Lattice calculation of BSM B-parameters using improved staggered fermions in $N_f = 2 + 1$ unquenched QCD

Boram Yoon

Los Alamos National Laboratory

Oct 1, 2013

Let me introduce my self

- **Boram Yoon**
- **Profile**
  - Ph. D. in Physics (Feb, 2013)
    Seoul National University, Korea (Adv: Prof. Weonjong Lee)
  - Los Alamos National Lab (Aug, 2013)
- **Research Interests**
  - Lattice Gauge Theory (QCD)
  - Chiral Perturbation Theory
  - Data Analysis
  - High Performance Computing
- **Projects in LANL**
  - Neutron Electric Dipole Moments (nEDM)
  - Illuminating the Origin of the Nucleon Spin

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Collaboration

- **Seoul National University**
  - Yong-Chull Jang, Hwancheol Jeong, Jangho Kim, Jongjeong Kim, Kwangwoo Kim, Seonghee Kim, Weonjong Lee, Jaehoon Leem, Boram Yoon
- **University of Washington**
  - Stephen R. Sharpe
- **Brookhaven National Laboratory**
  - Hyung-Jin Kim, Chulwoo Jung
- **Korea Institute of Science and Technology Information**
  - Taegil Bae

Beyond the Standard Model B-parameters

Motivation & Background

Neutral Kaon System

- Flavor eigenstates

$$K^0 = (\bar{s}d), \qquad \overline{K}^0 = (s\bar{d})$$

- CP eigenstates

$$K_\pm = \frac{1}{\sqrt{2}}(K^0 \pm \overline{K}^0), \qquad CP|K_\pm\rangle = \pm|K_\pm\rangle$$

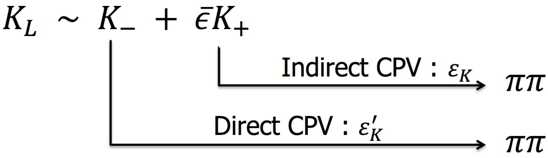
- Mass eigenstate

$$K_S = \frac{K_+ + \bar{\epsilon}K_-}{\sqrt{1+|\bar{\epsilon}|^2}}, \qquad K_L = \frac{K_- + \bar{\epsilon}K_+}{\sqrt{1+|\bar{\epsilon}|^2}}, \qquad |\bar{\epsilon}| \approx \mathcal{O}\left(10^{-3}\right)$$

- Preferable decays into pion states

$$K_S \to 2\pi(\text{ via } K_+, \text{ CP even })$$
$$K_L \to 3\pi(\text{ via } K_-, \text{ CP odd })$$

Direct / Indirect CP Violation



- CP violating $K_L \to \pi\pi$ can occur in two ways:
  - K_- (CP odd) $\to \pi\pi$ (CP even) : Direct CPV

$$\varepsilon'_K = \frac{1}{\sqrt{2}}\left(\frac{A[K_L \to (\pi\pi)_2]}{A[K_S \to (\pi\pi)_2]} - \varepsilon_K \frac{A[K_L \to (\pi\pi)_2]}{A[K_S \to (\pi\pi)_0]}\right)$$

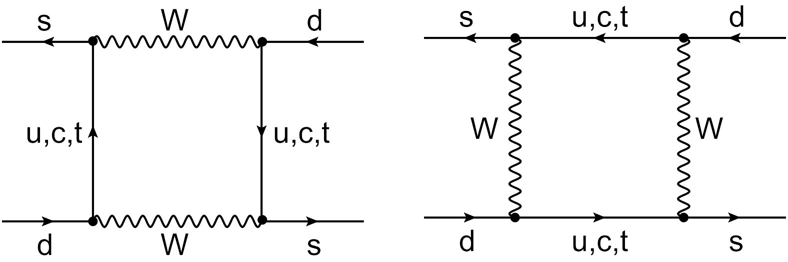
  - $\bar{\epsilon}K_+$ (CP even) $\to \pi\pi$ (CP even) : Indirect CPV

$$\varepsilon_K = \frac{1}{\sqrt{2}}\left(\frac{A[K_L \to (\pi\pi)_0]}{A[K_S \to (\pi\pi)_0]}\right)$$

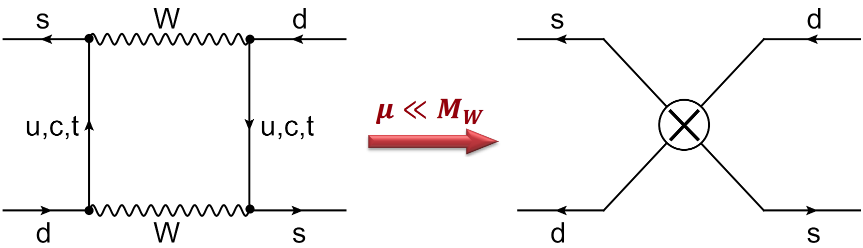
K_L can have small CP even component via $K^0 - \overline{K}^0$ mixing

$K^0 - \overline{K}^0$ Mixing in the Standard Model

- Arises from the $\Delta S = 2$, $s\bar{d} \to \bar{s}d$ FCNC
- Responsible for indirect CPV and $\Delta M_K \equiv M_{K_L} - M_{K_S}$
- Dominated by the following box diagrams:



