FIESTA 2017

Fission Experiments and Theoretical Advances

Santa Fe, 18-22 September, 2017

Fission dynamics with microscopic level densities

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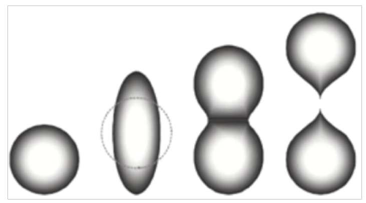
Editors' Suggestion:
Phys. Rev. C **95**, 024618 (2017)

Nuclear fission is a result of shape dynamics

Otto R. Frisch (1904-1979)

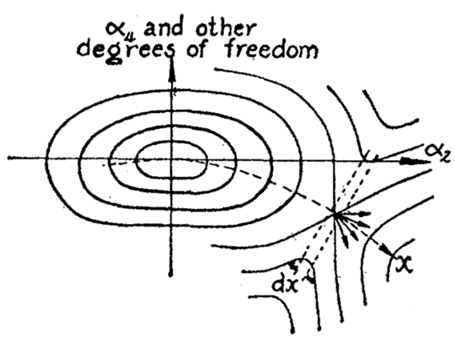
L. Meitner & J.A. O.R. Frisch, Nature **143** (1939) 239:
Disintegration of Uranium by Neutrons:
A New Type of Nuclear Reaction

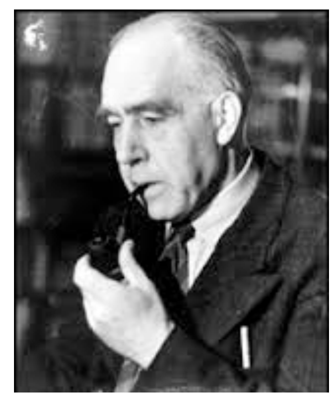


Lise Meitner (1878-1968)

John A. Wheeler (1911-2008)

N. Bohr & J.A. Wheeler, Phys Rev **56** (1939) 426:
The Mechanism of Nuclear Fission

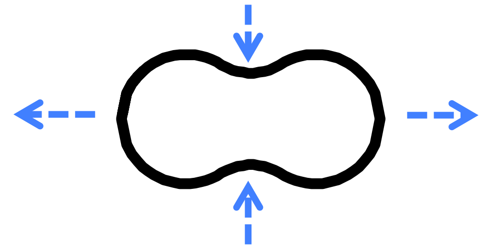


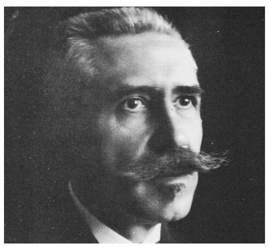


Niels Bohr (1885-1962)

Nuclear shape dynamics -> random walk

The time evolution of the nuclear shape parameters, $\boldsymbol{q}(t)$:





Paul Langevin (1872-1946)

Langevin equation: $d\boldsymbol{p}/dt = \boldsymbol{F}^{cons} + \boldsymbol{F}^{diss}$

$\boldsymbol{M}(\boldsymbol{q})$ $U(\boldsymbol{q})$ $\gamma(\boldsymbol{q})$

Marian Smoluchowski (1872 – 1917)

J. Randrup and P. Möller, PRL **106**, 132503 (2011):

The shape motion is *highly dissipative*:

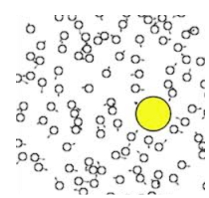
Smoluchowski equation: $\boldsymbol{0} = \boldsymbol{F}^{cons} + \boldsymbol{F}^{diss}$

$U(\boldsymbol{q})$ $\gamma(\boldsymbol{q})$

If $P(A_f)$ is ≈ *insensitive* to $\gamma(\boldsymbol{q})$:

Random walk on the energy landscape $U(\boldsymbol{q})$

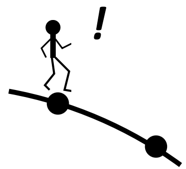
Brownian motion



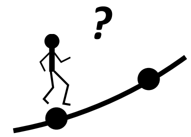
Metropolis, Rosenbluth[2], Teller[2],
J. Chem. Phys. **21** (1953) 1087

Nicholas C. Metropolis
(1915-1999)

Metropolis walk …



$P_{down} = 1$

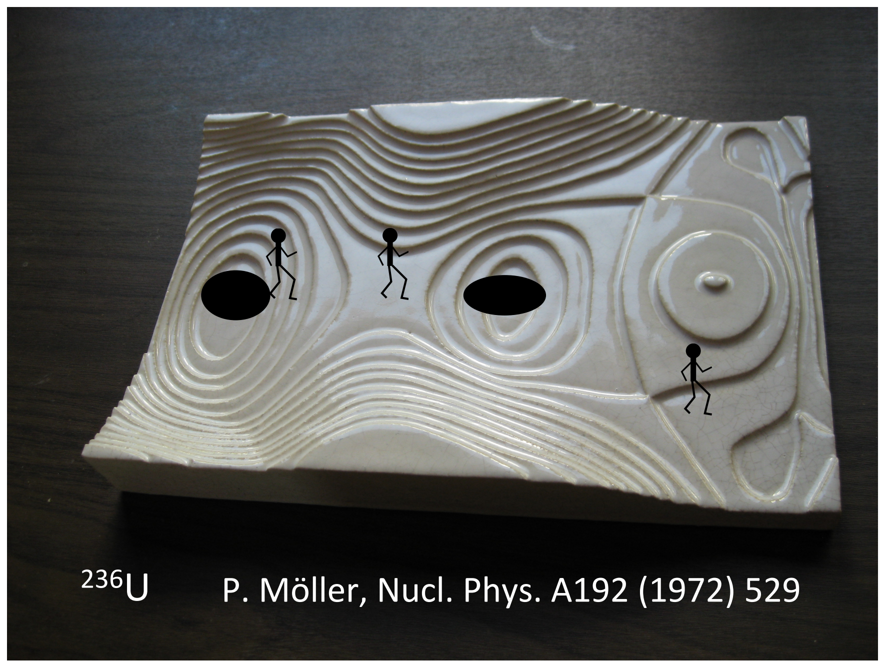


$P_{up} = \exp(-\Delta U/T)$

ΔU: Change in potential
T: Local temperature

… on the potential-energy surface:

