Resummed Photon spectrum from WIMP annihilation

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Outline

- **History**
  - Wino Dark Matter
  - Indirect dectection of Dark Matter
  - Previous results and need for a new calculation
- **Geography**
  - Kinematics
  - Set up of EFT
  - Resummation
- **Economics**
  - Differential photon spectra
  - Updated exclusion for the Wino

History

Wino Dark Matter

$$\mathcal{L} = \mathcal{L}_{\rm SM} + \bar{\chi}\left(i\not{D} + M_2\right)\chi$$

- $SU_L(L)$ triplet fermion χ, superpartner of weak gauge bosons, free mass parameter M_2 ~ TeV

- Mass eigenstates after electroweak breaking:

$$\boxed{\begin{aligned} \chi^0 &= \chi^3 \\ \chi^\pm &= \frac{1}{\sqrt{2}}(\chi^1 \mp i\chi^2) \end{aligned}}$$

Neutralino : Majorana fermion

Chargino

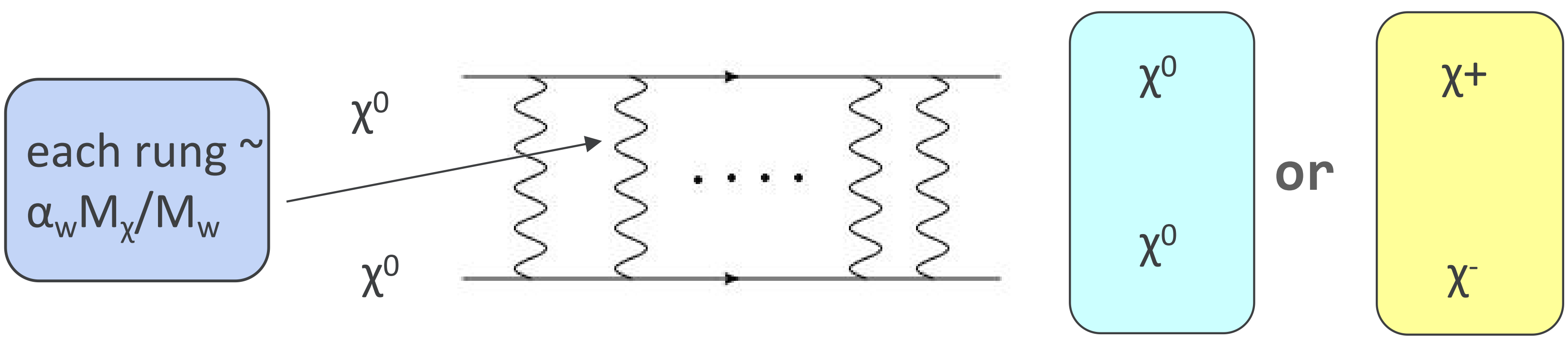
- Mass splitting of 170 MeV from electroweak radiative corrections

- Neutral state χ^0 is the LSP -> Dark matter WINO

Wino annihilation to photons

$$\chi^0\chi^0 \to \gamma + X$$

- Semi- inclusive cross section of annihilation to a hard photon .
- The wino's are non relativistic , v ~ 10^{-3}
- Interaction has two contributions :
- Phase 1 : Well separated slow moving fermions interact via gauge boson exchange

