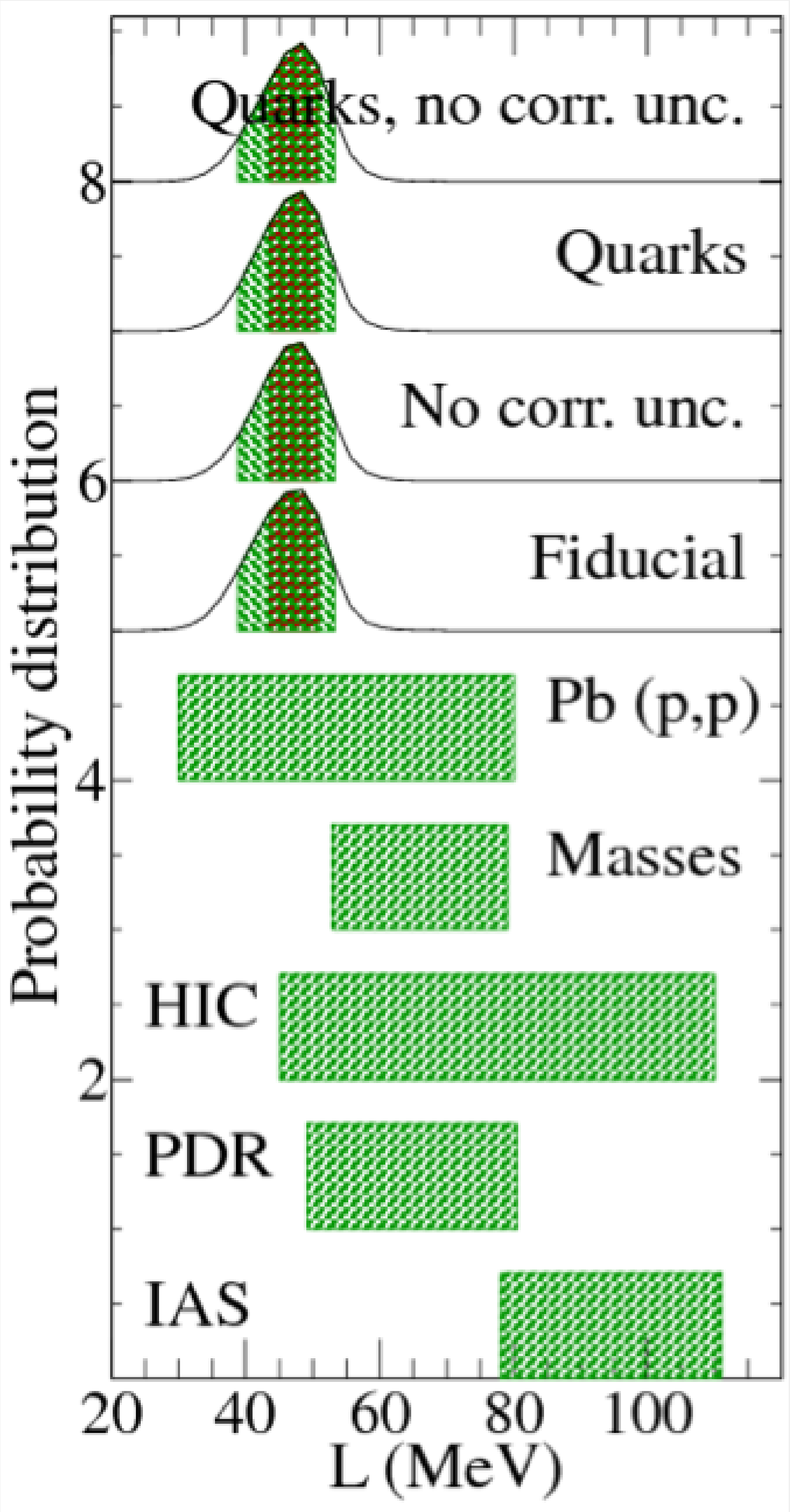
- Choose several different models, for every observable, find the region which encloses all ranges
- We find concordance between nuclear physics data and astronomical observations



