Four-Quark Mesons?

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A Mesonic Analog of the Deuteron

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Mesons Are Made of Quarks

I. They are colorless objects with $B = 0$.

II. Usually $q\,\bar{q}$.

III. But why not $q\,q\,\bar{q}\,\bar{q}$?

IV. Certainly allowed by QCD.

V. **Some hints** in the exotic spectrum, e.g.,

X(3872) has $J^{PC} = 1^{++}$, now confirmed.

Could it be $c\,\bar{c}\,u\,\bar{u}$? Or hybrid with gluons)?

Y(4260)? Z_c(3900)?

We'll Consider $b\, c\, \bar{u}\, \bar{d}$

- A bound state of a $B^- = (b\bar{u})$ and a $D^0 = (c\bar{d})$?
- Let them collide and see what happens.
- No need to antisymmetrize – quarks all different.
- The b and c quarks are **heavy** – 4180 MeV/c and 1500 MeV/c, heavier than a proton.
- They provide confining potentials for the light $\bar{u}$ and $\bar{d}$ quarks.
- For us ”light” means massless, hence **relativistic**.
- Like Hydrogen molecule in Born-Oppenheimer approximation.
- We work in the **relativistic** Los Alamos Model Potential of Goldman *et al*.

Take Confinement as Linear

Actually, there are **two** linear potentials:

$S(r) = \kappa^2 r \to r$, dimensionless, as is r

$V(r) = \kappa^2 (r - R) \to r - R$

$\kappa = 2.152 \text{ fm}^{-1}$ and $R = 1.92$ from fitting charmonia

S is a Lorentz scalar, V is 4th component of a Lorentz vector.

Parallel slopes to reduce spin-orbit contribution (PGG).

No Coulomb-like component in V. (see our "Convolve" paper).

Light Quark Wave Functions

Dirac's four-component wave function:

$$\Psi_{jlm} = \begin{bmatrix} \psi_{l,a}(r) \\ -i\vec{\sigma}\cdot\hat{r}\ \psi_{l',b}(r) \end{bmatrix}, \qquad l' = 2j - l$$

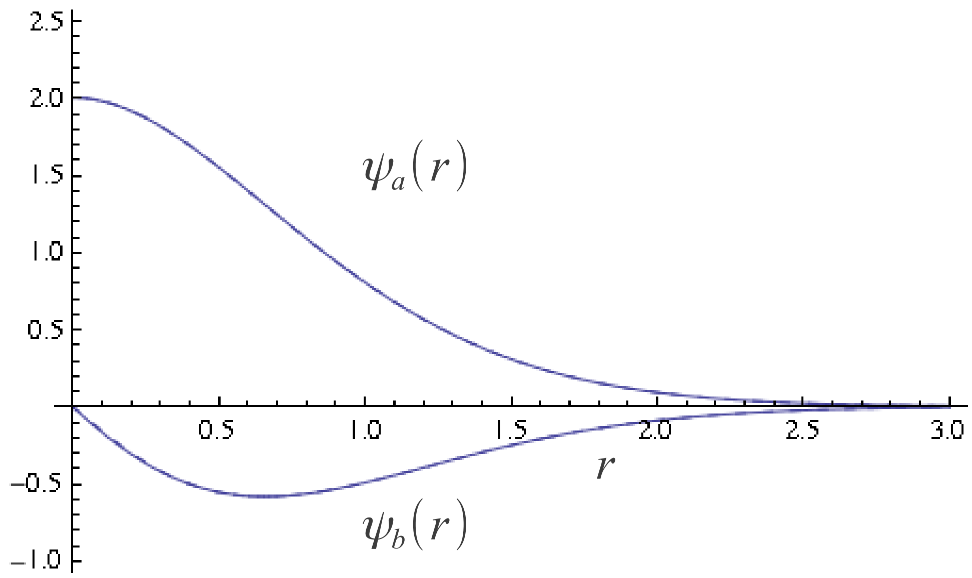
(times ang. mom. and spin factors)

We'll assume the *u* and *d* quarks are massless. Also, ignore small E&M corrections.

Solve the Dirac equation with *S*(*r*) and *V*(*r*) for the radial g.s. wave functions $\psi_a(r)$ and $\psi_b(r)$ for *u* or *d* in a single well.

Can chose ψ 's to be real.

The Light Quark W. Fcns. (II)



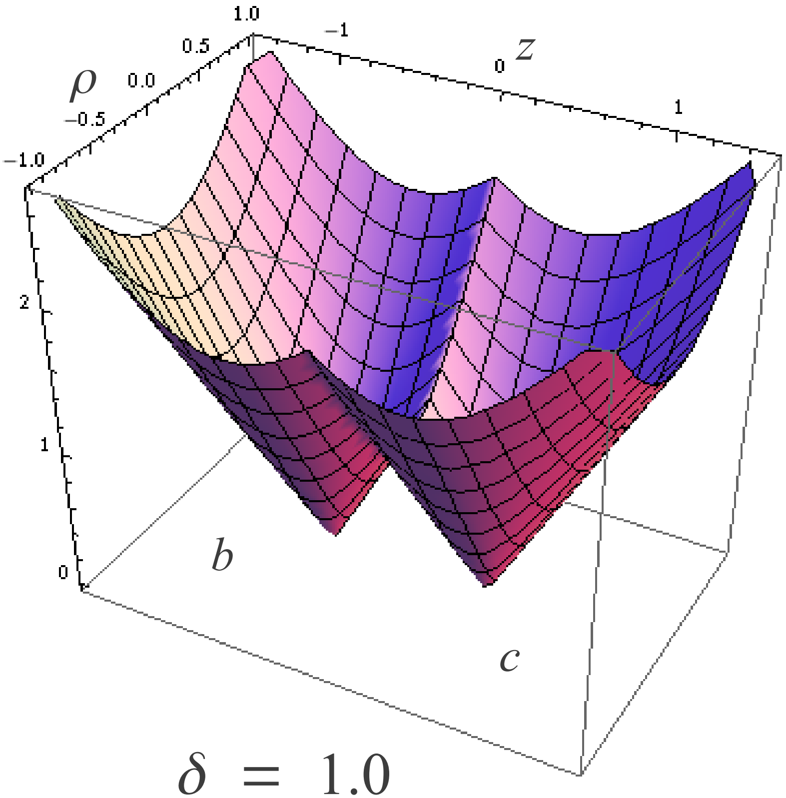
Fit the solutions as a sum of Gaussians:

$$\psi_a(r) = \sum_{i=1}^{6} a_i \exp(-\mu_i r^2/2) \qquad \psi_b(r) = \sum_{i=1}^{6} b_i \exp(-\mu_i r^2/2)$$

I won't bore you with the values of the parameters here.