Start at ground-state (or isomeric) minimum



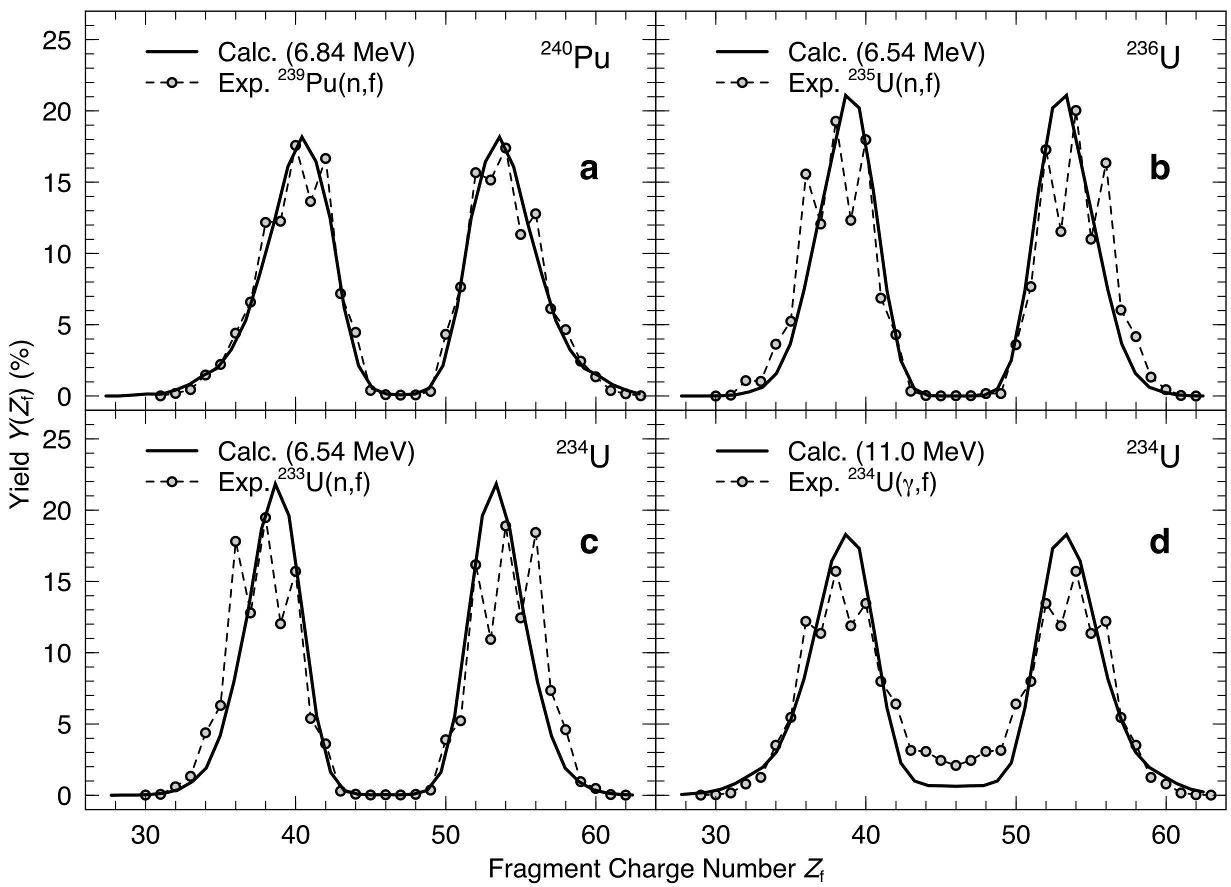
Elongation

Walk until the neck has become thin …

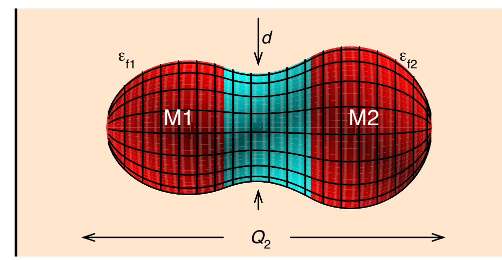
… then bin the mass asymmetry

Nuclear shape evolution as a random walk on the 5D potential energy landscape

J. Randrup and P. Möller, Phys. Rev. Lett. **106**, 132503 (2011)



5D shape family



$\boldsymbol{q}$ {
- Elongation Q_2
- Neck radius c
- Endcap defs ε_{f1} & ε_{f2}
- Mass asymmetry α