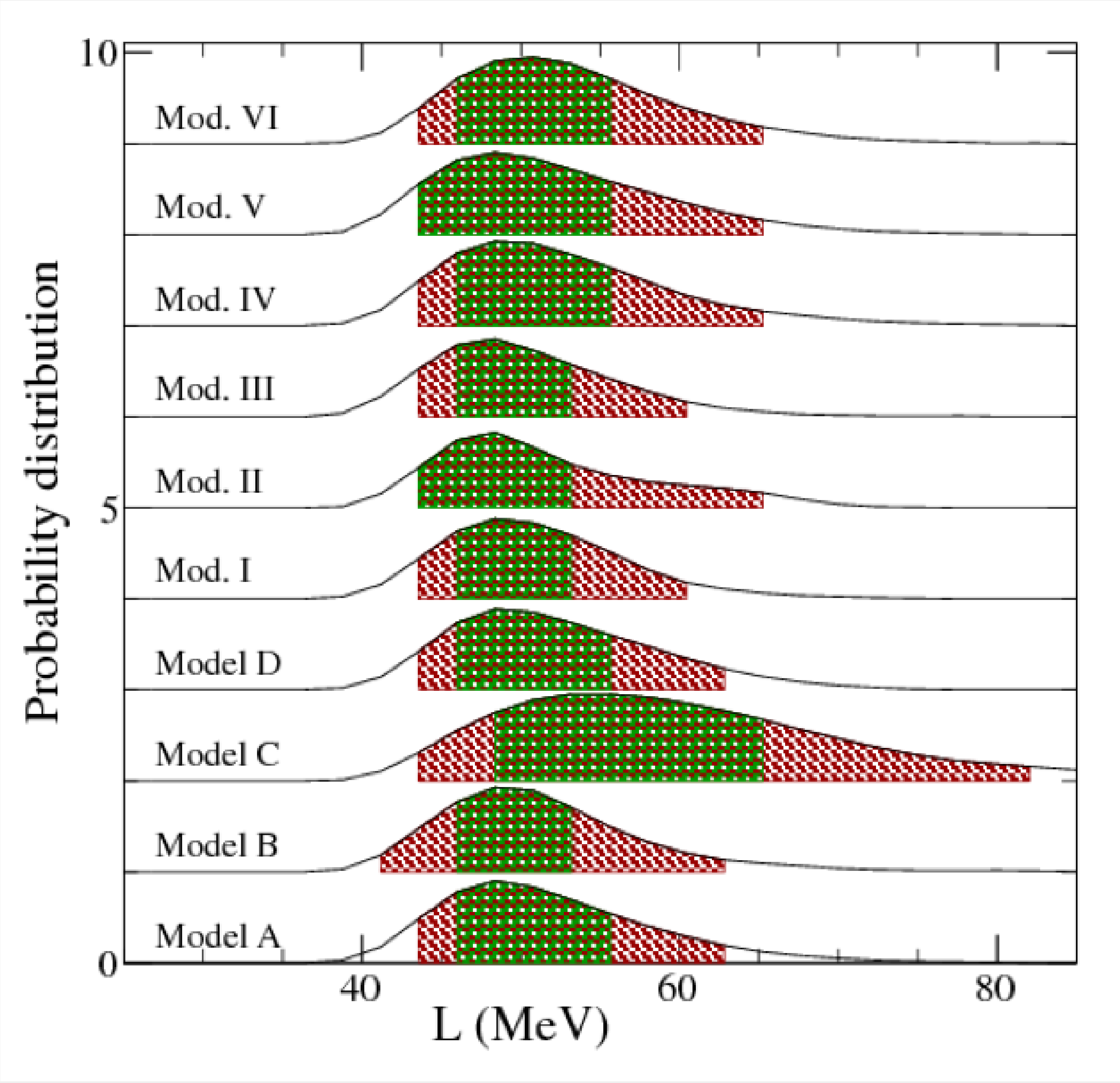
Steiner, Lattimer, and Brown (2013)

- Can determine pressure, but not composition
- Future: novel combinations of several observations with models and careful assessment of uncertainties

Neutron Star Constraints on L

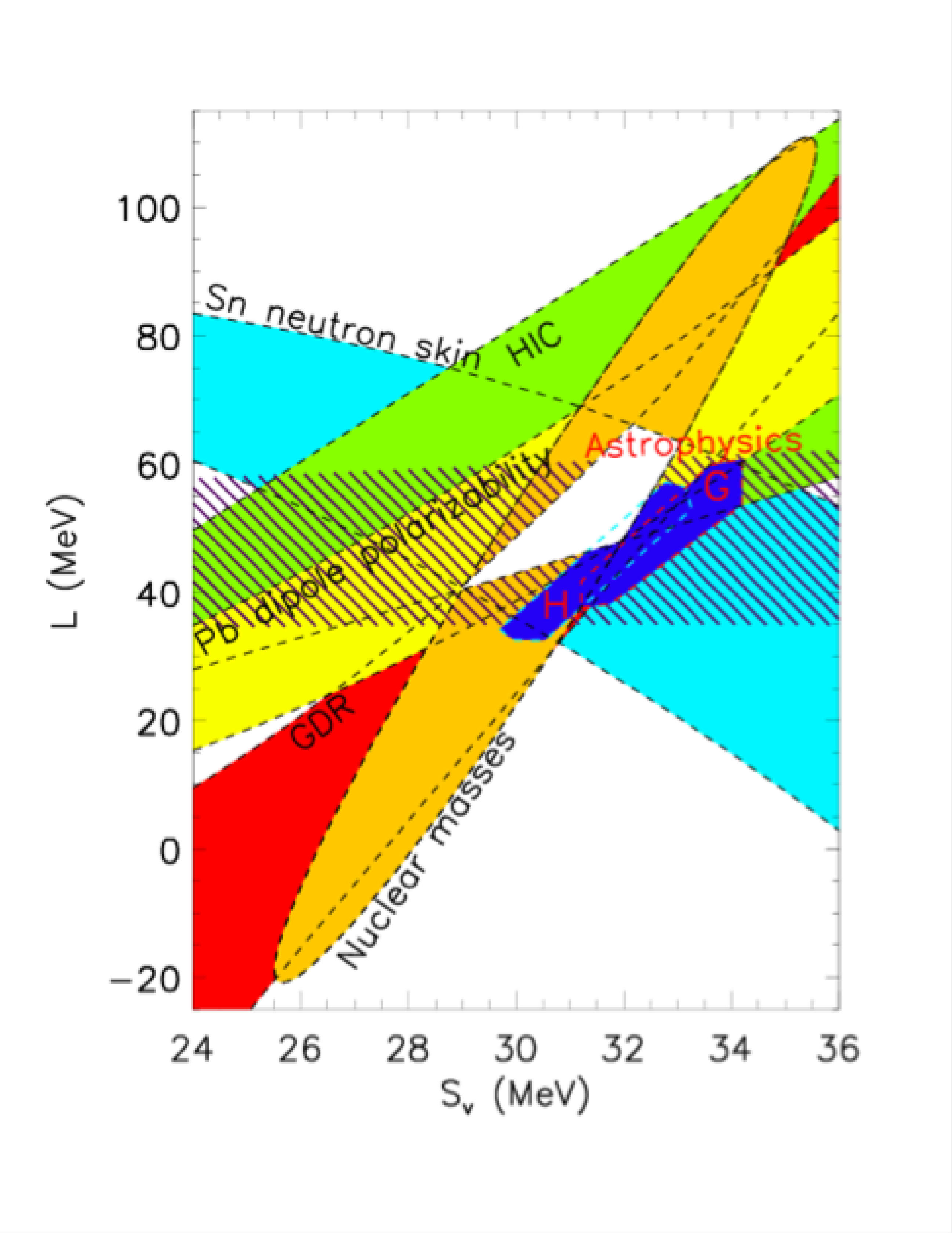


Steiner and Gandolfi (2012)