The fits (dashed) overlay the solutions (solid).

The Two-Well Potential – I



$\delta = 1.0$

Cylindrical coordinates, ρ and z

$$\mathbf{r}_\pm = \{x, y, z \pm \delta\}$$

$$\rho^2 = x^2 + y^2$$

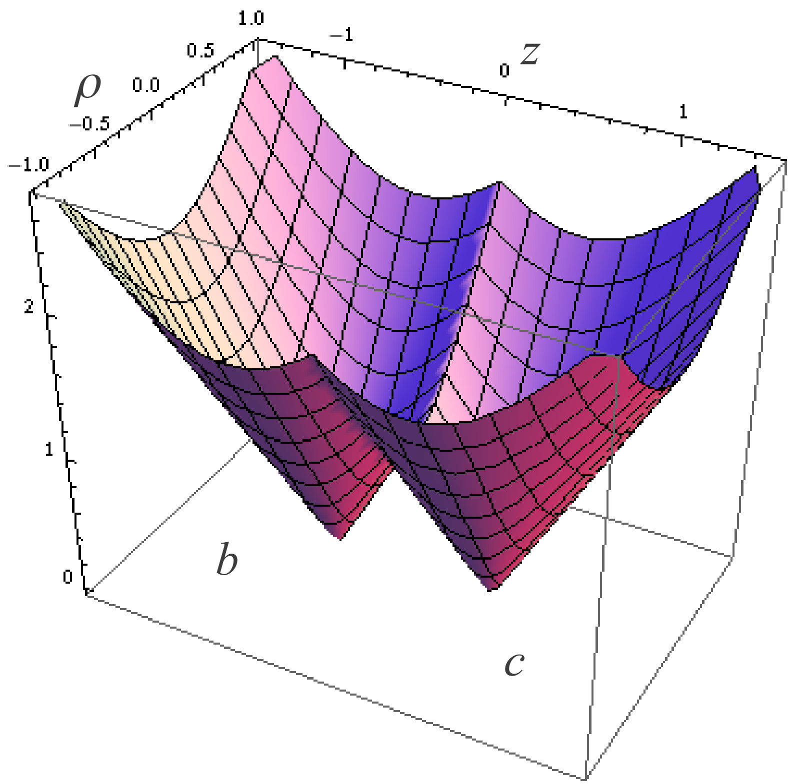
For the scalar potentials from the b at $\mathbf{r}_+$ and the c at $\mathbf{r}_-$

.

$$V(\mathbf{r}) = \begin{cases} r_- - R, & \text{if } z > 0 \\ r_+ - R, & \text{if } z < 0 \end{cases}$$

Similarly for $S(\mathbf{r})$, without the R.

The Two-Well Potential – II



Quark $\bar{u}$ on left (initially bound to *b*) can tunnel through to the *c* on the right. And vice versa for $\bar{d}$.

Delocalization can (might) lead to binding.

In principle, should solve for $\Psi(\vec{r})$ in this two-well potential for both $S(\vec{r})$ and $V(\vec{r})$.

That's very hard to do!

Go to a variational approximation.

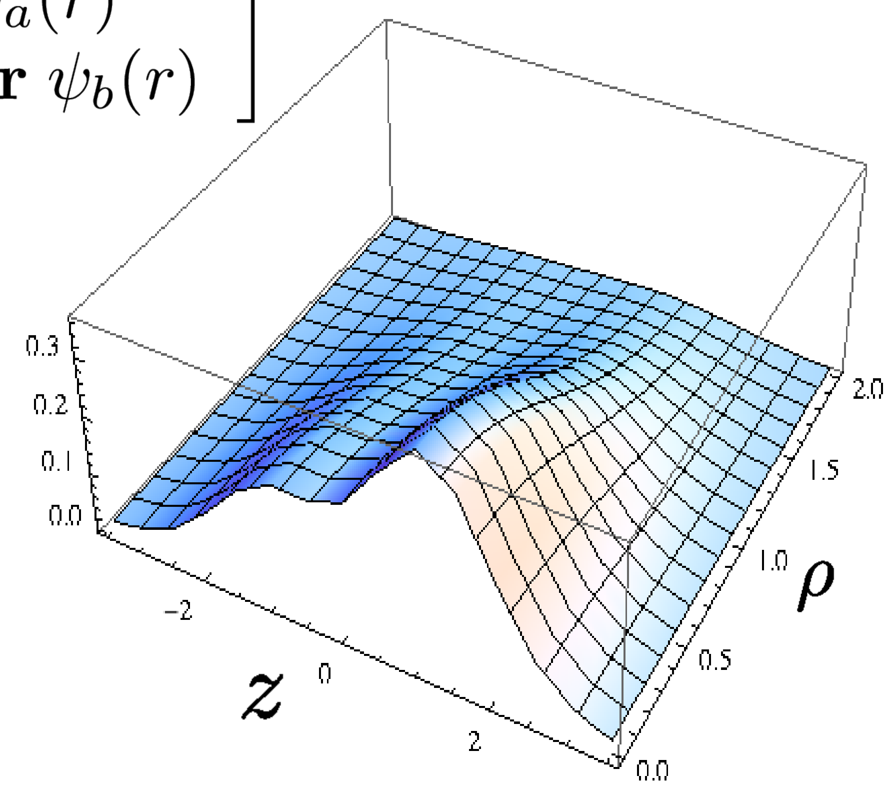
Our Variational Wave Function

Two parameters, ϵ and δ :

$$\Psi_{\text{trial}} = \Psi(\rho, z-\delta) + \epsilon\, \Psi(\rho, z+\delta)$$

1s g.s. $$\Psi(\mathbf{r}) = \frac{1}{\sqrt{4\pi}} \begin{bmatrix} \psi_a(r) \\ i\boldsymbol{\sigma}\cdot\mathbf{r}\,\psi_b(r) \end{bmatrix}$$

E.g., ψ_a for $\delta = 1.0$
and $\epsilon = 0.5$



What parameters minimize H_D^2?