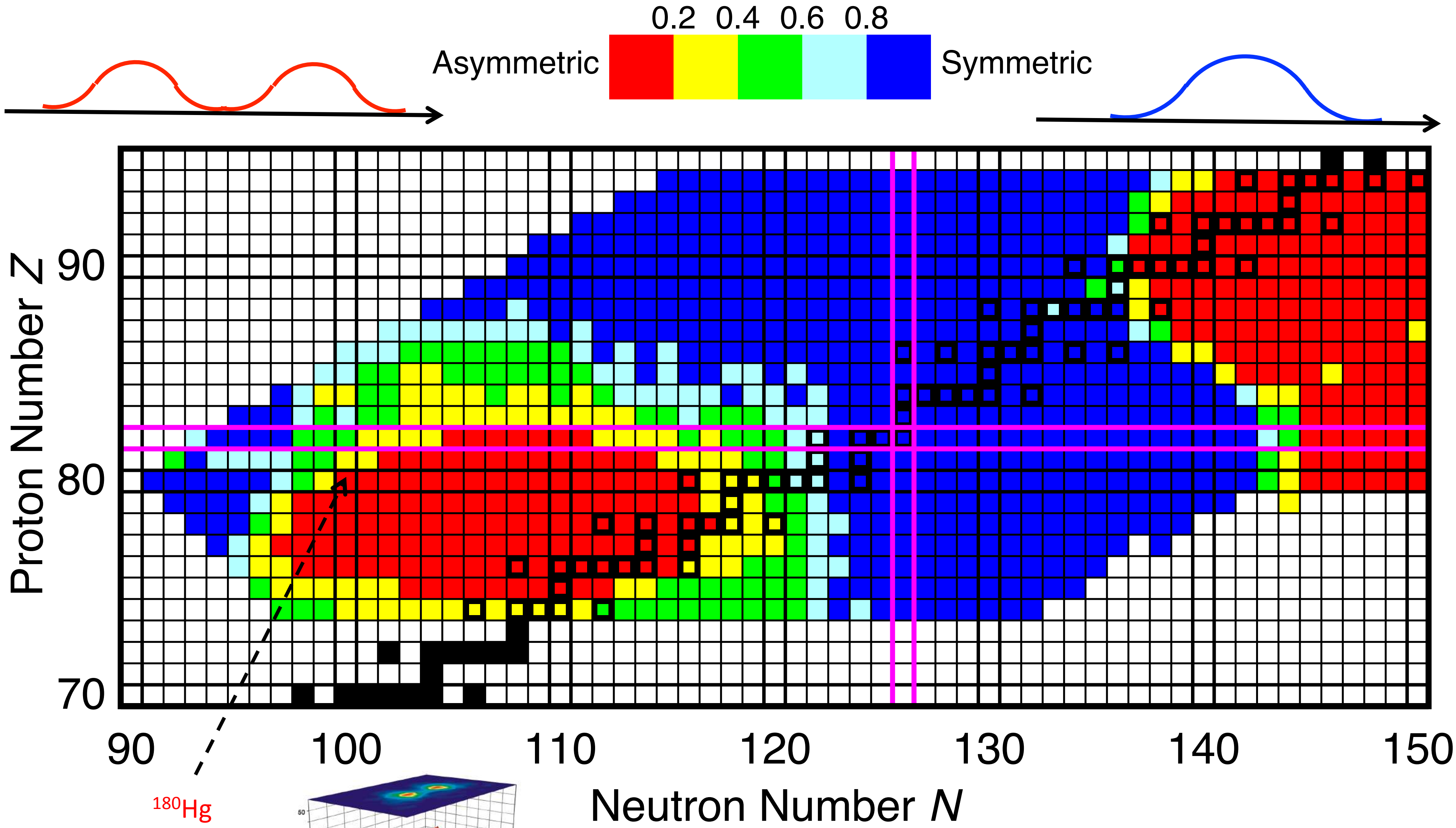
Ray Nix 1969

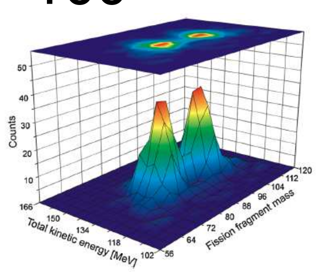
$U(\boldsymbol{q}) = U_{\text{macro}}(\boldsymbol{q}) + U_{\text{micro}}(\boldsymbol{q})$