Steiner, Lattimer, and Brown (2013)

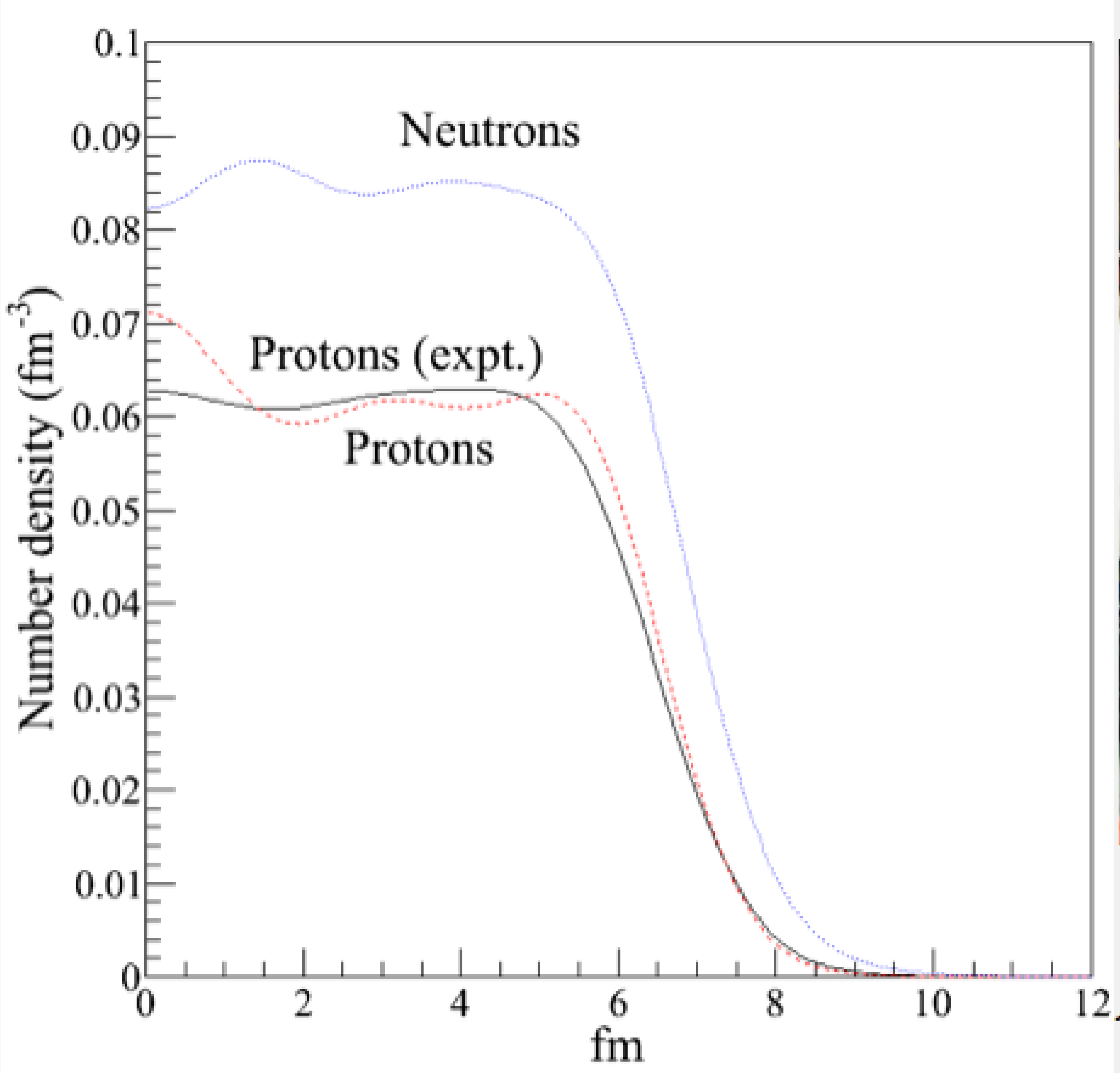
Nuclear Symmetry Energy



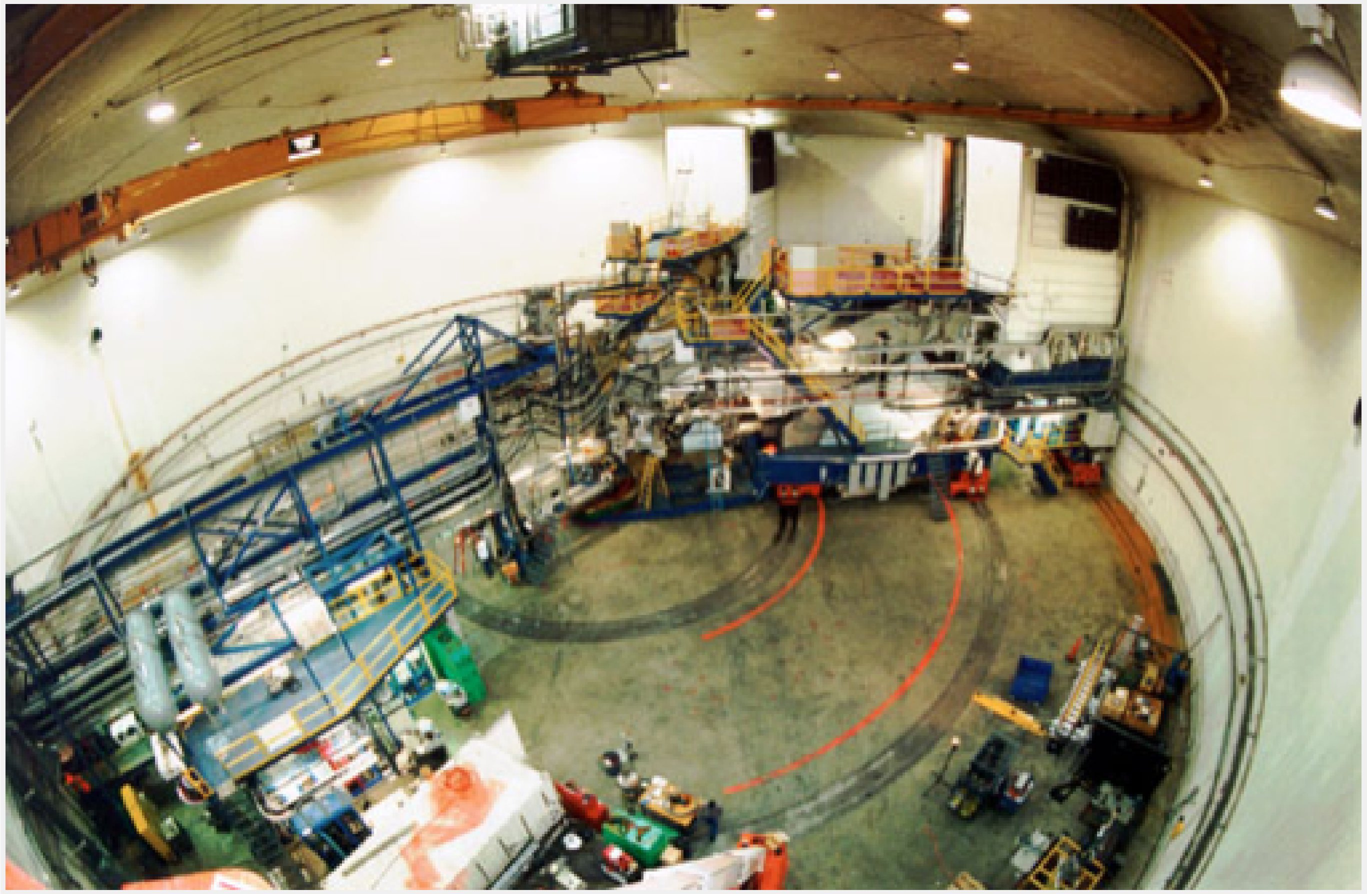
Taken from Lattimer and Steiner (2013)

The Neutron Skin Thickness of Lead

- Lead-208: 82 protons, 126 neutrons