$K^0 - \overline{K}^0$ Mixing in the Standard Model



- Integrating out heavy particles, the box diagram can be replaced by a local, four-quark operator

$$H_{\text{eff}}^{\Delta S=2} = \frac{G_F^2 M_W^2}{16\pi^2} F^0 Q_1 + h.c.$$

$$Q_1 = [\bar{s}\gamma_\mu(1-\gamma_5)d][\bar{s}\gamma_\mu(1-\gamma_5)d]$$

Kaon Bag Parameter – B_K

- In the SM, indirect CPV can be predicted as follows

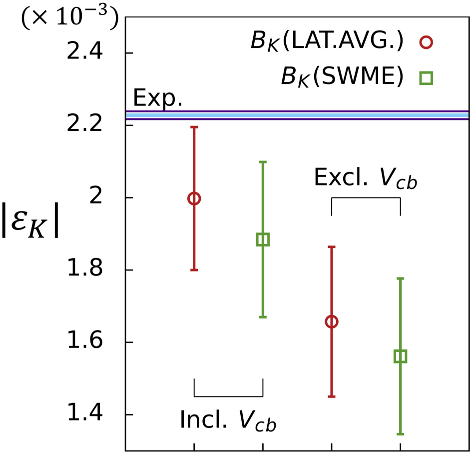
$$\varepsilon_K \sim \text{known factors} \times V_{\text{CKM}} \times \hat{B}_K$$

- $\hat{B}_K$ is the RG invariant form of B_K

$$B_K = \frac{\langle \overline{K}^0 | [\bar{s}\gamma_\mu(1-\gamma_5)d][\bar{s}\gamma_\mu(1-\gamma_5)d] | K^0 \rangle}{\frac{8}{3}\langle \overline{K}^0 | \bar{s}\gamma_\mu\gamma_5 d | 0 \rangle \langle 0 | \bar{s}\gamma_\mu\gamma_5 d | K^0 \rangle}$$

- B_K contains all the **non-perturbative QCD contribution** for ε_K, can be calculated from **lattice simulations**

Experiment vs SM prediction of ε_K



- There are two methods (exclusive, inclusive) to determine V_{cb}, whose results are somewhat **different**
- SM prediction of ε_K deviates from the experimental value about 3σ for exclusive V_{cb} channel

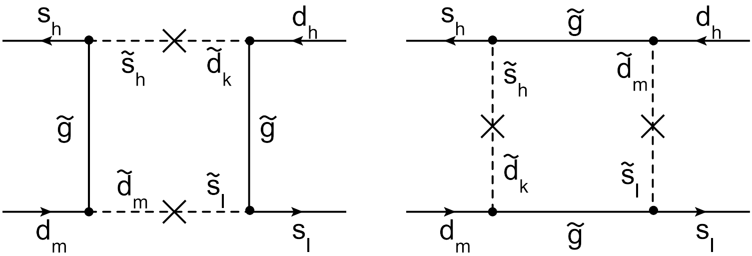
(Y. Jang & W. Lee, 2012)

BSM Contribution to $K^0 - \overline{K}^0$ Mixing

- In the Standard Model, only the "left–left" form contributes to the $K^0 - \overline{K}^0$ mixing box diagram

$$\langle \overline{K}^0 | [\bar{s}\gamma_\mu(1-\gamma_5)d][\bar{s}\gamma_\mu(1-\gamma_5)d] | K^0 \rangle$$

- Considering BSM physics, integrating out heavy particles (e.g. squarks and gluinos in supersymmetric models) leads to new operators with Dirac structures other than "left–left"



$h, k, l, m \in \{L, R\}$

BSM Operators

- Considering BSM, generic effective Hamiltonian is

$$H_{\text{eff}}^{\Delta S=2} = \sum_{i=1}^{5} C_i Q_i$$

$$\begin{aligned}
Q_1 &= [\bar{s}^a \gamma_\mu (1-\gamma_5) d^a][\bar{s}^b \gamma_\mu (1-\gamma_5) d^b] \\
Q_2 &= [\bar{s}^a (1-\gamma_5) d^a][\bar{s}^b (1-\gamma_5) d^b] \\
Q_3 &= [\bar{s}^a \sigma_{\mu\nu} (1-\gamma_5) d^a][\bar{s}^b \sigma_{\mu\nu} (1-\gamma_5) d^b] \\
Q_4 &= [\bar{s}^a (1-\gamma_5) d^a][\bar{s}^b (1+\gamma_5) d^b] \\
Q_5 &= [\bar{s}^a \gamma_\mu (1-\gamma_5) d^a][\bar{s}^b \gamma_\mu (1+\gamma_5) d^b]
\end{aligned}$$

- Once a BSM physics is chosen, C_i are determined
- If we know $\langle \overline{K}^0 | Q_i | K^0 \rangle$,
  we can calculate ε_K estimated by the BSM physics
- Comparing with experiments,
  we can give constraints on the BSM physics

