Dynamical Microscopic and Macro-Micro Approaches to Nuclear Fission

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Introduction and Background

• Even though the history of researches in nuclear fission is already more than 70 years-long, many mysteries still exist actually what is going on during the process of nuclear fission, especially before scission

• Due to the difficulties, accuracy of the fission-related nuclear data is still considered not high enough

• To resolve this situation and to understand many different aspects of nuclear fission in a consistent way, we think accurate dynamical treatment of nuclear fission is necessary

• We are making researches in fission by dynamical approaches with Langevin equation, AMD (antisymmetrized molecular dynamics) and TDHF

• Here we explain them briefly
Origin of extra spin that fission fragments have

spin of $^{148}$Pm from fission of $n+^{235}$U system as a function of excitation energy of compound nucleus

Do fission fragments rotate? If so, what is the origin of the rotational motion and how much angular momenta they have?

Twisting mode in fusion reactions

Twisting & Bending modes in fission


FIG. 3. Artist’s view of the spin excitation generated in a central collision of two $^{16}$O nuclei. Closely based on the numerical spin density vectors produced in the calculations.

PRC21, 204(1980)
Anomaly in the average Total Kinetic Energy of Fission Fragments
Our approaches (CN to scission)

**Macro-micro approach : Langevin model**
- PES from macro-micro, transport coefficients from macro or micro + random force
- 3D Langevin with macroscopic transport coefficients (with F.Ivaniuk)
- 3D Langevin with microscopic transport coefficients (by linear response theory)
- 4D Langevin with macroscopic transport coefficients
- 4D Langevin with microscopic transport coefficients
- 5D Langevin -- talk by A. Sierk

**Microscopic approaches**

**Antisymmetrized Molecular Dynamics (AMD)**
- Slater determinant with Gaussian wave packets (coherent state) as basis function
- mean field calculated with SLy4 interaction
- Stochastic nucleon-nucleon collisions to express branching of Slater determinant and deformation of Gaussian wave packet

**Time-dependent Hartree Fock (TDHF)**
- No restriction on single-particle wave function (3D mesh calculation)
- mean field by SV-Bas interaction
- based on OAK3D and SKY3D (no pairing)
- to obtain nuclear friction coefficient from dynamical model
Simulation of nuclear fission \((^{235}\text{U} + 140 \text{ MeV n})\) by JQMD

JQMD : JAERI Quantum Molecular Dynamics
= a \text{semiclassical} molecular dynamics for nuclear reactions (mean field + NN collision)

\[ t = 0 \text{ fm/c} \]
QMD simulation of nuclear fission ($^{235}$U + 140 MeV n)

Time evolution of $^{235}$U + 140 MeV n reaction by JQMD

Nuclear fission by Langevin equation

Nuclear shape evolution is driven by random kicks by nucleons in thermal equilibrium (microscopic d.o.f.) given to the nuclear surface (macroscopic d.o.f) from inside the surface.

These 2 different d.o.f have different time scales:
- nucleon motion: 1 to 10 fm/c
- shape motion: ~>10,000 fm/c
### Langevin Equations for nuclear fission

\[
\frac{dq_i}{dt} = \left( m^{-1} \right)_{ij} p_j \\
\frac{dp_i}{dt} = -\frac{\partial V}{\partial q_i} - \frac{1}{2} \frac{\partial}{\partial q_i} \left( m^{-1} \right)_{jk} p_j p_k - \gamma_{ij} \left( m^{-1} \right)_{jk} p_k + g_{ij} R_j(t)
\]

- \(q_i\): deformation coordinates (nuclear shape) in two-center shell-model parametrization \(\{q\}_{3D} = \{ZZ_0, \delta_1 = \delta_2, \alpha\}\) \(\{q\}_{4D} = \{ZZ_0, \delta_1, \delta_2, \alpha\}\)
- \(p_i\): momentum conjugate to \(q_i\)
- \(m_{ij}\): mass tensor, \textbf{Hydrodynamical mass (Werner-Wheeler)}
- \(\gamma_{ij}\): friction tensor, \textbf{Wall and Window formula (one-body dissipation)}

or from \textbf{Linear Response Theory} (microscopic)

**Random force**:
\[
\langle R_i(t) \rangle = 0, \quad \langle R_i(t_1) R_j(t_2) \rangle = 2\delta_{ij}\delta(t_1 - t_2)
\]
White noise
\[
\sum_k g_{ik} g_{jk} = T\gamma_{ij}
\]
Einstein relation

**Intrinsic energy and Temperature**:
\[
E_{\text{int}} = E^* - \frac{1}{2} \left( m^{-1} \right)_{ij} p_i p_j - V(q) = aT^2
\]
- \(E_{\text{int}}\): intrinsic energy, \(E^*\): excitation energy, \(T\): temperature
- \(V(q)\): potential at deformation \(q\)
- \(V_{\text{sh}}(q)\): Strutinsky shell correction energy
- \(a\): level density parameter
Shape parametrization