- Need H_D^2 not H_D to avoid negative energy states.
- 3D plot versus ϵ and δ to look for that minimum.
- Take square root to find best variational energy of the B and D system. Does it bind?

$$H_D = -i\boldsymbol{\alpha}\cdot\nabla + V(\mathbf{r}) + \beta\, S(\mathbf{r})$$

$$H_D^2 = -\nabla^2 + V^2(\mathbf{r}) + S^2(\mathbf{r}) + 2\beta\, V(\mathbf{r})\, S(\mathbf{r})$$
$$-i\boldsymbol{\alpha}\cdot\left[(\nabla V(\mathbf{r})) + \beta\,(\nabla S(\mathbf{r}))\right] - 2i\, V(\mathbf{r})\,\boldsymbol{\alpha}\cdot\nabla$$

Top line is diagonal.
Lower line is off-diagonal.

$$\boldsymbol{\alpha} = \begin{bmatrix} 0 & \boldsymbol{\sigma} \\ \boldsymbol{\sigma} & 0 \end{bmatrix} \qquad \beta = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

Need Expectation Values

$$< \Psi_{\mathrm{trial}}(\epsilon, \delta) \mid H_D^2 \mid \Psi_{\mathrm{trial}}(\epsilon, \delta) >$$

Proceed piece by piece, each term in H_D^2.

Integrals of Gaussians over ρ and z .

Diagonal upper-components easier (somewhat simpler) than diagonal lower-components. Off-diagonal pieces, connecting upper and lower components are the most difficult and the messiest.

Details in the archived paper (submitted to PRC).

Dependence on ϵ is Quadratic

$$\Psi_{\rm trial}(\mathbf{r}, \epsilon, \delta) = \Psi(\mathbf{r}_-) + \epsilon\, \Psi(\mathbf{r}_+)$$

$$\begin{aligned} < \Psi_{\rm trial} \mid \mathcal{O} \mid \Psi_{\rm trial} > &= < \Psi(\mathrm{r}_-) \mid \mathcal{O}^{(0)} \mid \Psi(\mathrm{r}_-) > + \, 2\, \epsilon < \Psi(\mathrm{r}_-) \mid \mathcal{O}^{(1)} \mid \Psi(\mathrm{r}_+) > \\ &\quad + \epsilon^2 \; < \Psi(\mathrm{r}_+) \mid \mathcal{O}^{(2)} \mid \Psi(\mathrm{r}_+) > \\ &= (1 + \epsilon^2) < \Psi(\mathrm{r}_-) \mid \mathcal{O}^{(0)} \mid \Psi(\mathrm{r}_-) > + \, 2\, \epsilon < \Psi(\mathrm{r}_-) \mid \mathcal{O}^{(1)} \mid \Psi(\mathrm{r}_+) > \end{aligned}$$

by symmetry under $\delta \rightleftharpoons -\delta$.

The direct expectation $< \mathcal{O}^{(0)} >$ is simpler than the cross-term expectation $< \mathcal{O}^{(1)} >$.

Three Kinds of Integrals

$$< \Psi(\mathbf{r}_-) \mid \mathcal{O}^{(n)}_{\rm diag} \mid \Psi\ (\mathbf{r}_-) > = \sum_{i,j} a_i a_j I^{(n)}_{ij} + \sum_{i,j} b_i b_j J^{(n)}_{ij}$$

$$< \Psi(\mathbf{r}_-) \mid \mathcal{O}^{(n)}_{\rm offdiag} \mid \Psi\ (\mathbf{r}_-) > = \sum_{i,j} a_i b_j K^{(n)}_{ij}$$, where

$$I^{(0)}_{ij} = \frac{1}{4\pi} \int d^3r\ e^{-\mu_i\, r_-^2/2}\ \mathcal{O}_{\rm diag}\ e^{-\mu_j\, r_-^2/2}$$

$$J^{(0)}_{ij} = \frac{1}{4\pi} \int d^3r\ e^{-\mu_i\, r_-^2/2}\ \boldsymbol{\sigma} \cdot \mathbf{r}_-\ \mathcal{O}_{\rm diag}\ \boldsymbol{\sigma} \cdot \mathbf{r}_-\ e^{-\mu_j\, r_-^2/2}$$

$$K^{(0)}_{ij} = \frac{1}{4\pi} \int d^3r\ e^{-\mu_i\, r_-^2/2}\ \mathcal{O}_{\rm offdiag}\ (-i\boldsymbol{\sigma} \cdot \mathbf{r}_-)\ e^{-\mu_j\, r_-^2/2}$$

and similarly for the (1) integrals.

Doing the Integrals

- Expectations are integrals over ρ and z .
- Do the ρ integration first; independent of δ .
- The z integration does dependent on δ .
- Split that integration into two halves.
- Do the $z > 0$ integration with $r_\pm \to r_-$.
- And the $z < 0$ integration with $r_\pm \to r_+$.
- Expect Erf's and Erfc's from the partial integrations over the Gaussians.
- As I said earlier, it can get pretty messy.