BSM B-parameters

- **BSM B-parameters**

$$B_i = \frac{\langle \bar{K}_0 | Q_i | K_0 \rangle}{N_i \langle \bar{K}_0 | \bar{s}\gamma_5 d | 0 \rangle \langle 0 | \bar{s}\gamma_5 d | K_0 \rangle}$$

$$\begin{aligned}
Q_2 &= [\bar{s}^a(1-\gamma_5)d^a][\bar{s}^b(1-\gamma_5)d^b] \\
Q_3 &= [\bar{s}^a\sigma_{\mu\nu}(1-\gamma_5)d^a][\bar{s}^b\sigma_{\mu\nu}(1-\gamma_5)d^b] \\
Q_4 &= [\bar{s}^a(1-\gamma_5)d^a][\bar{s}^b(1+\gamma_5)d^b] \\
Q_5 &= [\bar{s}^a\gamma_\mu(1-\gamma_5)d^a][\bar{s}^b\gamma_\mu(1+\gamma_5)d^b]
\end{aligned}$$

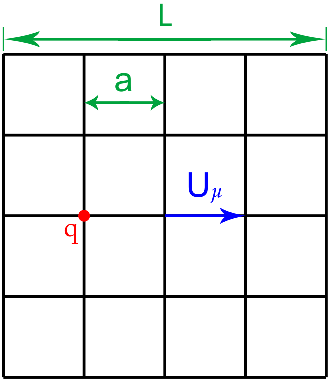
$$(N_2, N_3, N_4, N_5) = (5/3, 4, -2, 4/3)$$

- In the lattice calculation, forming dimensionless ratio reduces statistical and systematic error
- Chiral perturbation expression is simpler

Lattice QCD

Lattice QCD

- **Non-perturbative** approach to understand QCD
- Formulated on **discretized Euclidean space-time**
  - Hypercubic lattice
  - Lattice spacing “a”
  - Quark fields placed on sites
  - Gauge fields on the links between sites; U_μ



Lattice QCD

- **Expectation value**

$$
\begin{aligned}
&\langle \mathcal{O}(U, q, \bar{q}) \rangle \\
&\quad = \frac{1}{Z} \int \mathcal{D}\bar{q}\, \mathcal{D}q\, \mathcal{D}U \left[\mathcal{O}(U, q, \bar{q})\; e^{-S_g[U] - \sum_f \bar{q}_f \left(D[U] + m_f\right) q_f} \right] \\
&\quad = \frac{1}{Z} \int \mathcal{D}U \left[\mathcal{O}(U, (D[U] + m_f)^{-1})\; e^{-S_g[U]} \prod_f \det\left(D[U] + m_f\right) \right]
\end{aligned}
$$

- Integrating over the q and $\bar{q}$ gives
  determinant of Dirac operator, $\det\left(D[U] + m_f\right)$
  and quark propagators, $(D[U] + m_f)^{-1}$

Lattice QCD

- **Expectation value**

$$\langle \mathcal{O}(U, q, \bar{q}) \rangle$$
$$= \frac{1}{Z} \int \mathcal{D}U \left[\mathcal{O}(U, (D[U] + m_f)^{-1}) \, e^{-S_g[U]} \prod_f \det \left(D[U] + m_f \right) \right]$$

- **Numerical Integration**

  By generating random samples of gauge links, U_μ according to the probability distribution, one can perform the integration using the Monte Carlo method

$$\langle f(X) \rangle = \int dx \; f(x) p_X(x) \simeq \frac{1}{N} \sum_i f(x_i)$$

  where x_i are random samples of X

Lattice QCD

- Use **numerical method** (Monte Carlo simulation) to calculate integral

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int \mathcal{D}\bar{q}\, \mathcal{D}q\, \mathcal{D}U\ \mathcal{O}\ e^{-S}$$

- "**Lattice action**" is needed to simulate in discretized space-time

$$S[U, \bar{q}, q] = S_G[U] + S_F[U, \bar{q}, q]$$

- In this work, we use "**Staggered fermion**" for the lattice fermion
  - The **fastest** lattice fermion action
  - Suffered from "**taste symmetry breaking**", but manageable

Beyond the Standard Model B-parameters

Lattice Calculation of B-parameters & Data Analysis

Physical Results from Unphysical Simulations

- **Chiral Extrapolation**
  - In the lattice simulation, the smaller quark mass requires the exponentially larger computational cost

    $\Rightarrow$ Use light quark masses larger than physical light quark mass, and extrapolate to the physical light quark mass using chiral perturbation theory
  - Tuning the strange quark mass to precise physical quark mass is not practical

    $\Rightarrow$ Extrapolate to the physical strange quark mass
- **Continuum Extrapolation**
  - Simulation is done with finite lattice spacing ($a \gtrsim 0.045$ fm)

    $\Rightarrow$ Extrapolate to continuum limit, $a = 0$

Data Analysis Strategy

1. **Calculate raw data**
   Calculate BSM B-parameters for different quark mass combinations $(m_x,\ m_y)$

2. **Chiral fitting**
   X-fit: Fix strange quark mass, extrapolate $m_x \to m_d^{\text{phys}}$
   Y-fit: Extrapolate $m_y \to m_s^{\text{phys}}$