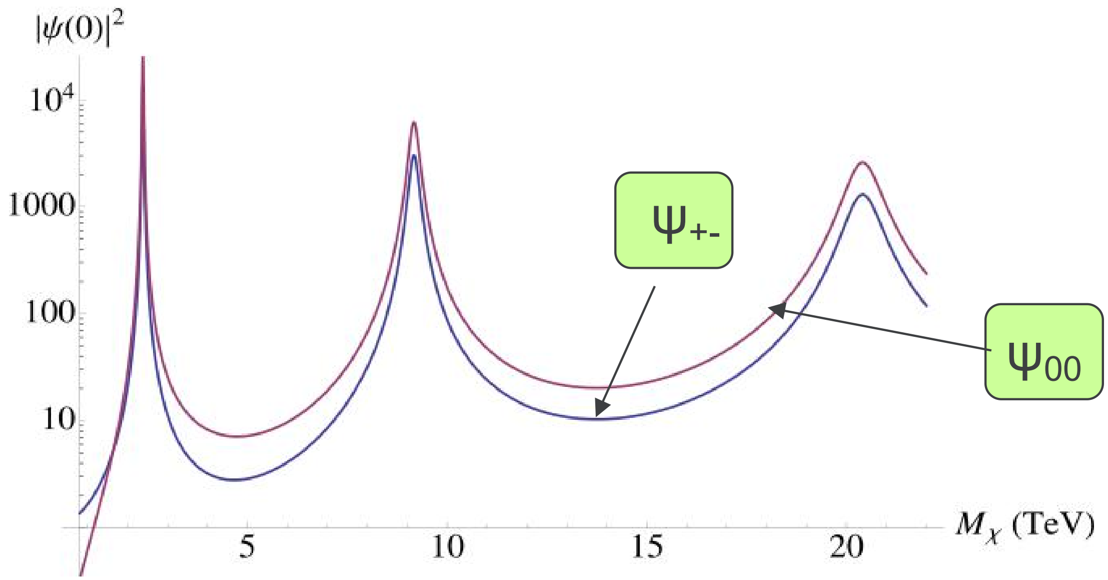


Sommerfeld enhancement

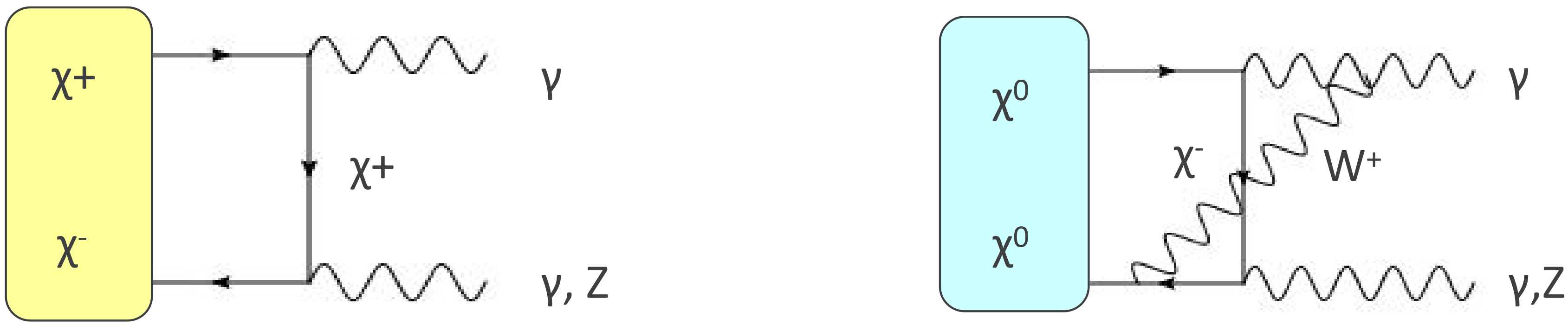
- Effect captured by solving Schrodinger equation with effective potential

$$V(r) = \begin{pmatrix} 2\delta M - \frac{\alpha}{r} - \alpha_W c_W^2 \frac{e^{-m_Z r}}{r} & -\sqrt{2}\alpha_W \frac{e^{-m_W r}}{r} \\ -\sqrt{2}\alpha_W \frac{e^{-m_W r}}{r} & 0 \end{pmatrix}, \quad \Psi = \begin{bmatrix} \Psi_{+-} \\ \Psi_{00} \end{bmatrix}$$