> 5M shapes per nucleus

> 5k nuclei

Fission-Fragment Symmetric-Yield to Peak-Yield Ratio



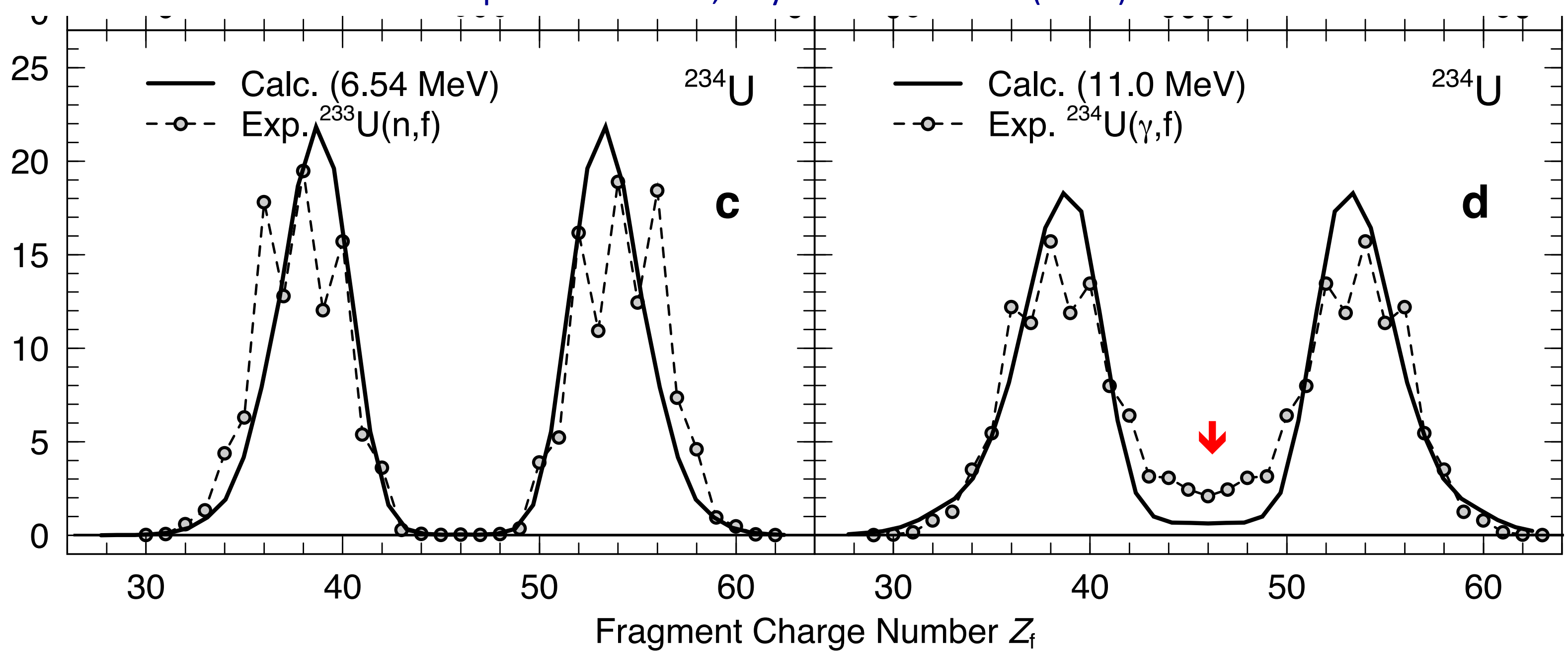


A.N. Andreyev *et al.*, PRL **105**, 252502 (2010)

P. Möller & J. Randrup, PRC **91**, 044316 (2015)

Energy dependence of the fission shape evolution

J. Randrup and P. Möller, Phys. Rev. Lett. **106** (2011) 132503



Use an effective energy landscape obtained by suppressing the microscopic terms

J. Randrup and P. Möller, Phys. Rev. C **88**, 064606 (2013): $U(\boldsymbol{q}) = U_{\text{macro}}(\boldsymbol{q}) + U_{\text{micro}}(\boldsymbol{q}) \times S(E^*(\boldsymbol{q}))$

Use shape-dependent microscopic level densities to guide the random walk: $\rho_{\text{micro}}(\boldsymbol{q})$

D.E. Ward, B.G. Carlsson, T. Døssing, P. Möller, J. Randrup, S. Åberg,
Phys. Rev. C **95**, 024618 (2017) *[Editors' Suggestion]*

Level densities in dynamics

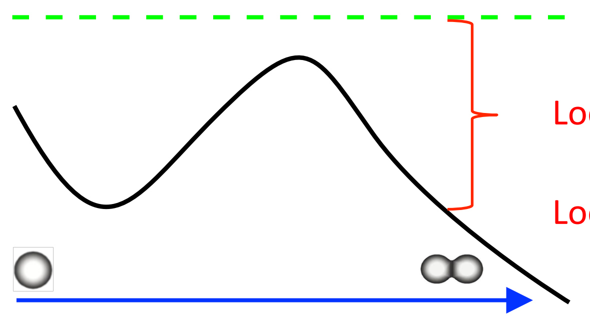
DETAILED BALANCE

$$\frac{\nu(\chi \to \chi')}{\nu(\chi' \to \chi)} = \frac{\rho(\chi')}{\rho(\chi)}$$

Total energy E

Potential energy $U(\chi)$

Shape coordinate χ



Local statistical excitation $E^*(\chi) = E - U(\chi)$

Local level density $\rho_E(\chi)$

$\chi \approx \chi'$ =>
$$\frac{\nu(\chi \to \chi')}{\nu(\chi' \to \chi)} = \frac{\rho_E(\chi + \delta\chi)}{\rho_E(\chi)} \approx 1 + \frac{\partial}{\partial\chi} \ln \rho_E(\chi) \cdot \delta\chi$$

If $\rho_E(\chi) = \rho(E(\chi))$
=> Metropolis:

Temperature: $\frac{\partial}{\partial E} \ln \rho(E) = 1/T(E)$

Driving force: $\frac{\partial}{\partial \chi} E^*(\chi) = -\frac{\partial}{\partial \chi} U(\chi) = F(\chi)$

$$\frac{\nu(\chi \to \chi')}{\nu(\chi' \to \chi)} \approx \exp(-\delta U/T)$$

But generally $\rho_E(\chi) \neq \rho(E(\chi))$ due to structure effects: *So use realistic level densities!*

Collaboration for the purpose of obtaining level densities for all relevant fission shapes:
Gillis Carlsson, Thomas Døssing, Peter Möller, Jørgen Randrup, David Ward, Sven Åberg

Combinatorial method for the nuclear level density

H. Uhrenholt, S. Åberg, A. Dobrowolski, Th. Døssing, T. Ichikawa, P. Möller: NPA **913** (2013) 127

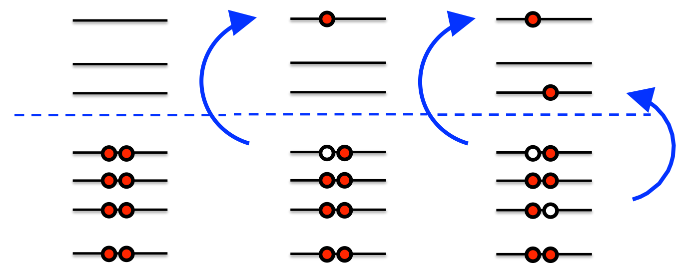
$$\rho(E,I,\pi) = \frac{1}{\Delta E}\int_E^{E+\Delta E}\sum_i \delta(E' - E_i(I,\pi))dE' \quad \Rightarrow \quad \rho(E) = \sum_{I\pi}\rho(E,I,\pi)$$

$$E = \mathcal{E}_p + \mathcal{E}_n + E_{rot}$$

Intrinsic states:

Consider all multiple p-h excitations for protons and neutrons separately

$$|i\rangle = \prod_{\alpha=1}^{n} a^{+}_{\nu_\alpha} a_{\nu'_\alpha} |0\rangle,$$