3. **RG Evolution**
   Obatin results at 2 GeV and 3 GeV from $\mu = 1/a$

4. **Continuum extrapolation**
   Repeat [1–3] for different lattices and extrapolate to $a = 0$

Analysis Data

Lattices generated with the $N_f = 2 + 1$ improved "asqtad" staggered action by the MILC collaboration

a (fm)	am_l/am_s	size	$1/a$ (GeV)	ens $\times$ meas	ID
0.09	$0.0062/0.031$	$28^3 \times 96$	2.3	995×9	F1
0.09	$0.0093/0.031$	$28^3 \times 96$	2.3	949×9	F2
0.09	$0.0031/0.031$	$40^3 \times 96$	2.3	959×9	F3
0.09	$0.0124/0.031$	$28^3 \times 96$	2.3	1995×9	F4
0.09	$0.00465/0.031$	$32^3 \times 96$	2.3	651×9	F5
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0.045	$0.0028/0.014$	$64^3 \times 192$	4.5	747×1	U1

Operator Matching

- To find continuum (NDR with $\overline{\text{MS}}$) results from those regularized on the lattice, "operator matching" is needed

- We use **one-loop** matching factors

  (J. Kim, W. Lee and S. Sharpe, 2011)

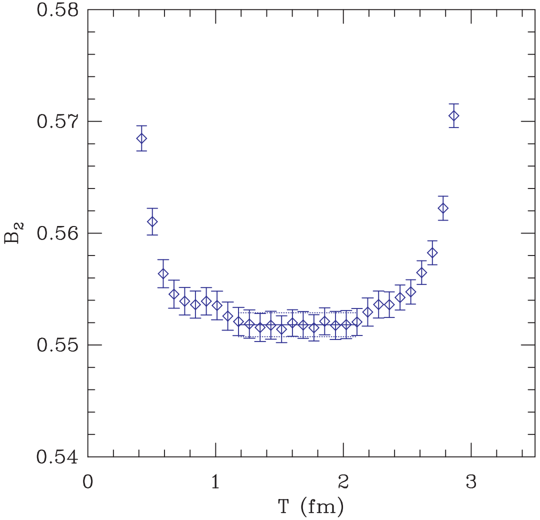
- Matching scale $\mu = 1/a$

$$\mathcal{O}_i^{\text{Cont}} = \sum_{j\in(A)} z_{ij}\mathcal{O}_j^{\text{Lat}} - \frac{g^2}{(4\pi)^2}\sum_{k\in(B)} d_{ik}^{\text{Lat}}\mathcal{O}_k^{\text{Lat}}$$

$$z_{ij} = b_{ij} + \frac{g^2}{(4\pi)^2}\Big(-\gamma_{ij}\log(\mu a) + d_{ij}^{\text{Cont}} - d_{ij}^{\text{Lat}} - C_F I_{MF} T_{ij}\Big)$$

Calculation of BSM B-parameters

$$B_i = \frac{\langle \bar{K}_0 | Q_i | K_0 \rangle}{N_i \langle \bar{K}_0 | \bar{s}\gamma_5 d | 0 \rangle \langle 0 | \bar{s}\gamma_5 d | K_0 \rangle} \quad \rightarrow \quad \frac{\langle W(t_1) Q_i(t) W(t_2) \rangle}{N'_i \, \langle W(t_1) P(t) \rangle \, \langle P(t) W(t_2) \rangle}$$



- B_2 calculated on F1 ($a = 0.09$ fm)
- Valence quark :
  $m_x = \frac{1}{10} m_s$
  $m_y = m_s$

Chiral Extrapolation of Valence Quark Masses

- **Valence quark masses**
  - $m_x = \left\{ \frac{n}{10} \times m_s \middle| n = 1, 2, 3, 4 \right\}$
  - $m_y = \left\{ \frac{n}{10} \times m_s \middle| n = 8, 9, 10 \right\}$
- **X-fit**
  - $m_x \to m_d^{\text{phys}}$ for fixed m_y
  - Use SU(2) Staggered ChPT ($m_x \ll m_y \sim m_s$)
- **Y-fit**
  - $m_y \to m_s^{\text{phys}}$
  - Assuming B_j are smooth functions of $m_y \propto Y_P = m_{y\bar{y}}^2$,

$$B_j = c_1 + c_2 Y_P$$

Chiral Extrapolation of Valence Quark Masses

- **Fitting functions for X-fit**

$$B_i(\text{NNNLO}) = c_1 F_0(j) + c_2 X_P + c_3 X_P^2 + c_4 X_P^2 \big(\ln(X_P)\big)^2 + c_5 X_P^2 \ln(X_P) + c_6 X_P^3$$

where $X_P = m_{x\bar{x}}^2 \; (= m_\pi^2)$

$$F_0(j) = 1 \pm \frac{1}{32\pi^2 f^2} \left\{ \ell(X_I) + (L_I - X_I)\tilde{\ell}(X_I) - 2\langle \ell(X_B) \rangle \right\}$$

($+$ for $j = 2, 3, K$, $-$ for $j = 4, 5$)