Two-center oscillator model
(Maruhn and Greiner, Z. Phys. 251(1972) 431)

Collective coordinates (4 dynamical variables)

\[ \{q\}_{3D} = \{ZZ_0, \delta_1 = \delta_2, \alpha\} \quad \{q\}_{4D} = \{ZZ_0, \delta_1, \delta_2, \alpha\} \]

- \( ZZ_0 = \frac{Z_0}{R} \) \quad Elongation
  
  \[ R : \text{Radius of compound nucleus} = 1.2 A_{CN}^{1/3} \]

- \( \delta_i = \frac{3(a_i - b_i)}{2a_i + b_i} \), \( i = 1, 2 \) \quad Deformation of fragments at outer tips
  
  4D : \( \delta_1, \delta_2 \) are independent, 3D : \( \delta_1 = \delta_2 \)

- \( \alpha = \frac{A_1 - A_2}{A_1 + A_2} \) \quad Mass asymmetry
  
  \( A_1 : \text{mass of the right fragment} \quad A_2 : \text{mass of the left fragment} \)

- \( \varepsilon = 0.35 \) \quad neck parameter : fixed

Note that neck radius depends on all the 5 parameters
Initial condition of Langevin calculation

We start from here or here with **zero initial momenta** (there is no distribution in the initial state) $P(q, p) = \delta(q - q_0)\delta(p)$

Many people start from this region

- Neutron-induced fission
- Tunneling fission
- Fission from isomer
- Spontaneous fission

$P(q, p) = \delta(q - q_0)\delta(p)$
Example of 4D Langevin trajectories ($^{236}$U)

\[ ZZ_0 = \frac{z_0}{R} \]
Mass Distribution of Fission Fragments at Ex=20MeV

- **234U**
- **238Np**
- **240Pu**
- **258Fm**
Distributions of TKE (total kinetic energy of fission fragments)
Systematics of $\langle \text{TKE} \rangle$ by 3D Langevin
Extension to 4D Langevin $\{q\}_{4D} = \{ZZ_0, \delta_1, \delta_2, \alpha\}$
$\sigma_{TKE}$ by 4D Langevin

$^{235}\text{U} + n$ system

1st chance
Nuclear fission by Antisymmetrized Molecular Dynamics: AMD

- Even though the Langevin-model description gives satisfactory results in many aspects of nuclear fission, it has only 3 to 4 degrees-of-freedom to express nuclear shape. It should be better to have ways to describe the nuclear fission by microscopic theories having d.o.f. of all the nucleons.


BE/A for U isotopes

![Graph showing binding energy per nucleon for U isotopes against mass number.]
AMD-1 : Basic formulation

Single-particle wave function : Gaussian coherent state

\[ <r|\phi Z|l> = \left( \frac{2v}{\pi} \right)^{3/4} \exp[-v(r-Zl)/\sqrt{v}] \chi a l \]

Total wave function

\[ |\Phi(r_1,r_2,...,r_A)> = \frac{1}{\sqrt{A}} \det[\phi(rj)] \]

\[ \delta \int t_1 t_2 dt \Phi \frac{\hbar d}{dt} - H \Phi / \Phi = 0 \]

Time-dependent variational principle:

\[ i\hbar \sum j t \chi C_{l j \sigma} Z_{l j} = \partial K / \partial Z_{l \sigma} \]

Equation of motion of the mean field:

\[ H = <\Phi|H|\Phi>/<\Phi|\phi l i o j t r = \partial t_2 / \partial Z_{l \sigma} \partial Z_{l j} \log <\Phi|\Phi> \]

where
AMD-2 : NN interactions

• Effective N-N interaction (origin of nuclear mean-field)

\[ v \downarrow ij = t \downarrow 0 (1 + x \downarrow 0 P \downarrow \sigma ) \delta(r) + 1/2 \ t \downarrow 1 (1 + x \downarrow 1 P \downarrow \sigma ) [\delta(r) k \uparrow 2 + r \delta(r)] + t \downarrow 2 (1 + x \downarrow 2 P \downarrow \sigma ) k \cdot \delta(r) k + t \downarrow 3 (1 + x \downarrow 3 P \downarrow \sigma ) [\rho (r \downarrow l i )] \uparrow \alpha \delta(r) \]

where

\[ \rho (r) = \left( \frac{2 \nu}{\pi} \right)^{3/2} \sum_{i=1}^{A} \sum_{j=1}^{A} e^{-2\nu(r-R_{ij})^2} B_{ij} B^{-1}_{ji} \]

\[ B_{ij} = \langle \varphi_i | \varphi_j \rangle \]

\[ R_{ij} = \frac{1}{2\sqrt{\nu}} (Z^*_i + Z_j) \]

• Stochastic NN collision as a part of residual interaction
  (A test particles from Wigner function)
  mimics random force in Langevin theory

  ✓ In-medium NN cross section
  ✓ Pauli Blocking after the collision
  ✓ branching and deformation of wave functions--> tunneling, distribution
Initial state of $^{236}\text{U}$ ($^{235}\text{U} + \text{n}$)

As a first step to simulation of nuclear fission by AMD, here we simply give a boost momentum to each nucleon, to the opposite directions for nucleons staying at the left region and right region.