Example: First Off-Diagonal Term

$$\nabla V(\mathbf{r}) \;=\; \nabla S(\mathbf{r}) = \begin{cases} \hat{\mathbf{r}}_- & \text{if } z > 0 \\ \hat{\mathbf{r}}_+ & \text{if } z < 0 \end{cases}$$

$$\begin{aligned}\mathcal{O}_{\text{offdiag}} &= -i\boldsymbol{\alpha}\cdot\left[(\nabla V(\mathbf{r})) + \beta\,(\nabla S(\mathbf{r}))\right] \\ &= \begin{bmatrix} 0 & 0 \\ -2i\,\boldsymbol{\sigma}\cdot\hat{\mathbf{r}}_\pm & 0 \end{bmatrix}\end{aligned}$$

$$\begin{aligned} K^{(0)}_{ij,<\nabla VS>} &= -2\,\frac{1}{4\pi}\int d^3r\; e^{-\mu_i\, r_-^2/2}\;\left[(\boldsymbol{\sigma}\cdot\mathbf{r}_-)(\boldsymbol{\sigma}\cdot\hat{\mathbf{r}}_\pm)\right]\; e^{-\mu_j\, r_-^2/2} \\ &= \; -\frac{2}{(\mu_i+\mu_j)^2}\left[2 - e^{-(\mu_i+\mu_j)\,\delta^2/2}\right] \\ &\quad -\frac{1}{\delta}\left[\frac{2\pi}{(\mu_i+\mu_j)^5}\right]^{1/2}\left[\mathrm{Erf}\left(\sqrt{2(\mu_i+\mu_j)}\,\delta\right) - \mathrm{Erf}\left(\sqrt{(\mu_i+\mu_j)/2}\,\delta\right)\right]\end{aligned}$$

Another Example:

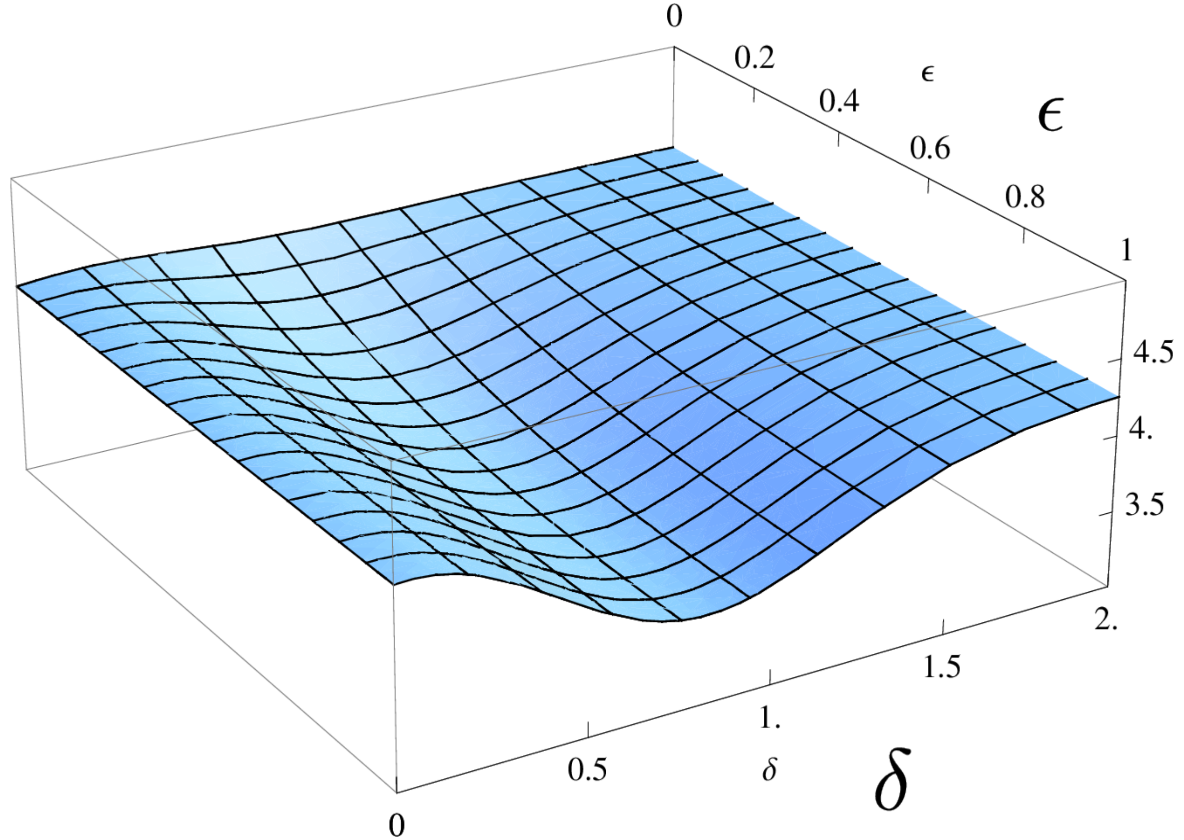
$$
\begin{aligned}
K^{(1)}_{ij,<\nabla VS>} = \; & \frac{1}{\mu_i \mu_j (\mu_i + \mu_j)^2} \Big[2\,\mu_j\,(\mu_j - \mu_i)\,e^{-2\mu_i\,\delta^2} + 2\,\mu_i\,(\mu_i - \mu_j)\,e^{-2\mu_j\,\delta^2} \\
& \qquad - (\mu_i - \mu_j)^2\,e^{-(\mu_i+\mu_j)\,\delta^2/2} \Big] \\
& - \frac{1}{2\,\delta\,\mu_i^2\mu_j^2} \left[\frac{\pi}{2(\mu_i + \mu_j)^5} \right]^{1/2} \times \\
& \Big\{ (\mu_i + \mu_j)^3 \left(\mu_i + \mu_j - 2\mu_i\mu_j\,\delta^2 \right) \,\mathrm{Erfc}\left(\sqrt{(\mu_i + \mu_j)/2}\,\delta \right) \\
& + 2\,\mu_i^2 \left[(\mu_i^2 + 4\mu_i\mu_j + 3\mu_j^2) - 4\mu_j^2(\mu_i - \mu_j)\,\delta^2 \right] \\
& \qquad \times e^{-2\mu_i\mu_j\,\delta^2/(\mu_i+\mu_j)}\,\mathrm{Erf}\left(\sqrt{2/(\mu_i + \mu_j)}\,\mu_j\,\delta \right) \\
& - 2\,\mu_j^2 \left[3\mu_i^2 + 4\mu_i\mu_j + \mu_j^2 - 4\mu_i^2(\mu_j - \mu_i)\,\delta^2 \right] \\
& \qquad \times e^{-2\mu_i\mu_j\,\delta^2/(\mu_i+\mu_j)}\,\mathrm{Erfc}\left(\sqrt{2/(\mu_i + \mu_j)}\,\mu_i\,\delta \right) \\
& - \left[(\mu_i^3 + 5\,\mu_i^2\mu_j + 5\,\mu_i\mu_j^2 + \mu_j^3) - 8\,\mu_i^2\mu_j^2\,\delta^2 \right] \\
& \qquad \times (\mu_i - \mu_j)\,e^{-2\mu_i\mu_j)\,\delta^2/(\mu_i+\mu_j)}\,\mathrm{Erfc}\left(\frac{(\mu_i - \mu_j)\,\delta}{\sqrt{2(\mu_i + \mu_j)}} \right) \Big\}
\end{aligned}
$$