Bayesian constrained fitting with priors $c_{4-6} = 0 \pm 1$

Golden Combinations

- **Golden combinations**
  Combinations that cancel the leading chiral logarithms

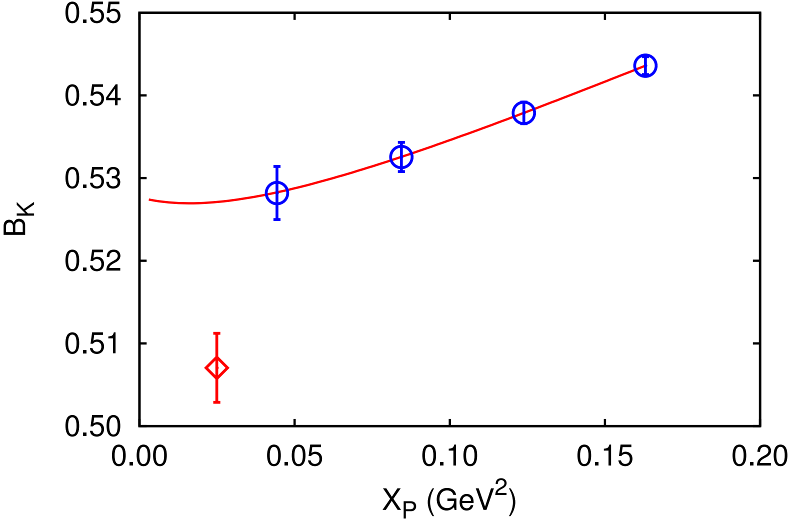
$$\frac{B_2}{B_3}, \quad \frac{B_4}{B_5}, \quad B_2 \cdot B_4, \quad \frac{B_2}{B_K}$$

- **Chiral fitting function for golden combinations**

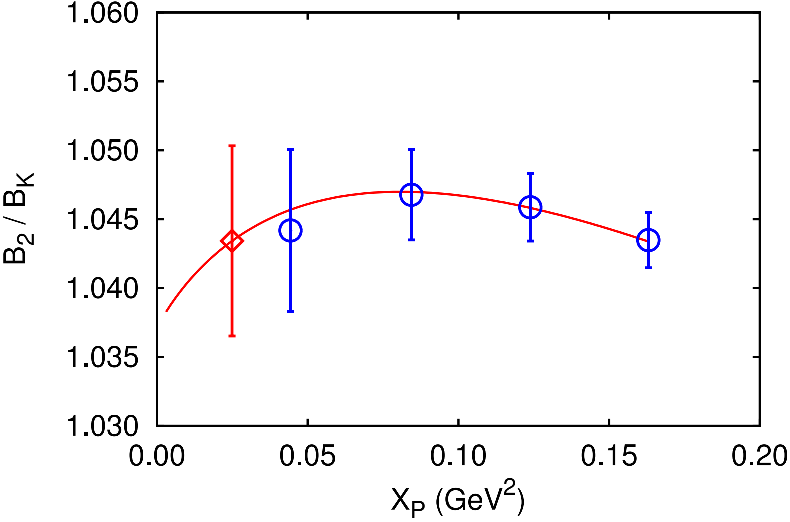
$$G_i(\text{NNNLO}) = c_1 + c_2 X_P + c_3 X_P^2 + c_4 X_P^2 \big(\ln(X_P)\big)^2 + c_5 X_P^2 \ln(X_P) + c_6 X_P^3.$$

- **BSM B-parameters from B_K and the Golden combinations**
  - Systematic error of the Golden combinations are small
  - We calculate BSM B-parameters from B_K and Golden combs

Chiral Fitting : X-fit of B_K on F1 Ensemble



Chiral Fitting : X-fit of B_2/B_K on F1 Ensemble



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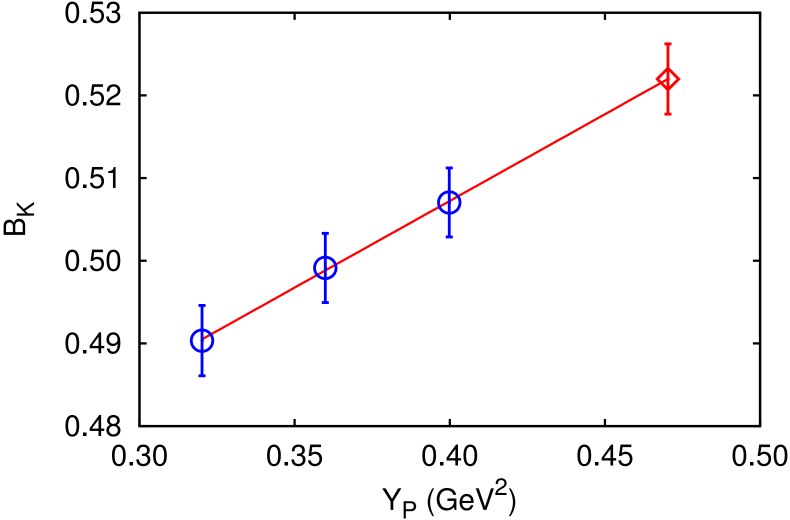