$E^+ \sim 1.3\text{MeV/u}$
Time evolution by AMD, $^{236}\text{U} \ (^{235}\text{U} + \text{n})$
Examination of phenomenological assumptions

• Brosa's random neck rupture condition due to hydrodynamical Rayleigh instability

$$2\ell = 11r$$
$$\ell = 5.5r$$


**AMD**: $l \sim 5r$

This verifies that Brosa's estimation of neck rupture condition is sensible
Results: FPY (left) and rotational angular momenta (left)

\[ 2^{36}U, \text{ boost fission by AMD} \]

- Due to our specific way of boost, the mass distribution corresponds to symmetric fission, which is categorized as super-long mode in Brosa's terminology (low TKE component).
- Each fragment has, on the average 5 to 10\( \hbar \) (on the average around 7\( \hbar \)) of rotational angular momentum, which is enough to account for the origin of extra spins of fission fragments as explained in the introduction.
Results: Mass-TKE correlation (left) and excitation energy of each fragment (left)

exp: $^{236}$U at excitation energy of 12 MeV

- Our result, which corresponds to the symmetric components, has the same TKE values deduced from experiment. This is in some sense quite satisfactory as well as surprising if we consider the difference of excitation energy of the data (20 MeV) and our calculation (300 MeV).
- This model has a capability to deduce excitation energy of each fragment, which is not the case in Langevin-type approaches. The results $E^* \sim A$ denotes equilibrium temperature Fermi-gas scheme.
Results from AMD calculation for $^{236}\text{U} (^{235}\text{U} + \text{n})$

1. There seems to be no oscillation in the elongation direction at all even though the fact that the way we gave the initial condition is expected to result in strong oscillation. However, there do exist surface oscillation.

2. This result is in contradiction to Langevin theory predictions, but in accord with TDHF and QMD results.

3. We notice that neck is thicker than we expected, and particle exchange through the neck region seems to be quite important, which may result in rotation of fragments.

4. Nuclear shape is quite complicated during fission, which is out of reach if we use only 3, 4 or 5 shape parameters (such as used in Langevin description). In this respect, description of nuclear fission by microscopic theories is recognized to be very important.
Nuclear friction by TDHF

\[ \frac{i\hbar}{\text{d}t} \varphi_i = h \downarrow \varphi_i \downarrow \rho(r) = \sum_i \uparrow N |\varphi_i\rangle \langle \varphi_i| \]

\[ R = |\int \mathcal{R} \rho(r) \text{d}r| \]

- We calculate the relative distance of fragment centers.
- From its time evolution, we derive average friction coefficient as explained in the next slide.
How to obtain friction coefficients

From the macroscopic e.o.m. for the center distance $R$

$$\mu R + \frac{dV}{dR} + \gamma R = 0$$

We can integrate

$$\int I^F\mu R \, dR + \int I^F \frac{dV}{dR} \, dR + \gamma \int I^F R \, dR = 0$$

$$\gamma = \left( \frac{1}{2} \mu R_{BF} R_{BF}^2 + V_{BF} \right) - \left( \frac{1}{2} \mu R_{AF} R_{AF}^2 \right)$$

This is the average friction coefficients, which depends on collision energy

$BF$: Before Fusion  $AF$: After Fission

Numbers in red characters indicate $\gamma/A$ at $E_{cm}/A = 7$MeV
For $^{236}\text{U} (=^{118}\text{Pd} + ^{118}\text{Pd})$ system

Wall-and-Window friction is a (static and semiclassical) hydrodynamical result, whereas our results are purely dynamical and quantal ones, but they coincide quite well.
Possibility of describing ternary and quarterly fission by TDHF

\[ 208^{\text{Pb}} + 208^{\text{Pb}} \]

DB: 000000.silo
Cycle: 0
Time: 0

6 MeV/A

7 MeV/A
Summary

• We have described 3 different dynamical approaches to nuclear fission currently studies at Tokyo Tech:

1. Dynamical description of fission by Langevin equation is explained
   • Calculations are performed for actinides to Fm region with both 3D and 4D versions, macro and micro transport coefficients
   • Mass and TKE distribution of fission fragments are reproduced with a high accuracy, including their systematical trends
   • With this degree of accuracy, this method can be considered to be one of the data production tool

2. Our first attempt to describe nuclear fission by AMD was described
   • Quite promising
   • Rotational angular momentum could be already derived to be finite
   • It can account for the extra spin which fission fragment has at the onset

3. TDHF approach was also described
   • to deduce nuclear friction from quantal dynamical model
   • possibility of describing ternary or quarterly fission
Thank you for your attention
Asymmetric and fine structures of mass distribution of primary fission products from neutron-induced fission.
Nuclear Theories which can describe various aspects of fission

Coupled-Channels Method

<table>
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<th>Dynamical Theory</th>
<th>Static Theory</th>
<th>Brosa-inspired</th>
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<td>RMF</td>
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</tbody>
</table>

- Isotope distribution
- Sharing of $E^*$ between 2 fragments,
- $J^\pi$ distribution

Scission

Statistical theory
- Weisskopf-Ewing
- Madland-Nix (LAM)
- Hauser-Feshbach

$\beta$-decay + statistical theory
- Gross Theory
- QRPA
- Shell Model

Weisskopf-Ewing
Hauser-Feshbach
Time evolution of fission

- **Compound nucleus (複合核の形成)**
- **Saddle, 2nd minima (サドル点)**
- **Scission (断裂点)**

**Isotope distribution, sharing of excitation energy, spin-parity distributions determine act as initial conditions for the following processes, but are not know well.**

- **Fission Fragment (核分裂片)**
- **Prompt n and γ (即発中性子及びγ)**
- **Primary yield (1次収率)**
- **Cumulative yield (累積収率)**
- **Independent yield (独立収率)**

**Nuclear Data**

- **Delayed n·γ**
- **β-decay**

**courtesy F. Minato Constrained Hartree-Fock**
Simulation of nuclear fission ($^{235}$U + 140 MeV n) by JQMD

JQMD : JAERI Quantum Molecular Dynamics
= a semiclassical molecular dynamics for nuclear reactions

K.Niita, T. Maruyama, Y. Nara, S. Chiba and A. Iwamoto, JAERI-Data/Code 99-042(1999)
QMD simulation of nuclear fission ($^{235}$U + 140 MeV n)

Time evolution of $^{235}$U + 140 MeV n reaction by JQMD

K.Niita, T. Maruyama, Y. Nara, S. Chiba and A. Iwamoto, JAERI-Data/Code 99-042(1999)
QMD simulation of nuclear fission ($^{235}\text{U} + 140\text{ MeV}\ n$)

Nuclear elongation seems to evolve monotonically (not oscillatory) in semi-classical molecular dynamics. This is also seen in TDHF calculation by Bulgac et al. (Univ. Washington)

Fission time scale $\sim 10,000\text{fm/c}$

Time evolution of $^{235}\text{U} + 140\text{ MeV}\ n$ reaction by JQMD

K.Niita, T. Maruyama, Y. Nara, S. Chiba and A. Iwamoto, JAERI-Data/Code 99-042(1999)
Binding energy/A of medium and heavy nuclei by AMD

Sn isotopes

U isotopes
Time evolution of $\langle V \rangle$ before (left) and after (right) scission
Essence of computational method

- **Shape parametrization**: Two-center model in 3 and 4 dimensions

- **Potential energy** -- macroscopic–microscopic method
  - Macroscopic part: Krappe–Nix (double-folded Yukawa model)
  - Microscopic part: two-center shell model or two-center Woods–Saxon model + Strutinsky + BCS
  - Excitation energy dependence of the shell damping: Ignatyuk or Randrup–Moeller prescription

- **Transport coefficients**
  - Werner–Wheeler method for mass tensor (3D and 4D)
  - Wall–and–Window model for friction tensor (3D and 4D)
  - Linear response theory for microscopic mass and friction tensors (3D only) (Ivaniuk et al., JNST DOI: 10.1080/00223131.2015.1070111)
\( p(180\text{MeV}) + {}^{27}\text{Al} \) by AMD

impact parameter: \( 0 < b < 6 \)
force: \( \text{SLy4} \)

cross section (mb)
Nuclear fission by Antisymmetrized Molecular Dynamics: AMD

- Even though the Langevin-model description gives satisfactory results in many aspects of nuclear fission, it has only 3 to 4 degrees-of-freedom to express nuclear shape. It should be better to have ways to describe the nuclear fission by microscopic theories having d.o.f. of all the nucleons.

- AMD is a very successful microscopic model of nuclear reactions and structures (A. Ono et al, Prog. Theor. Phys., 87(5), (1992), 1185-1206)

![Binding Energy](image1)

![Charge distribution](image2)