Putting It All Together

- So, find all the *I* 's, *J* 's, and *K* 's for all the terms in H_D^2 .
- Need also to calculate the normalization of $\Psi_{\mathbf{trial}}$ as a function of ϵ and δ .
- Call it $N(\epsilon, \delta)$.
- Don't forget to divide by $N^2(\epsilon, \delta)$.
- And finally make 3D plots to look for a minimum in ϵ and δ .

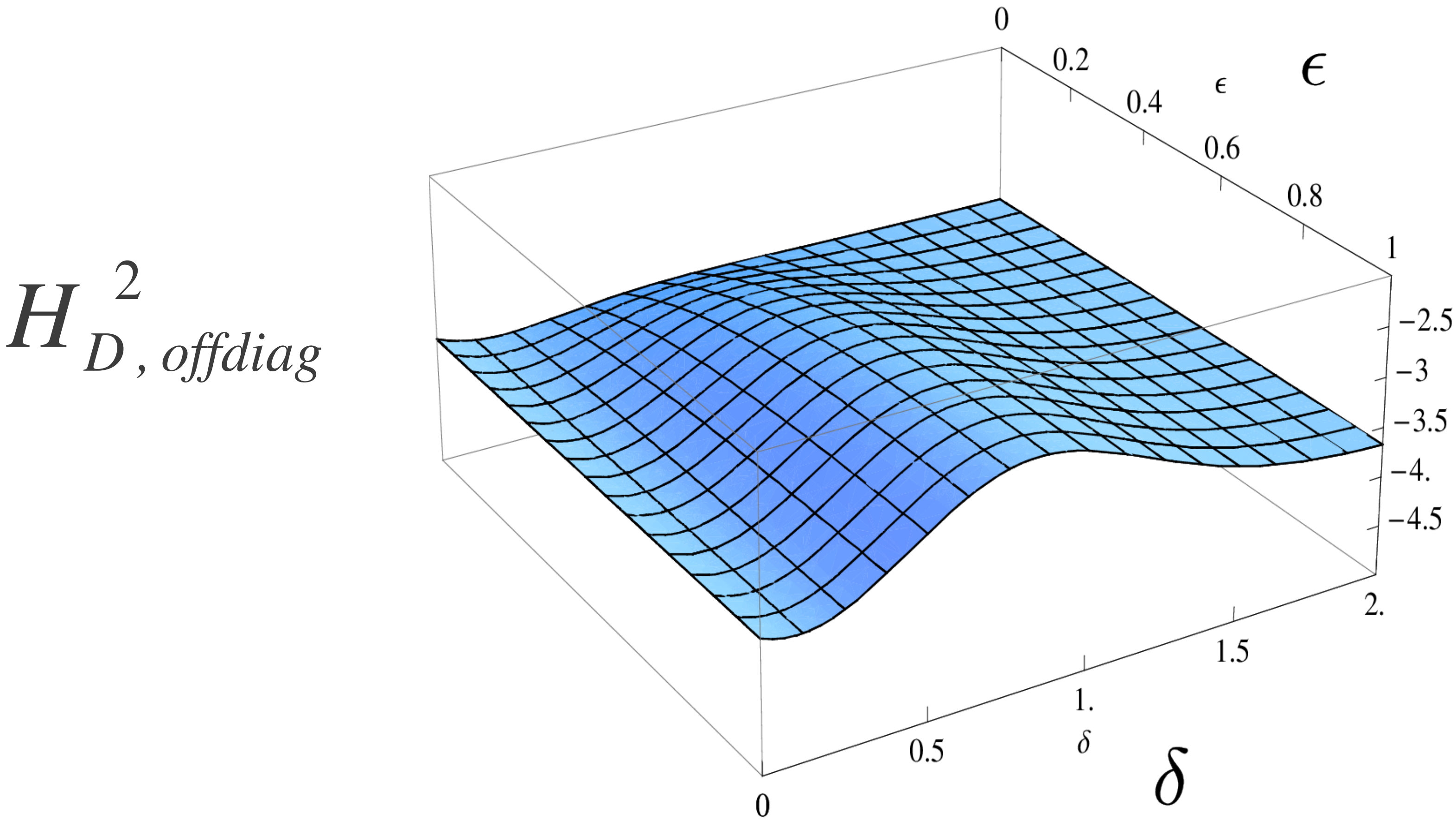
The 3D Plot of Diagonal Terms

$H^2_{D,diag}$



Shallow valley at $\delta \approx 0.9$, deepest at $\epsilon = 1$.