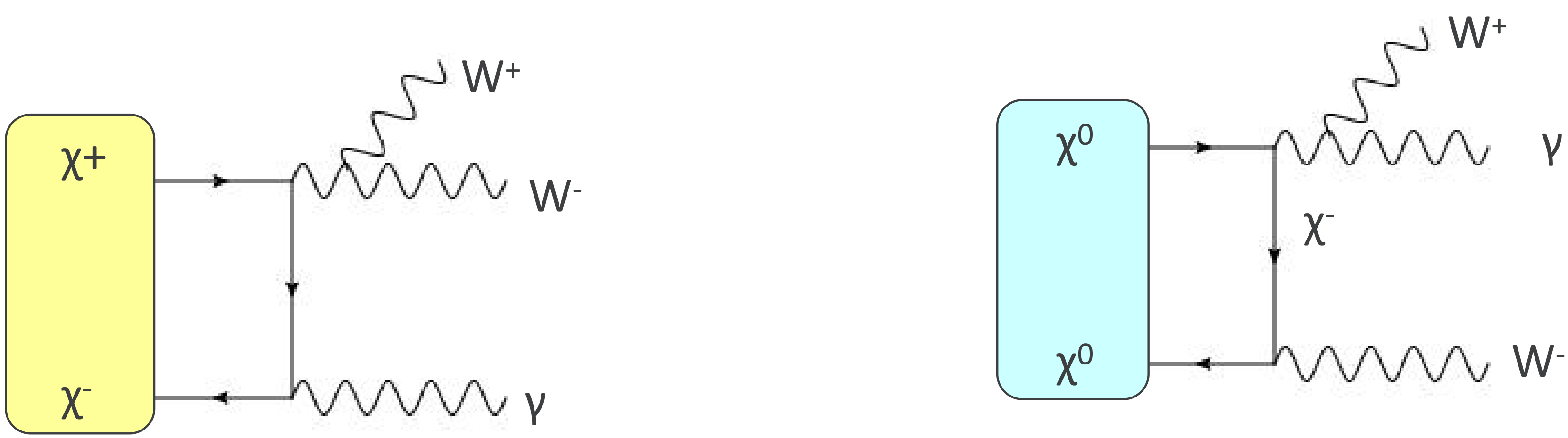
- Sommerfeld enhancement for two channels



Hard scattering to photon

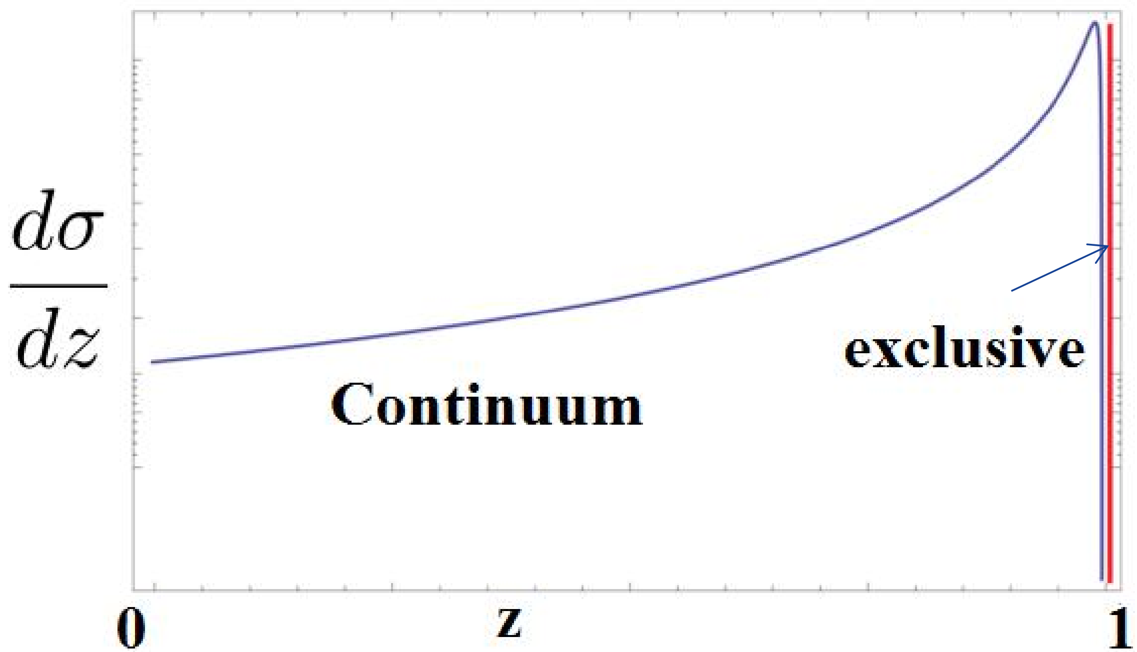


Exclusive cross section



Continuum cross section

What does the spectrum look like?