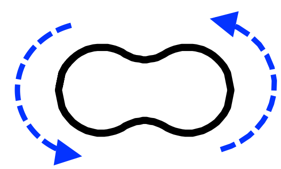
Pairing

Calculate BCS pairing for each one

Rotational enhancement

Rotational band built on each intrinsic state:

$$E_{rot}(I,K) = [I(I+1)-K^2]/2\mathcal{I}_{perp}(\chi,\Delta_p,\Delta_n)$$



OBS: Higher I => Lower E_{intr}

Vibrational enhancement

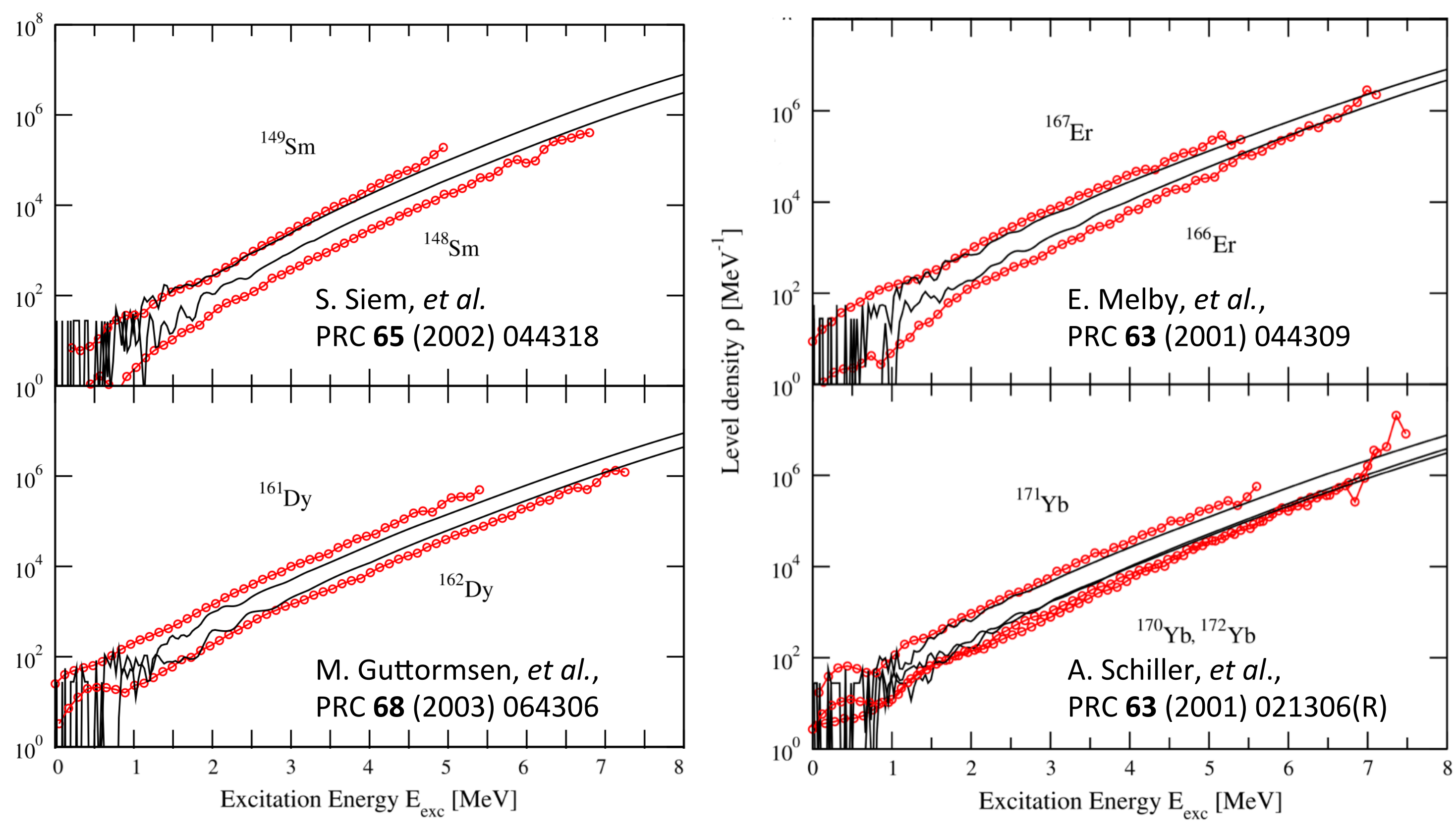
Expected to be unimportant => ignored

Note: The single-particle levels are the *same* as those used to get the shell and pairing energies in $U(\boldsymbol{q})$!

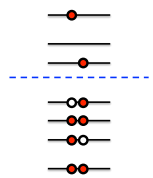
Combinatorial model for the nuclear level density

H. Uhrenholt, S. Åberg, A. Dobrowolski, Th. Døssing, T. Ichikawa, P. Möller: NPA **913** (2013) 127

$$\rho(E,I,\pi)=\frac{1}{\Delta E}\int_{E}^{E+\Delta E}\sum_{i}\delta(E'-E_i(I,\pi))dE' \qquad \Rightarrow \qquad \rho(E)=\sum_{I\pi}\rho(E,I,\pi)$$



Project:

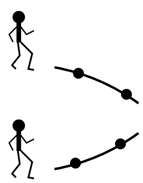


Use the combinatorial method to obtain the microscopic level density for *all* (>5M) 3QS shapes for which the potential has been tabulated:

$$\rho_{ZA}(E,I,\text{shape})$$

for each individual fissioning nucleus ^{A}Z (U_{ZA}(shape) exists for >5k ^{A}Z)

Asymmetric shapes
Replace $\{\varepsilon_n\}$ by 3QS
Get all s.p. levels (PM)

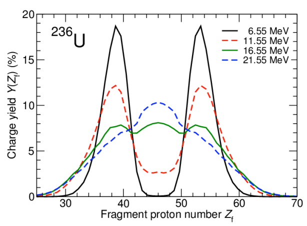


Use those as the basis for the random walk:

P_{down}: $P(U' \leq U) = 1$ --> $P(\rho' \geq \rho) = 1$

P_{up}: $P(U' \geq U) = \exp(-\Delta U/T)$ --> $P(\rho' \leq \rho) = \rho'/\rho$

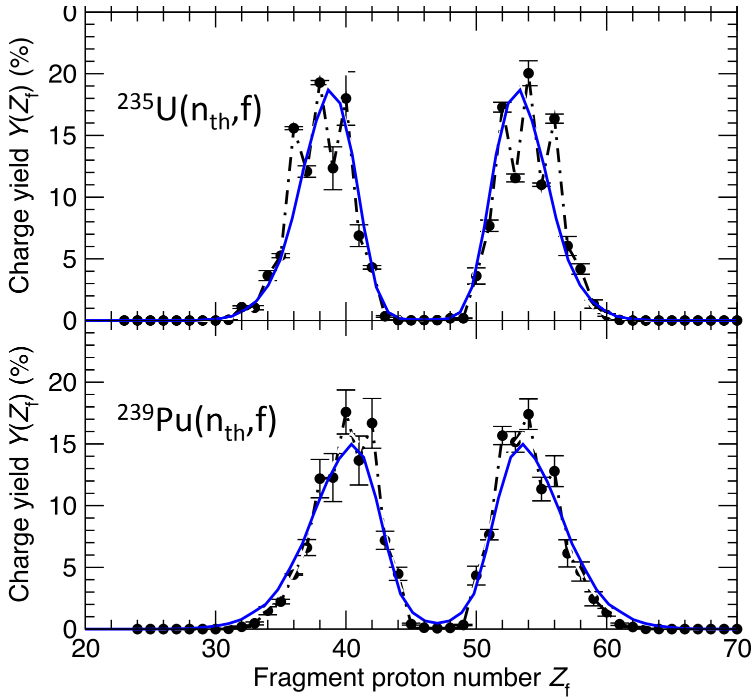
Trivial code modification

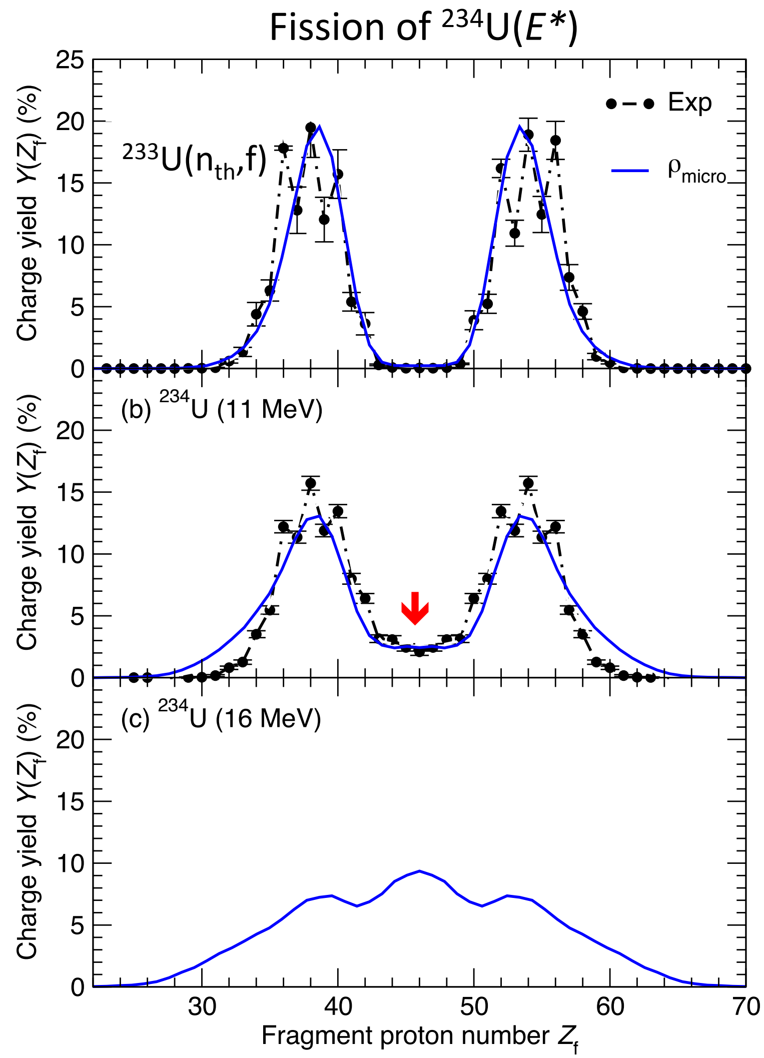


Then the gradual disappearance of pairing and shell effects with excitation is *automatically* included in the shape evolution

Fully consistent:
same s.p. levels
used for U and ρ
(no parameters)