The Off-Diagonal Plot

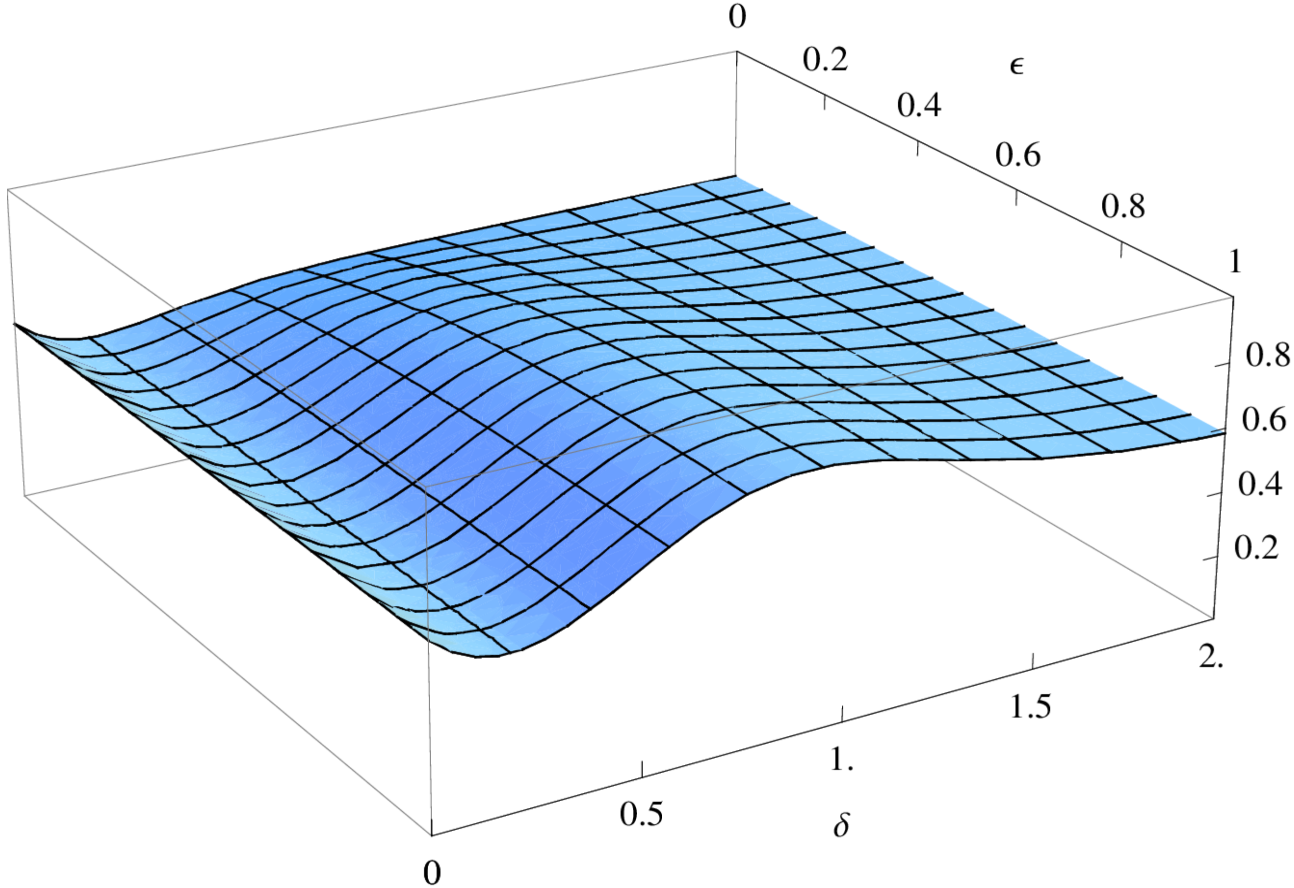


Shallow valley at $\delta \approx 0.2$, a **hump (!)** at $\delta \approx 1.0$.

Combining D and OD Terms

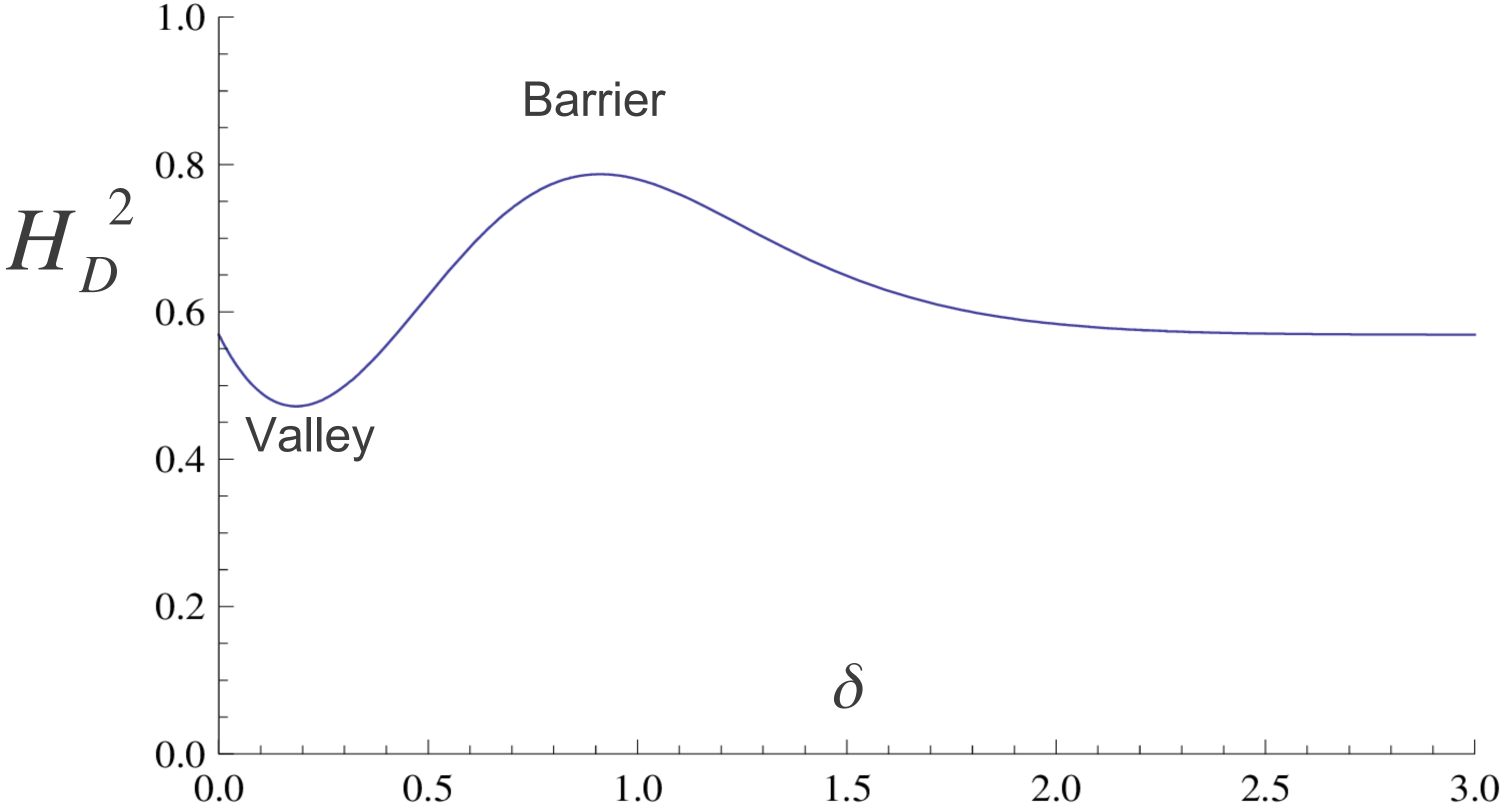
- Both are large: $H^2_{D,diag} \approx 4$ and $H^2_{D,offdiag} \approx -3.5$.
- But for the one-well case, $H_D \psi_D = E \psi_D$ with $E = 0.7540$ (i.e. 375 MeV)
- They ***do*** need to cancel so that $E^2 \approx 0.5685$, i.e., positive.
- The shallow valley in $H^2_{D,diag}$ is more than filled in by the bigger hump (”fission barrier”) in $H^2_{D,offdiag}$ around $\delta \approx 1.0$.
- There remains a long shallow valley in their sum at $\delta \approx 0.2$.