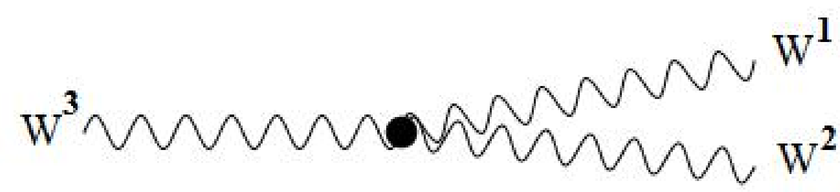
$$\frac{d\sigma}{dz} = \delta(1-z)\left(\sigma_{\gamma\gamma} + \frac{1}{2}\sigma_{\gamma Z}\right) + \mathrm{Continuum}(z)$$

What does the spectrum look like?

- KLN* : An IR safe observable needs to be suffciently inclusive over its final states.

  Assume there is no EW breaking

  A W^3 is indistinguishable from a W^1-W^2 pair created from its collinear splitting



- These apear as **Sudakov double logs** $\alpha_w \ln^2(M_\chi/M_W)$

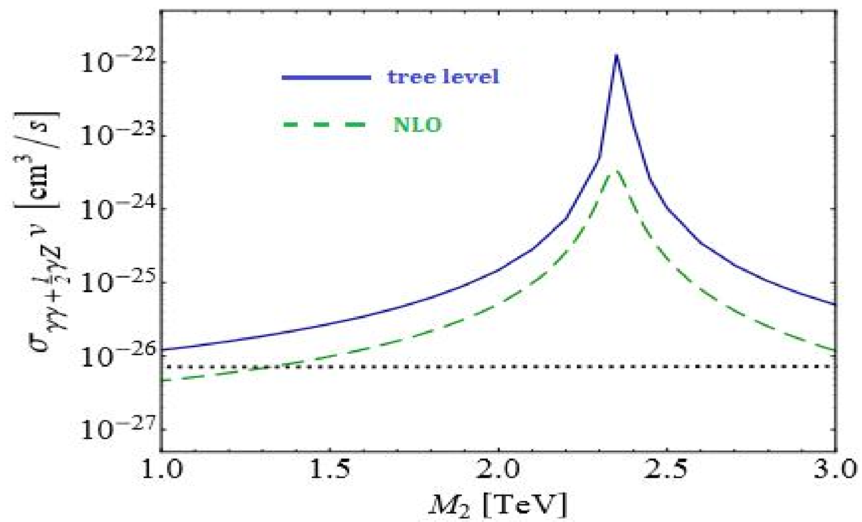
- Also contains $\alpha_w \ln^2(1\text{-}z)$ from incomplete phase space cancellations