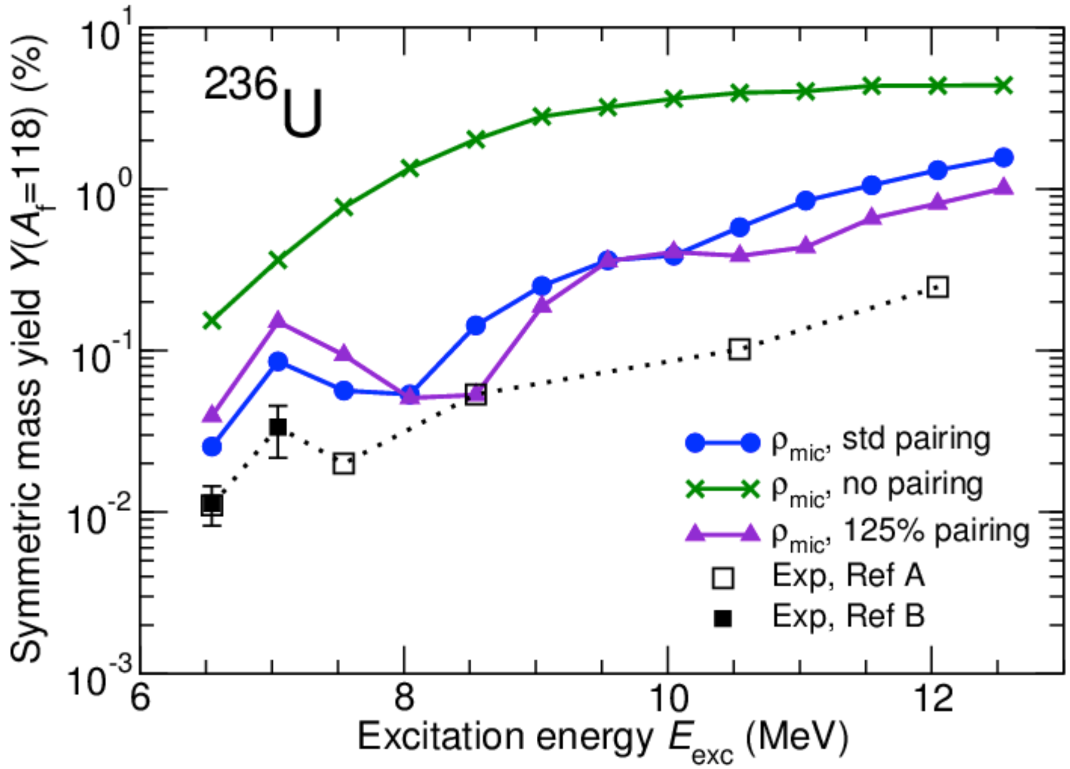
Mass yields using microscopic level densities





✔

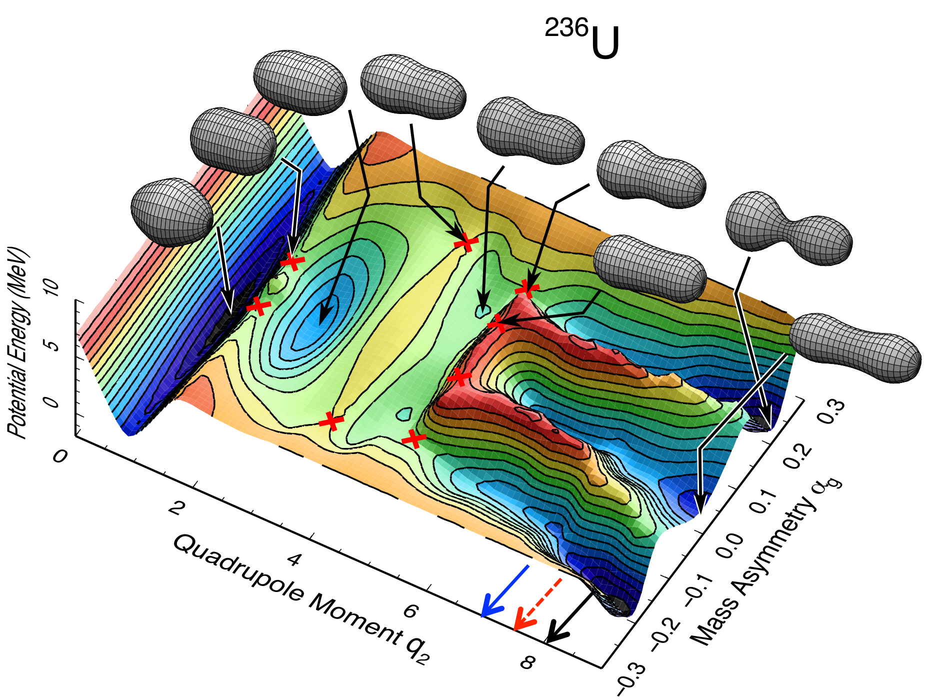
Energy dependence of fission yields:
Non-monotonic behavior of the symmetric yield



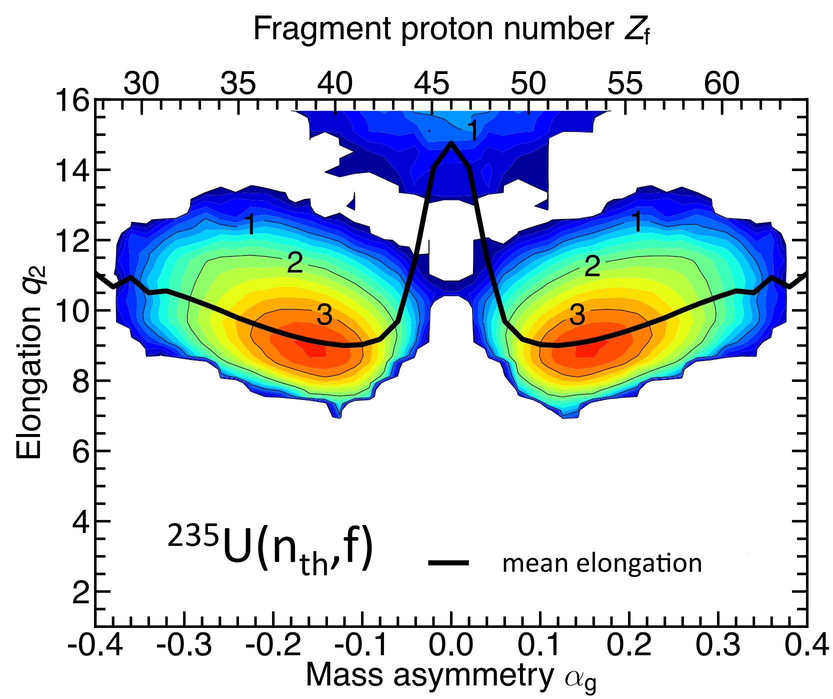
[A] L.E. Glendenin *et al.*, Physical Review C **24**, 2600 (1981)
[B] M.B. Chadwick *et al.*, Nuclear Data Sheets **112**, 2887 (2011)