So, the Final Plot of H_D^2



There should be binding of the B and D along the valley!

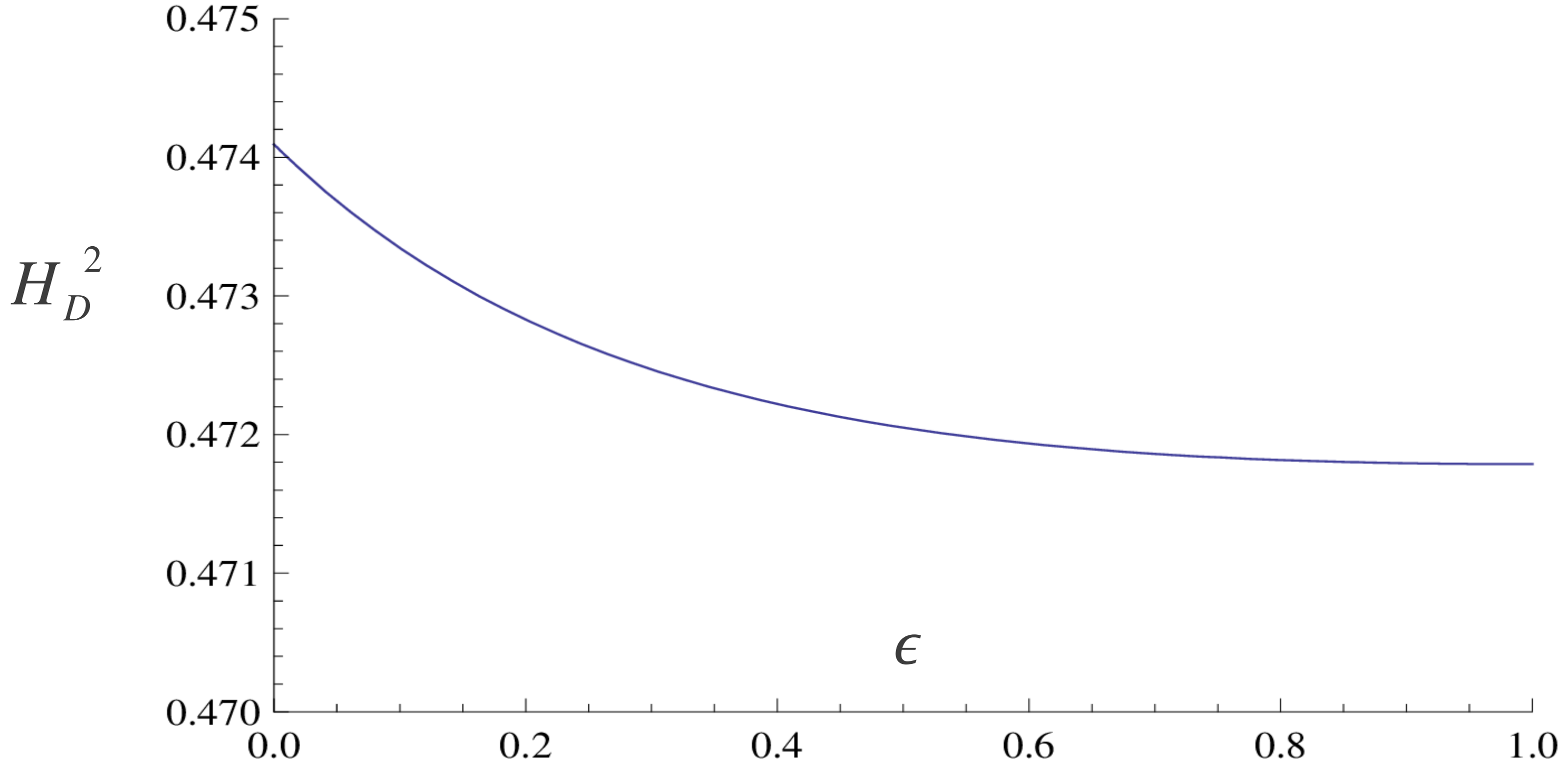
The End View



Dependence on δ at $\epsilon = 1$.