*Kinoshita–Lee–Nauenberg theorem

Previous results

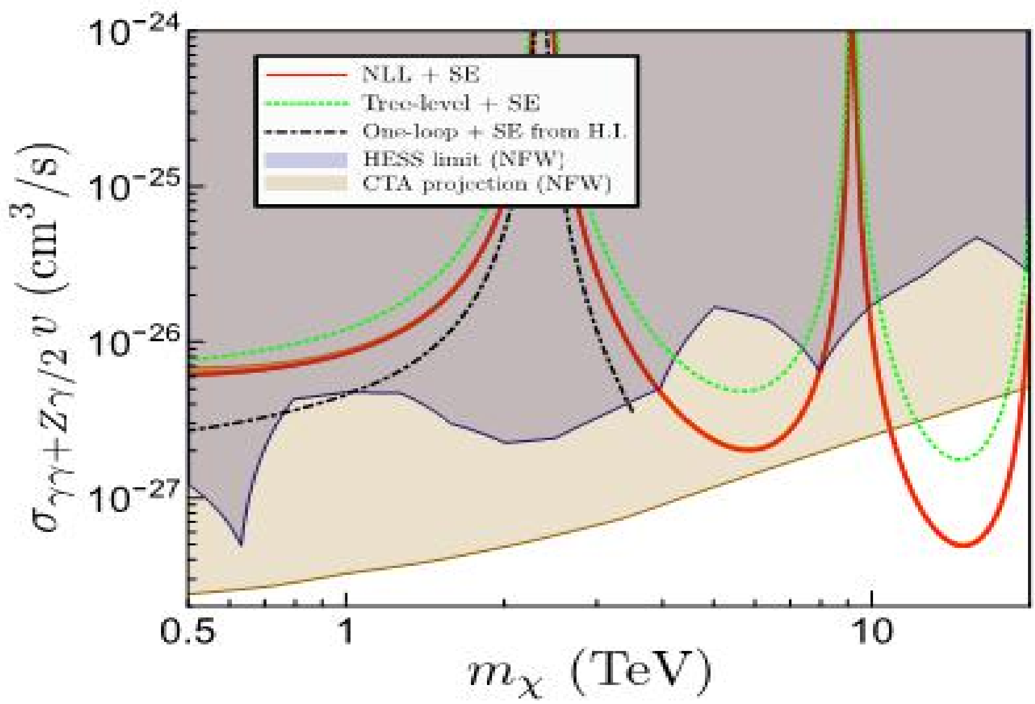
- **Wino under siege ：**

  T. Cohen, M. Lisanti, A. Pierce and T. R. Slatyer, JCAP { 1310}, 061 (2013)



- **Exclusive cross section at NLL**

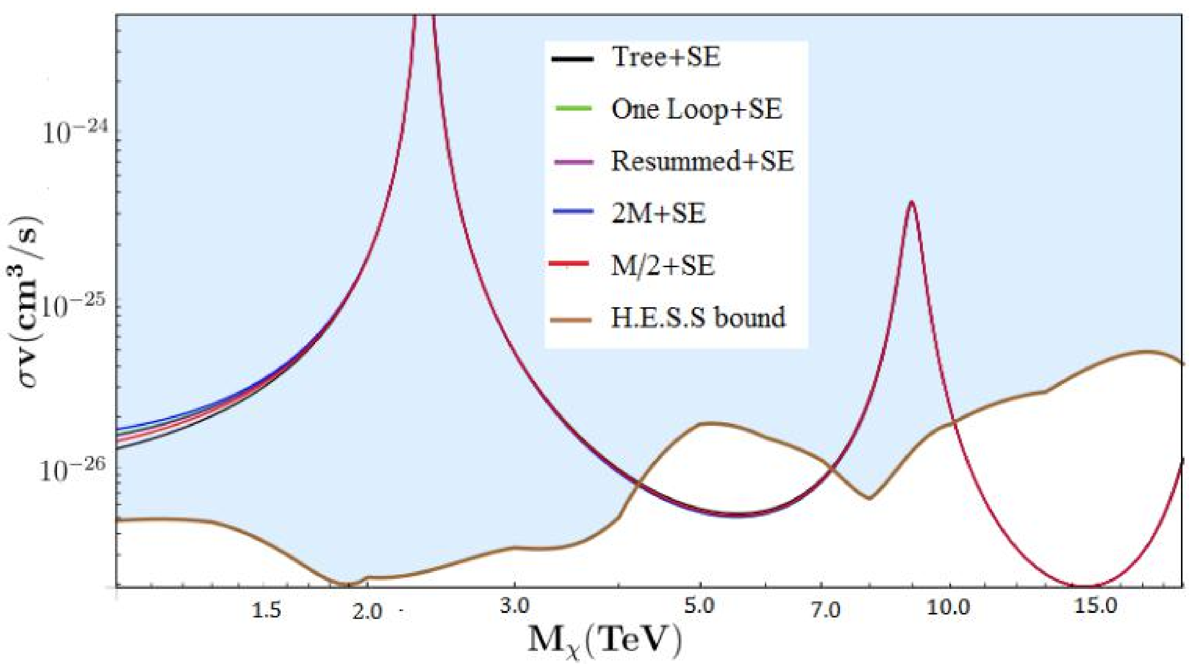
  G. Ovanesyan, T. R. Slatyer and I. W. Stewart, Phys. Rev. Lett. 114, no. 21, 211302 (2015)