Fission of ^{236}U:

Potential-energy surface



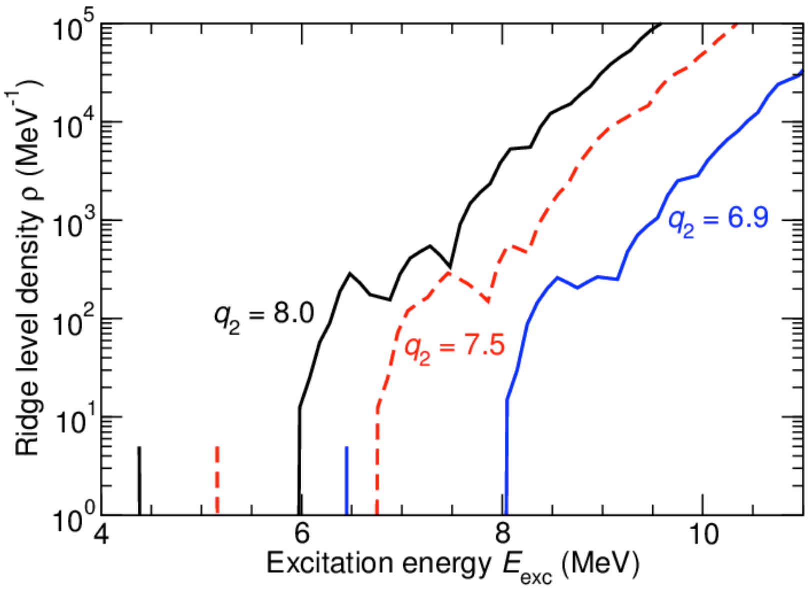
Terminal shapes

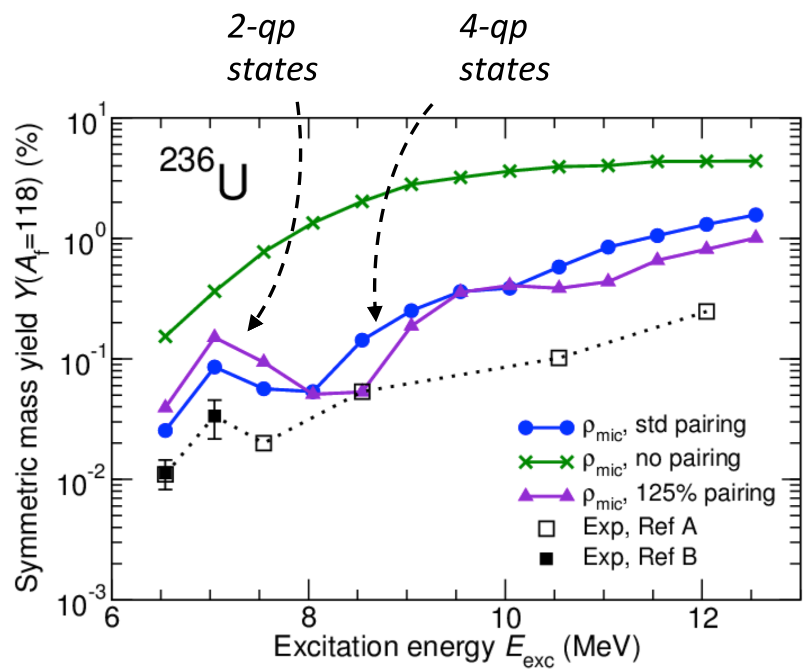


T. Ichikawa, A. Iwamoto, P. Möller, A.J. Sierk,
Physical Review C **86**, 024610 (2012)

Energy dependence of fission yields:
Non-monotonic behavior of the symmetric yield

Level density for three shapes along the ridge;
all have a dense single-particle spectrum, hence
a positive shell energy and a large pairing gap:





OBS: M.E. Gooden *et al.*, Nuclear Data Sheets **131**, 319 (2016):
Non-monotonic energy dependence of $Y(A_f)$ from ^{240}Pu*

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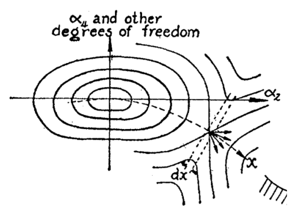
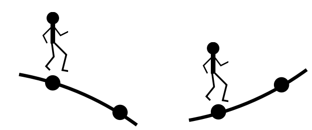
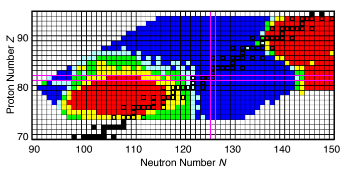
Fission dynamics with microscopic level densities



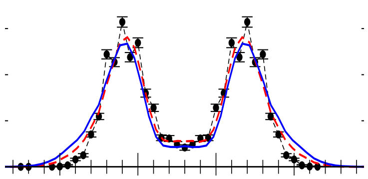
The nuclear shape evolution is akin to Brownian motion and can be approximately described as a random walk on the multi-dimensional deformation-energy surface
(no adjustable parameters, computationally fast)





This conceptually simple treatment makes it possible to calculate fission fragment mass and charge yields for any nucleus for which suitable potential-energy surfaces exist
(5D surfaces exist for over 5,000 nuclei)

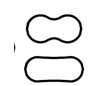
U_E(shape) = U_{macro} + U_{micro}



A general & consistent description was obtained by using the *microscopic* level densities calculated for *each shape* by means of a recently developed combinatorial method; the gradual disappearance of shell and pairing effects is then *automatically* ensured *without any new parameters*

ρ_{micro}(shape)

$$\frac{\nu(\chi \to \chi')}{\nu(\chi' \to \chi)} = \frac{\rho(\chi')}{\rho(\chi)}$$



A marriage between nuclear structure & dynamics!

Editors' Suggestion:
Phys. Rev. C **95**, 024618 (2017)