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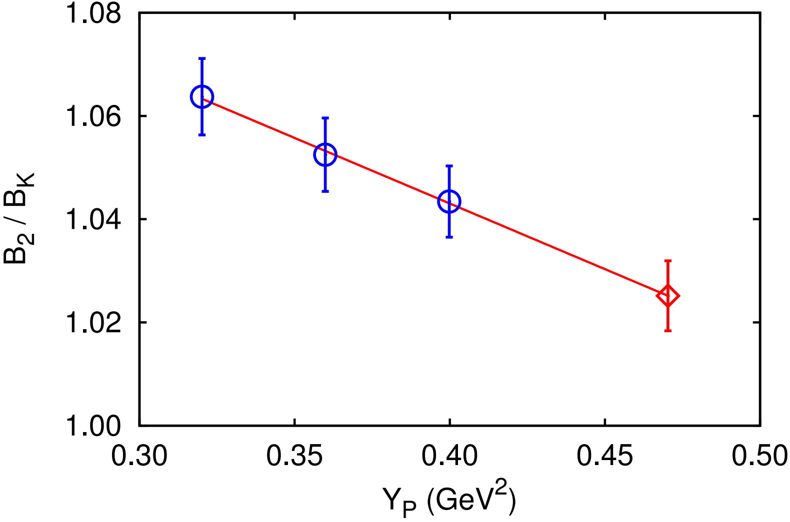

RG Evolution

- Now we have B-parameters evaluated at $\mu = 1/a$
- For continuum extrapolation with different lattice spacings, we need B-parameters at a common scale

---

$$B_i = \frac{\langle \bar{K}_0 | Q_i | K_0 \rangle}{N_i \langle \bar{K}_0 | \bar{s}\gamma_5 d | 0 \rangle \langle 0 | \bar{s}\gamma_5 d | K_0 \rangle}$$

- Remove N_i dependence by defining $R_i \equiv N_i B_i$
- RG running from $\mu_a(1/a)$ to $\mu_b(2\ \text{GeV}, 3\ \text{GeV})$

$$B_j(\mu_b) = \sum_k \frac{1}{N_j} W^R(\mu_b, \mu_a)_{jk} R_k(\mu_a)$$

where $W^R(\mu_b, \mu_a) = \dfrac{W^Q(\mu_b, \mu_a)}{[W^P(\mu_b, \mu_a)]^2}$

RG Evolution

- Evolution kernels satisfy the RG equation

$$\frac{dW(\mu_b, \mu_a)}{d \ln \mu_b} = -\gamma(\mu_b) W(\mu_b, \mu_a), \qquad W(\mu_a, \mu_a) = 1$$

$$\gamma(\mu) = \frac{\alpha(\mu)}{4\pi} \gamma^{(0)} + \left(\frac{\alpha(\mu)}{4\pi} \right)^2 \gamma^{(1)} + \cdots$$

- We use two-loop anomalous dimension (Buras, *et al.*, 2000)
- In the running, operator mixing arises in pairs: (Q_2, Q_3) and (Q_4, Q_5)
- RG evolution of the Golden combinations are calculated by using the similar method

Continuum & Sea Quark Mass Extrapolation

- **What we have done up to now**
  - Extrapolation of valence quark masses (m_x, m_y)
  - RG running to a common scale

- **What we need to do**
  - Extrapolation to continuum limit of $a = 0$
  - Extrapolation to physical sea quark masses

Lattice Ensembles

a (fm)	am_l / am_s	size	$1/a$ (GeV)	ens $\times$ meas	ID
0.09	0.0062 / 0.031	$28^3 \times 96$	2.3	995×9	F1
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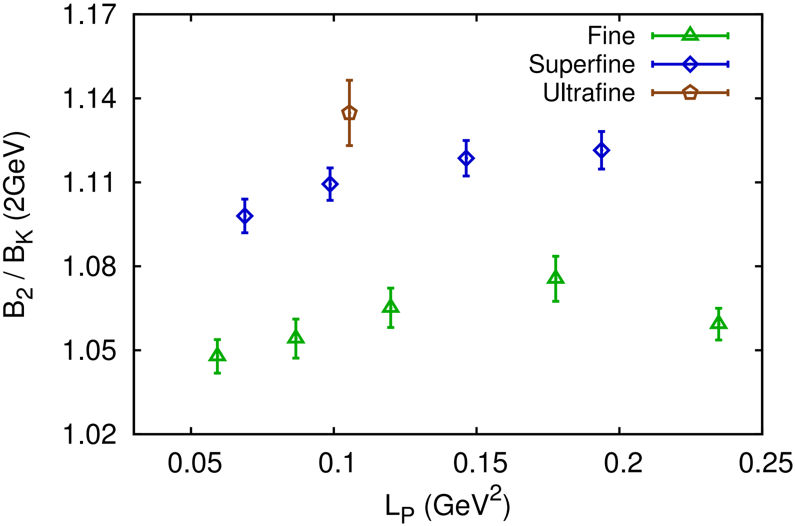
B_2/B_K Results on each Ensemble