- **Semi-inclusive annihilation**

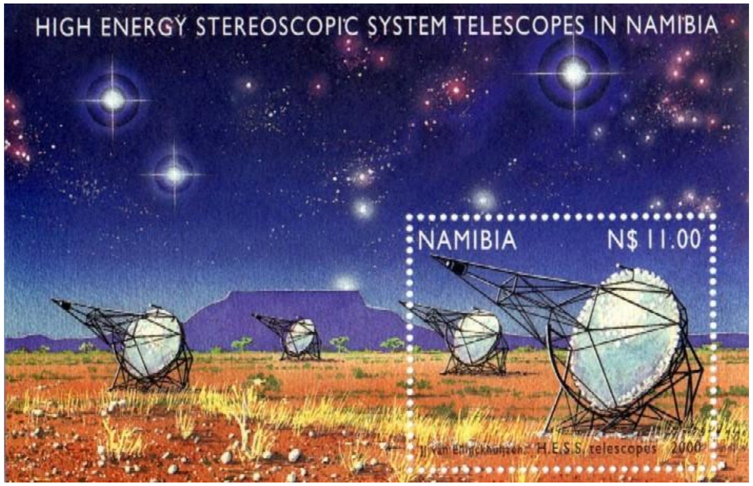
M. Baumgart, I. Z. Rothstein and V. Vaidya, JHEP 1504, 106 (2015)

$$\sigma = \left(\sigma_{\gamma\gamma} + \frac{1}{2}\sigma_{\gamma Z}\right) + \int_0^1 dz \mathrm{Continuum}(z)$$



Indirect Dark Matter Detection

- Detection of high energy gamma rays from annihilation of cold Dark Matter particles



- Air Cherenkov Telescope : HESS - High energy stereoscopic system , Namibia

- HAWC: High altitude water Cherenkov Observatory, Mexico

What does HESS see?

$$\frac{d(\sigma v)_\gamma}{dE_\gamma} n^2 \longrightarrow$$

Number of photons per unit volume of dark matter annihilation per unit time

$$\frac{d\Phi_\gamma}{dE} = \frac{d(\sigma v)_\gamma}{dE_\gamma} \int d\Omega \int_{LOS} n^2 \longrightarrow$$

Flux of photons from the direction of the galactic center

$$\frac{d\Phi_\gamma}{dE} = \frac{1}{m_{DM}^2} \frac{d(\sigma v)_\gamma}{dE_\gamma} \boxed{\int d\Omega \int_{LOS} \rho^2}$$

J Factor

- The telescope has a finite energy resolution which ranges from 11- 20% of the DM mass over the range 1-20 TeV

HESS Constraints

- HESS assumes a line spectrum while setting constraints and ignores the continuum inspite of having a finite resolution.

$$\Phi_\gamma^{\mathrm{HL}} = \int_{\mathrm{bin}} dE \, \frac{d\Phi_\gamma}{dE} =$$

$$\frac{(\sigma v)_{\gamma\gamma+1/2\gamma Z} J}{m_{DM}^2} \int_{bin} dE \delta(E - m_{DM}) = \frac{(\sigma v)_{\gamma\gamma+1/2\gamma Z} J}{m_{DM}^2}$$

- Set limits on $(\sigma v)_{\gamma\gamma+1/2\gamma Z} J$ at a given DM mass

- Two changes are needed
  - ✓ Include continuum contribution near z = 1 while setting limits
  - ✓ A theory calculation that gives accurate spectrum at and near z=1.

Geography

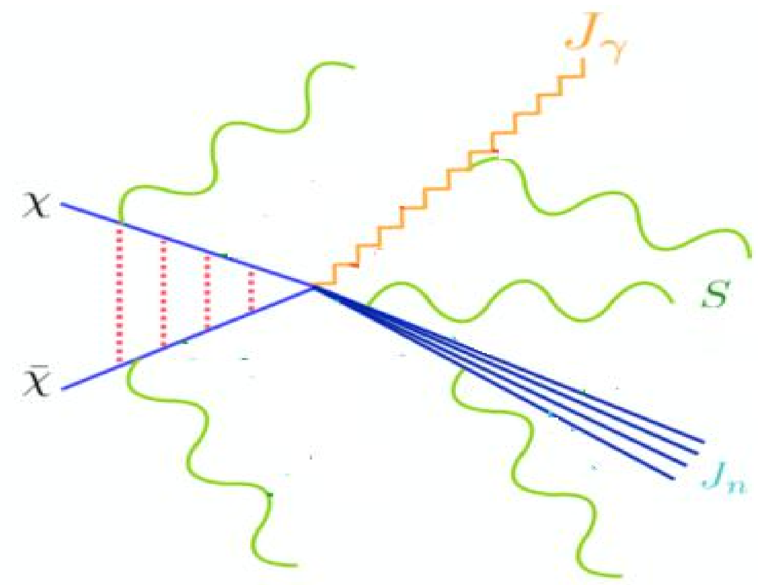
Factorization

- **Kinematics: Light-cone co-ordinates**

$$k^\mu = (k^+, k^-, k_\perp) \equiv (k^0 - k^3, k_0 + k^3, k_\perp)$$

$$\text{photon momentum}$$
$$q^\mu = 2M_\chi(z, 0, 0) \implies q^2 = 0$$

$$\text{recoiling jet}$$
$$p^\mu = 2M_\chi((1-z), 1, 0) \implies p^2 = (4M_\chi^2(1-z))$$