  $R_n^2 \equiv \int r^2 n_n(r)\ d^3r \qquad R_p^2 \equiv \int r^2 n_p(r)\ d^3r$
- Neutron radii are hard to measure, use parity-violating electron scattering
- Weak charge of neutron $\gg$ weak charge of proton, i.e.
  $|-1| \gg 1 - 4\sin^2\theta_W$



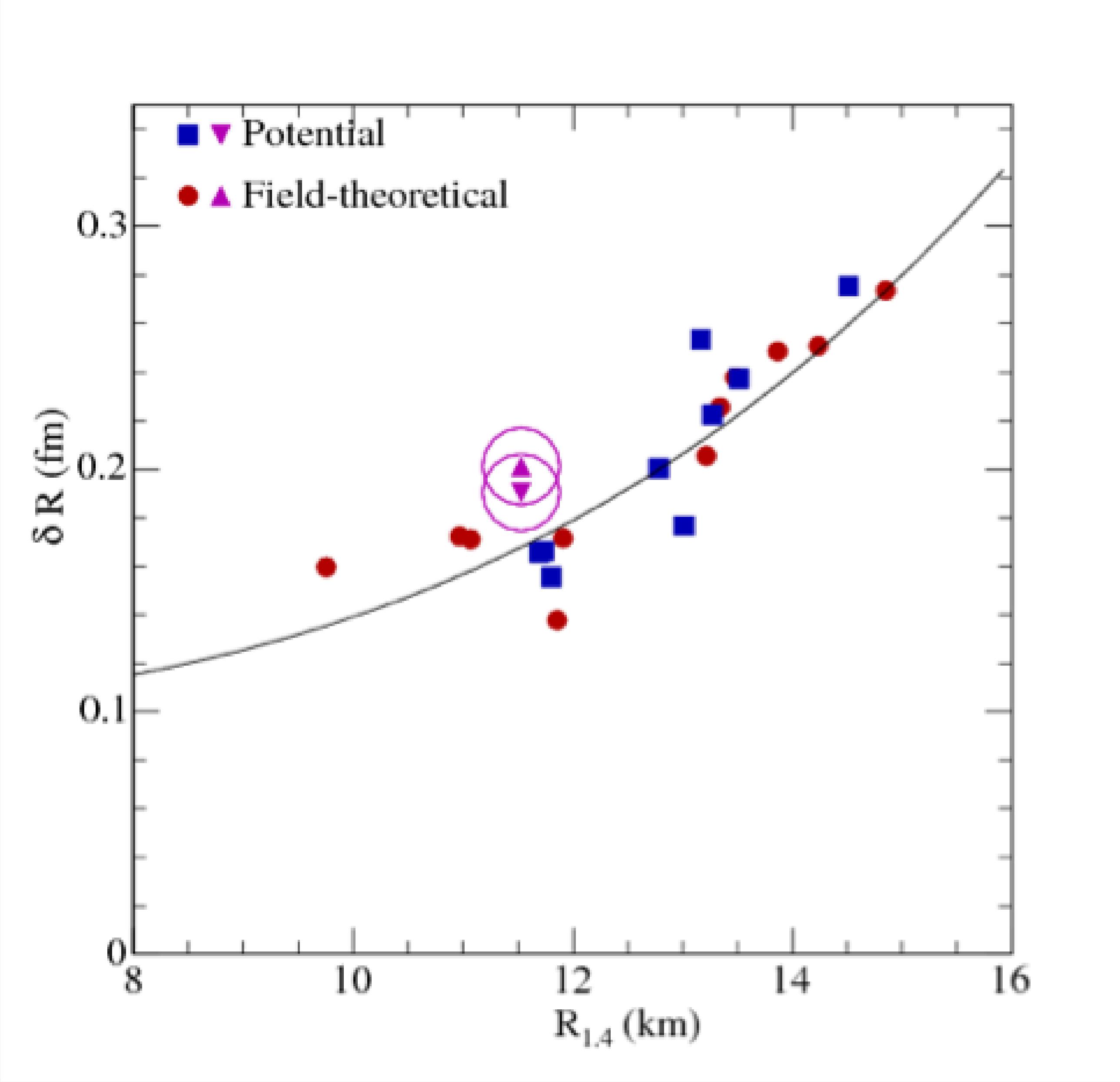
Steiner, Prakash, Lattimer and Ellis (2005)