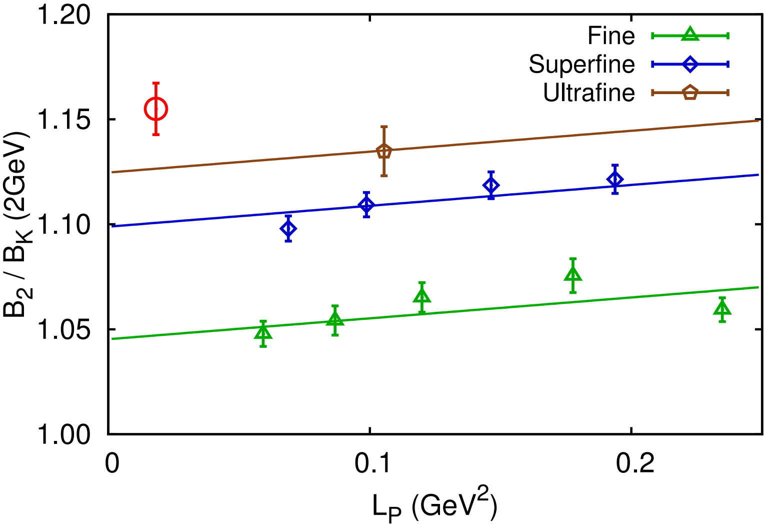


Continuum & Sea Quark Mass Extrapolation

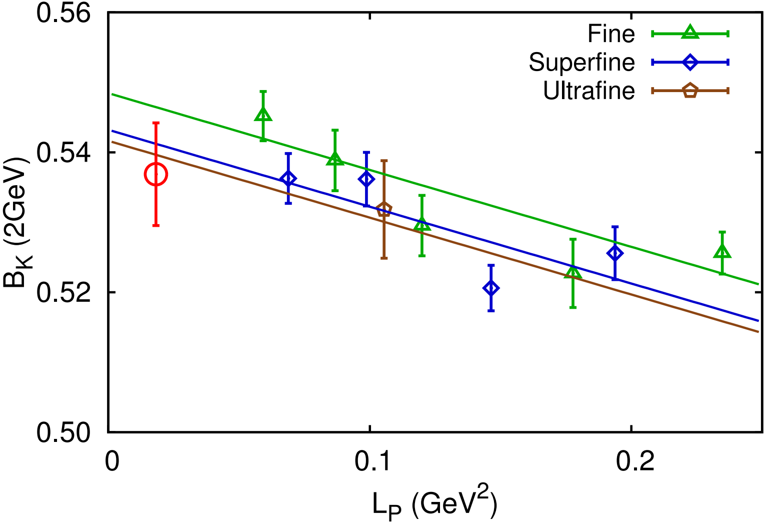
- **Simultaneous extrapolation**
  - (a, m_l, m_s) are extrapolated to their physical values, simultaneously
  - As proxies of quark masses (m_l and m_s), $L_P (= m^2_{l\bar{l}} \propto m_l)$ and $S_P (= m^2_{s\bar{s}} \propto m_s)$ are used
  - $a \to 0, \quad L_P \to m^2_{\pi_0}, \quad S_P \to m^2_{s\bar{s}}$
- **Fitting function**
  - Leading a and quark mass dependence is obtained by the Staggered Chiral Perturbation Theory (SChPT)
  - Power counding : $a^2 \sim m_q \sim m^2_{q\bar{q}}$

$$f_1 = c_1 + c_2\, a^2 + c_3\, L_P + c_4\, S_P$$

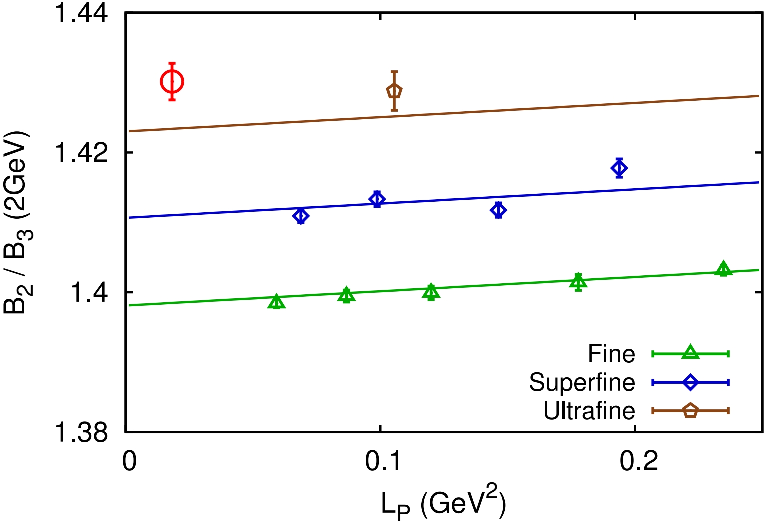
Continuum & Sea Quark Mass Extrapolation : $B_2 \ / \ B_K$