Valley depth here is – 155 MeV. Barrier height is + 212 MeV.

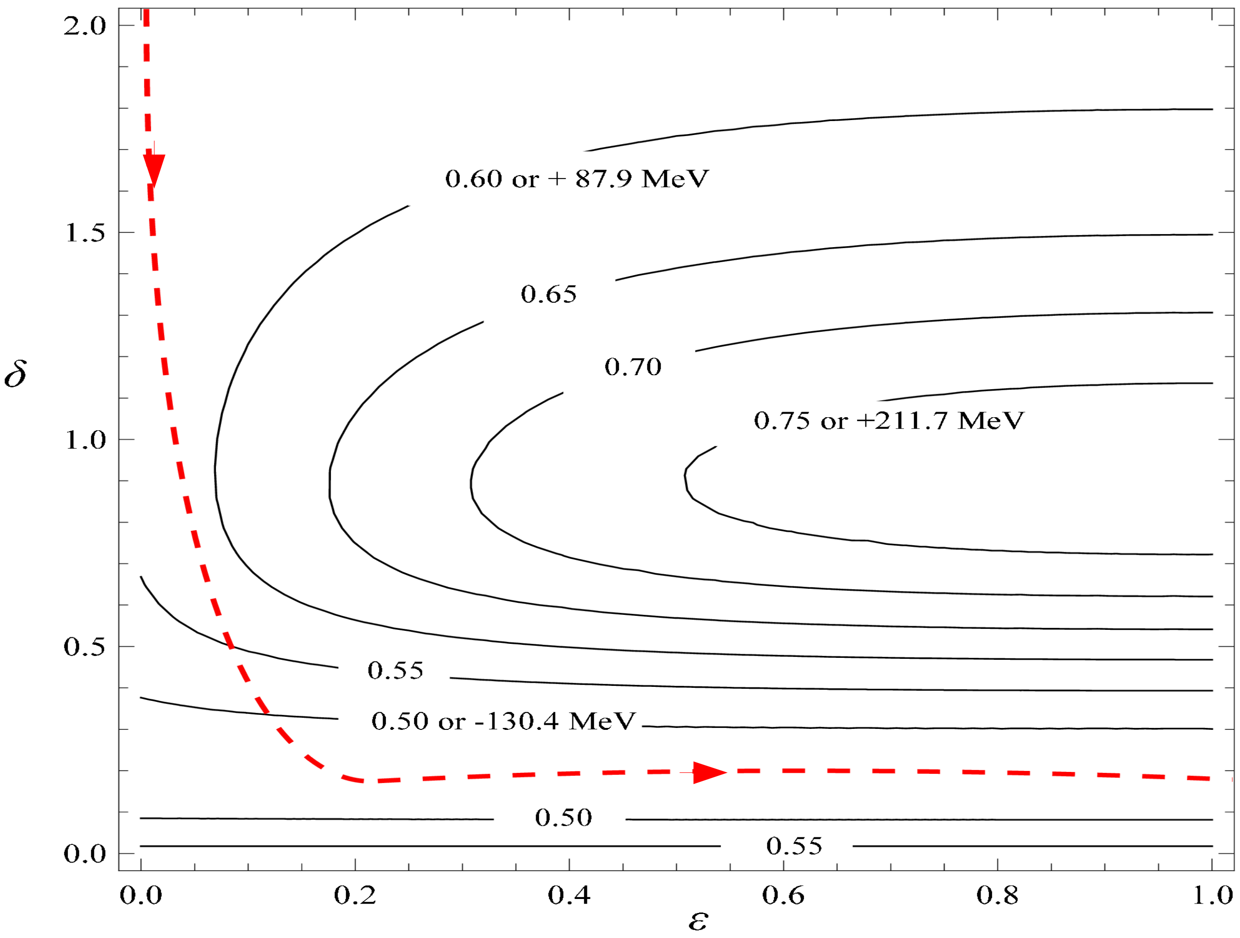
The Valley Is Surprisingly Flat



Dependence on ϵ at $\delta = 0.18$.

Note the fine scale. Drop in E is about 20 MeV.

How B and D Coalesce



Molecular or Tight 4-Quark Binding?

- So, where along the long, flat valley at delta around 0.2 (or 0.45 fm) will the four quarks end up?
- Molecular-like binding would correspond to a small near-zero value of epsilon.
- Tight four-quark binding would be at epsilon = 1, the light quarks equally shared between both of the two heavy quarks.
- The small 20 MeV energy difference between the top and bottom of the valley may allow Zitterbewegung to make the difference between these two descriptions indistinguishable.

What About *q q* Interactions?

- Called color-magnetic (or, hyperfine) interactions.
- Non-relativistically $E_{\mathrm{CMI}} \propto \langle \boldsymbol{\sigma_1} \cdot \boldsymbol{\sigma_2} \rangle / (m_1 m_2)$.
- If m_1 or m_2 is heavy, E_{CMI} is negligible.
- So, only the E_{CMI} between the light quarks matters. Typically these are about $\pm$ 50 MeV, depending on $\langle \boldsymbol{\sigma_1} \cdot \boldsymbol{\sigma_2} \rangle$.
- Relativistically, off-diagonal $\boldsymbol{\alpha}$ connects upper to lower components. For a heavy mass particle, the smaller the lower component is relative to the upper. Hence, negligible, again.
- For two light (massless) particles, lower component is comparable to the upper. Thus, again, they contribute the most to the E_{CMI}.

Conclusions

- It looks like *B* and *D* mesons *can* coalesce into a bound state.
- It may not be easy to distinguish between molecular-like and tight four-quark binding – the valley for binding is long and flat with a separation between the *b* and *c* quarks of about 0.45 fm.
- Binding energy is about 150 MeV.
- The barrier of 212 MeV will act to prevent fission of the bound state into separate B and D mesons.
- Color-magnetic interactions may be small, of order 50 MeV, and come mostly from the interaction between the two light quarks. Not enough to destroy the binding.
- But, they need to be calculated! Presently in progress.

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