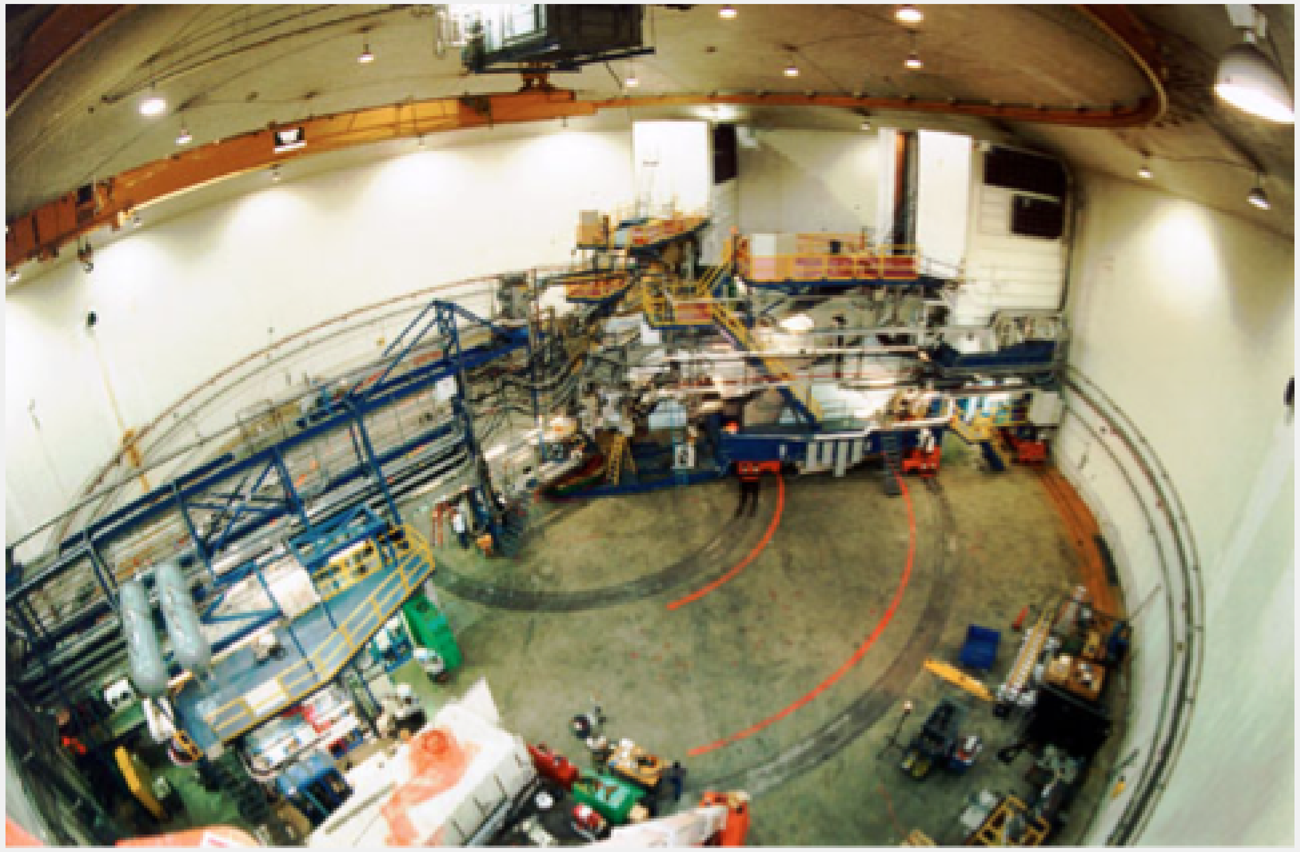


Jefferson Lab's Hall A
Measured $R_n - R_p = 0.33 \pm 0.16$ fm

The Neutron Skin Thickness of Lead

- The quantity $\delta R \equiv R_n - R_p$ is related to L as are neutron star radii