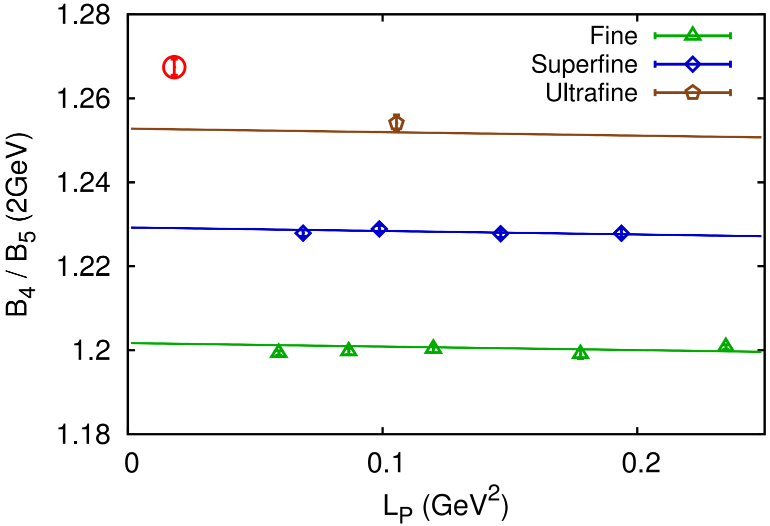
Continuum & Sea Quark Mass Extrapolation : B_K



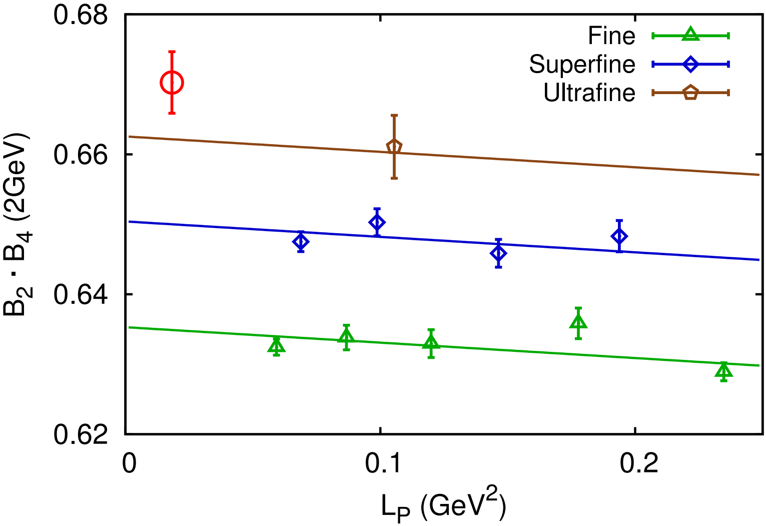
Continuum & Sea Quark Mass Extrapolation : B_2/B_3



Continuum & Sea Quark Mass Extrapolation : B_4/B_5



Continuum & Sea Quark Mass Extrapolation : $B_2 \cdot B_4$



Results

BSM B-parameters at 2GeV and 3GeV

	$\mu = 2$ GeV	$\mu = 3$ GeV
B_2	0.620 (4)(31)	0.549 (3)(28)
B_3	0.433 (3)(19)	0.390 (2)(17)
B_4	1.081 (6)(48)	1.033 (6)(46)
B_5	0.853 (6)(49)	0.855 (6)(43)

Error Budget

(unit: %)

cause	B_2	B_3	B_4	B_5	memo
statistics	0.64	0.63	0.60	0.66	Statistical
{ matching cont-extrap. }	4.95	4.40	4.40	5.69	(f_1 vs. f_2) or α_s^2
fitting (1)	0.10	0.10	0.12	0.12	X-fit
fitting (2)	0.12	0.19	0.22	0.16	Y-fit
finite volume	0.50	0.50	0.50	0.50	$m_\pi L = 4.4$ vs. 6.27
r_1	0.18	0.17	0.05	0.02	$r_1 = 0.3117(22)$ fm
f_π	0.46	0.46	0.46	0.46	132MeV vs. 124.2MeV

Matching and Continuum Extrapolation Error

- **Matching Error**
  - Operator matching has been done at one-loop level
  - Ignored highest order term is order $\mathcal{O}(\alpha_s^2)$
  - On the ultrafine lattice, $\alpha_s^2 = 4.4\%$
- **Continuum Extrapolation Error**
  - We fit the data with

$$f_1 = c_1 + c_2\, a^2 + c_3\, L_P + c_4\, S_P$$

$$f_2 = c_1 + c_2\, a^2 + c_3\, L_P + c_4\, S_P + c_5\, a^2\alpha_s + c_6\, \alpha_s^2 + c_7\, a^4$$

  - The difference of f_1 and f_2 can be considered as a systematic error in continuum extrapolation
- **Matching and Continuum Extrapolation Error**
  - Since "$f_1 - f_2$" and "α_s^2" are highly correlated, we quote the bigger of them as matching–continuum extrapolation error

Summary

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- BSM physics leads to new $\Delta S = 2$ four-fermion operators that contribute to $K^0 - \overline{K}^0$ mixing
- Calculating corresponding hadronic matrix elements, $\langle \overline{K}^0 | Q_i | K^0 \rangle$, can impose strong constraints on BSM physics
- We calculate BSM B-parameters on the lattice with about 5% error