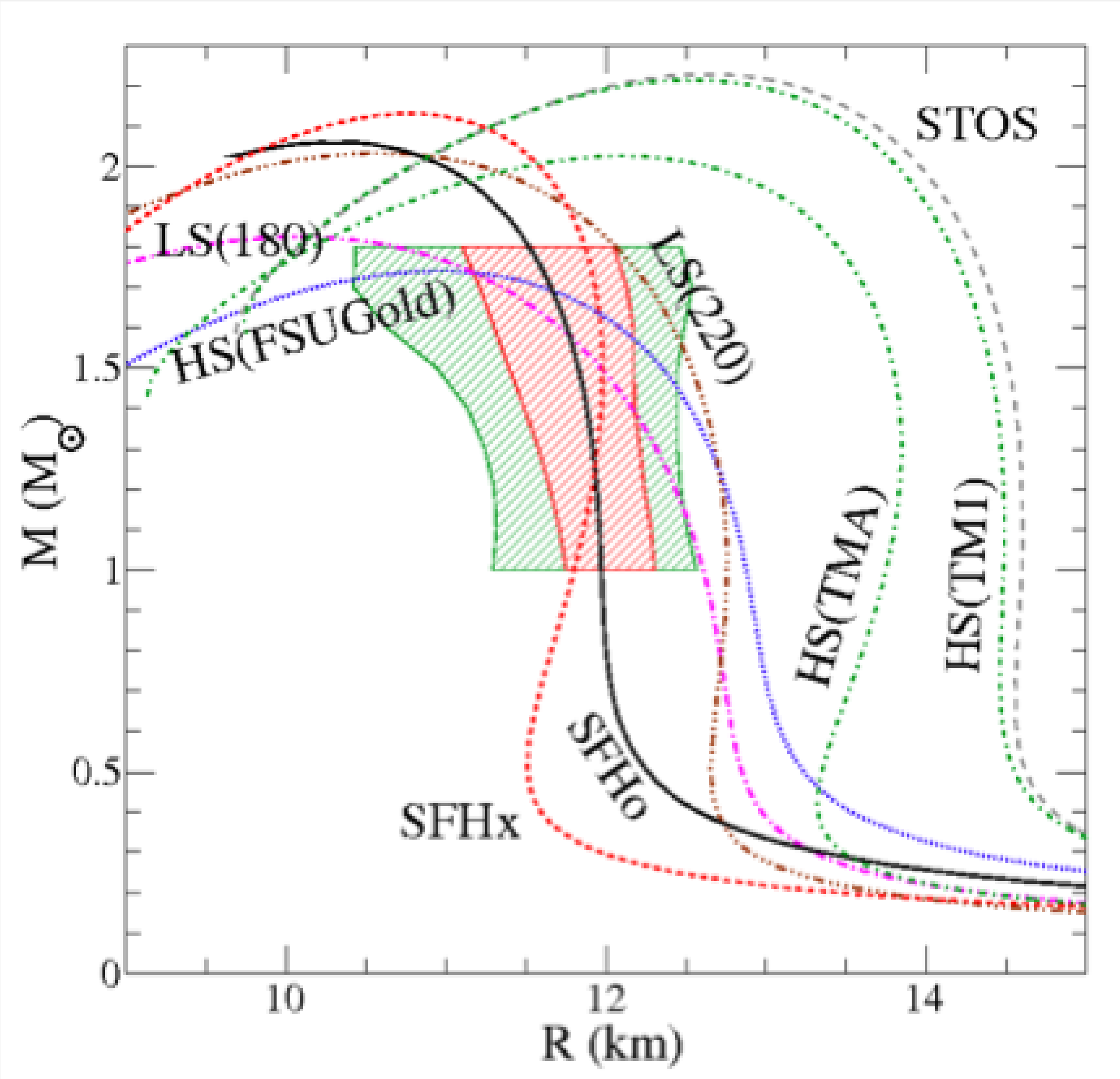


Jefferson Lab's Hall A: Measuring R_n

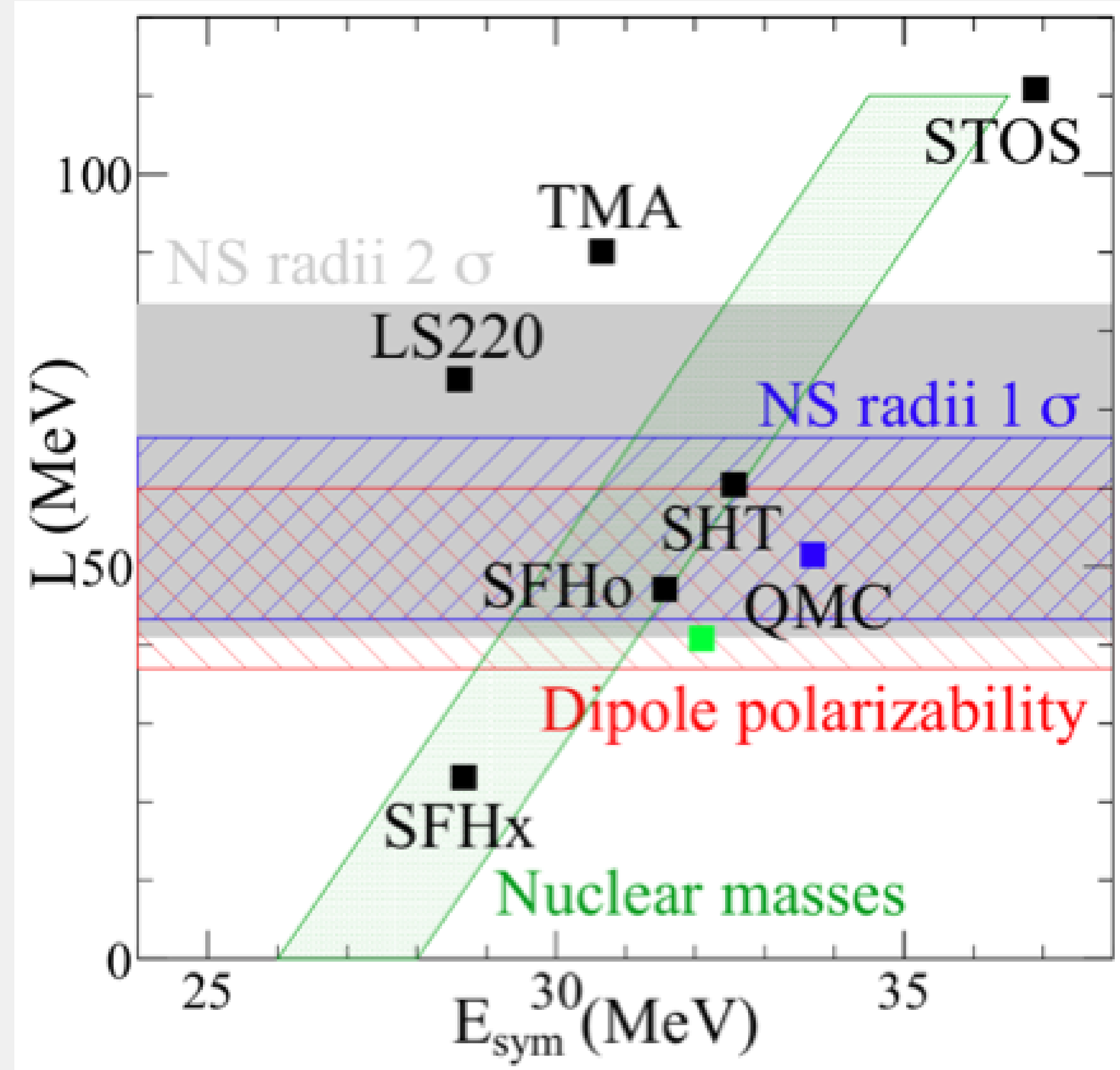
Steiner, Prakash, Lattimer, and Ellis (2005), based on Horowitz and Piekarewicz (2001)

- We find $\delta R < 0.2$ fm from neutron star observations

Supernova EOS and the Symmetry Energy



Steiner, Hempel, and Fischer (2013)

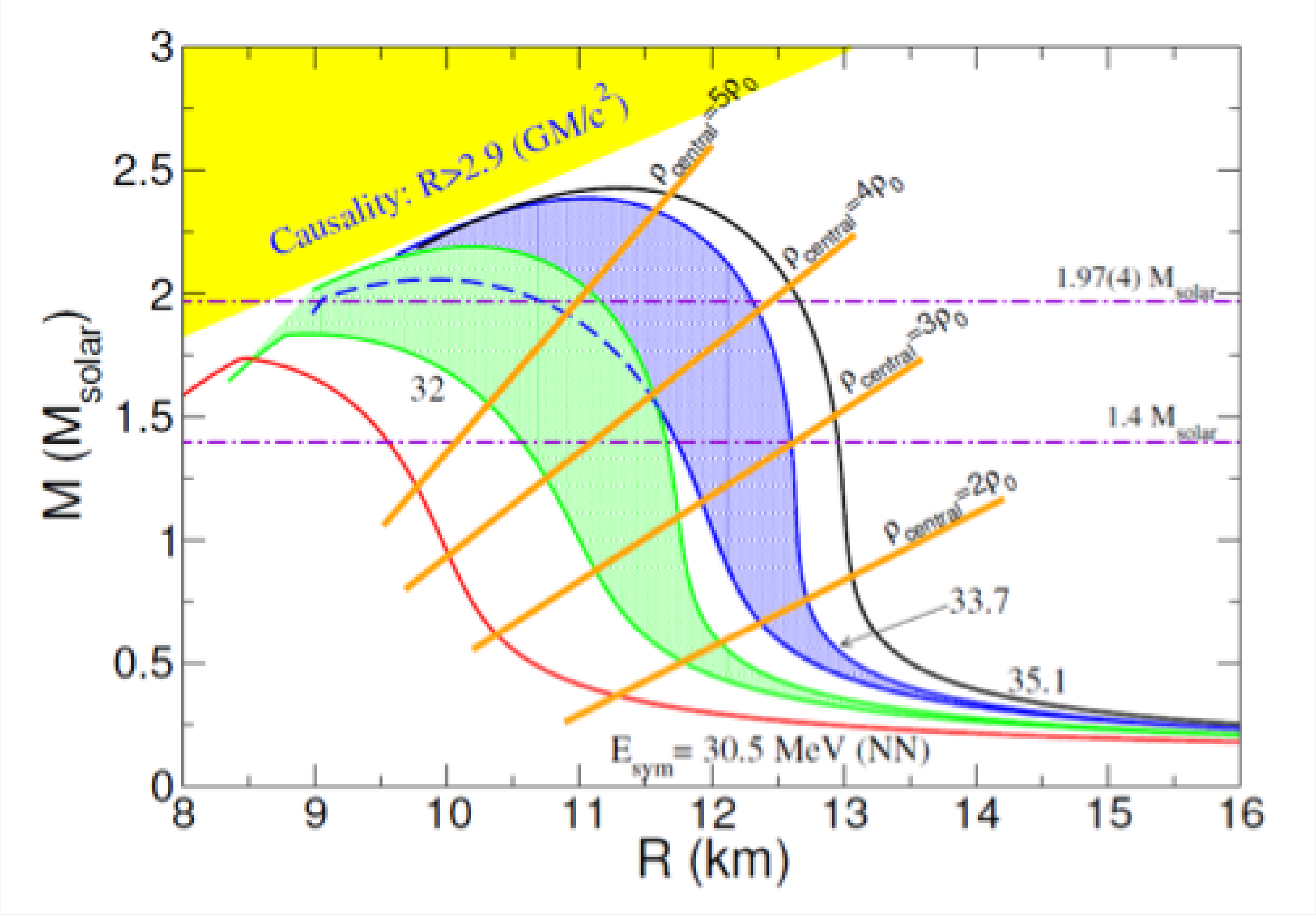


Based on Steiner, Hempel, and Fischer (2013)

- Limited number of supernova EOSs which satisfy $M-R$ constraints and the $S-L$ correlation
- Current EOS uncertainties too small to explain explosion
- Many simulation properties are weakly correlated with the symmetry energy

Connection to Nuclear Three-Body Forces

- Neutrons and protons are composite
- Build up a hierarchy: two-nucleon interactions, three-nucleon interactions, etc.



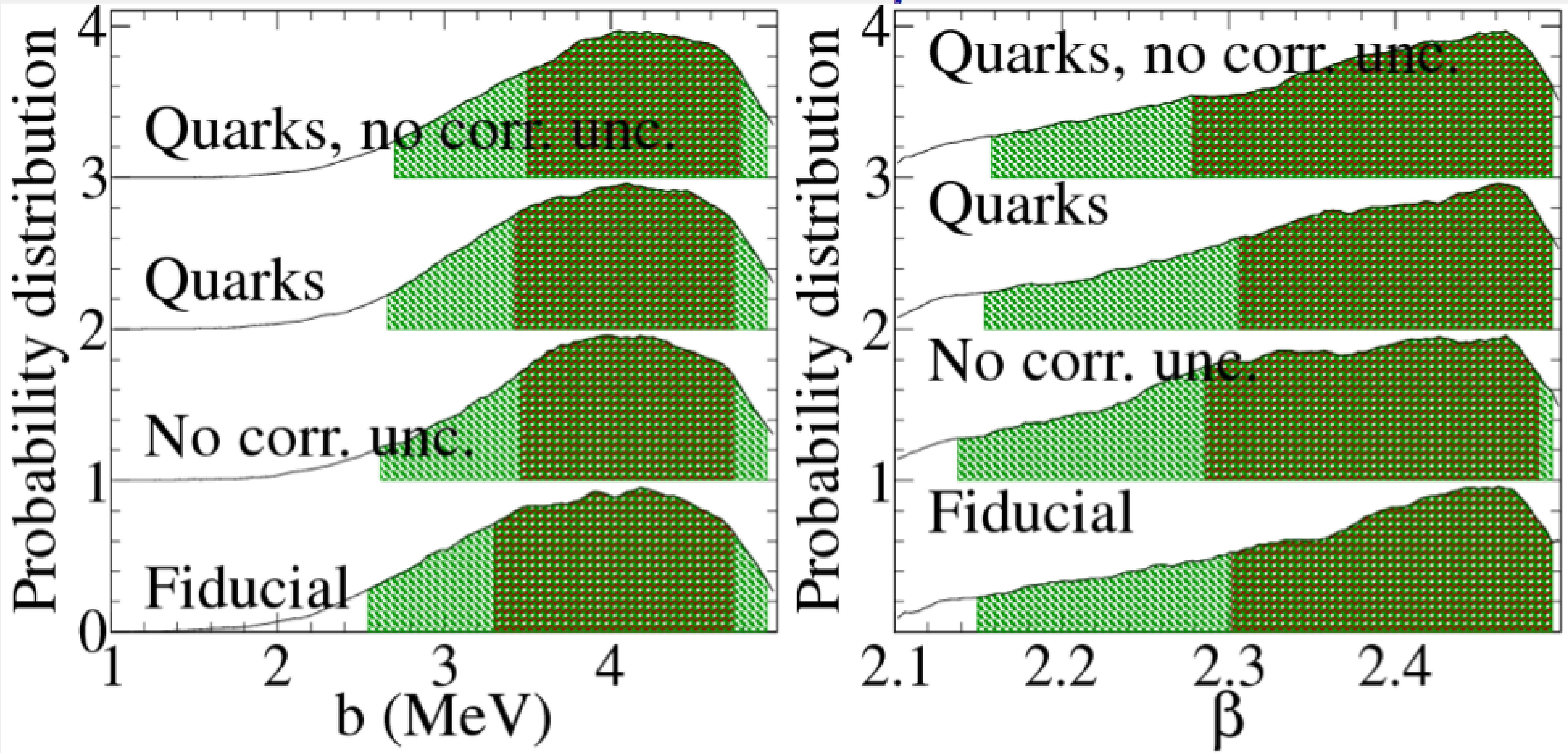
Colored regions denote different three-body forces

$$E_{\rm neut} = a\left(\frac{n}{n_0}\right)^{\alpha} + b\left(\frac{n}{n_0}\right)^{\beta}$$

Gandolfi, Carlson, and Reddy (2012)

- Three-nucleon interactions are important nuclei and neutron star radii

Constraints on Three-Body Force Parameters



Steiner and Gandolfi (2012)

- $E_{\mathrm{neut}} = a\left(\frac{n}{n_0}\right)^{\alpha} + b\left(\frac{n}{n_0}\right)^{\beta}$
- Values of a and α are unconstrained, but constraints on b and β
- Neutron star radii are constraining nuclear three-body forces

Summary

- Currently available neutron star mass and radius observations constrain the universal neutron star $M-R$ curve
  - Neutron star radii are likely between 10.4 and 13 km
- Constrain the nucleon-nucleon interaction and QCD.
  - 35 MeV $< L <$ 80 MeV
  - Neutron skin thickness is small, $<$ 0.2 fm
- Must attempt to address systematic uncertainties
- New EOS tables which respect neutron star observations
- Tension between large masses, small radii, and stiff EOSs
- More observations are needed
- ...in the mean time, statistical methods can help us connect experiment and observations