SCET (Soft Collinear effective theory) I

$$\lambda = \sqrt{1-z}$$

$$\text{Anti-Collinear}: \ 2M_\chi(\lambda^2, 1, \lambda)$$

$$\text{ultrasoft} \ 2M_\chi(\lambda^2, \lambda^2, \lambda^2)$$

Factorized cross section

$$\frac{1}{E_\gamma}\frac{d\sigma}{dE} = \sigma_0 L^{a'b'ab} J_\gamma \int \frac{dk^+}{2\pi} J_n(k^+) \int \frac{dq^+}{2\pi} \left(\sum_{(i,j)=1}^{2} C_{ij} S_{ij}^{a'b'ab}(q^+) \right) \delta(2M_\chi(1-z) - k^+ - q^+)$$

$$L^{a'b'ab} = \langle p_1 p_2 | \left(\chi_v^{a'T} i\sigma_2 \chi_v^{b'} \right)^\dagger |0\rangle\langle 0| \left(\chi_v^{aT} i\sigma_2 \chi_v^{b} \right) |p_1 p_2\rangle$$

DM Wavefunction

$$J_n(k^+) = \langle 0| B_{n\perp}^d(0) \delta(k^+ - \mathcal{P}^+) \delta(M_\chi - \mathcal{P}^-/2) \delta^2(\vec{\mathcal{P}}_\perp) |X_n\rangle\langle X_n| B_{n\perp}^d(0)|0\rangle$$

Recoil jet function

$$J_\gamma = \langle 0| B_{\bar{n}\perp}^c(0)|\gamma\rangle\langle\gamma| B_{\bar{n}\perp}^c(0)|0\rangle$$

Photon jet function

$$S_{11}^{a'b'ab} = \Pi_{X_{US}} \langle 0| \left(Y_n^{3f'} Y_{\bar{n}}^{dg'} \right)^\dagger (x) |X_{US}\rangle\langle X_{US}| \left(Y_n^{3f} Y_{\bar{n}}^{dg} \right)(0)|0\rangle \delta^{f'g'} \delta^{a'b'} \delta^{fg} \delta^{ab}$$

U-soft function

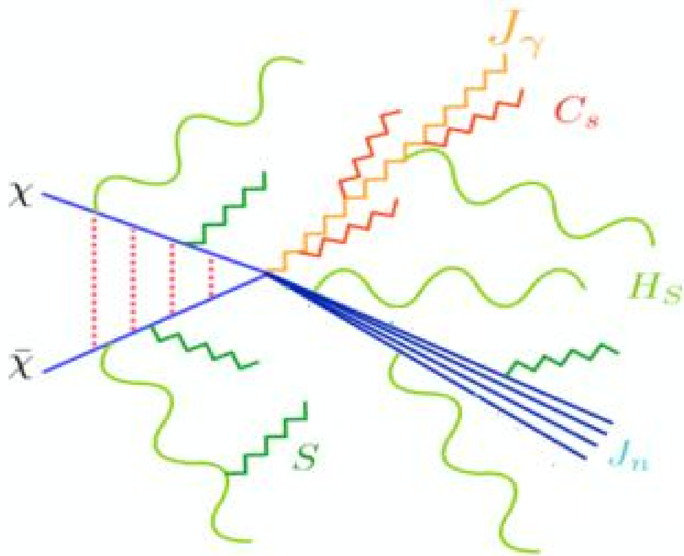
$$Y_n^{(r)}(x) = \mathbf{P} \exp\left[ig \int_{-\infty}^{0} \mathrm{d}s\, n \cdot A_{us}^a(x+sn) T_{(r)}^a \right]$$

Ultrasoft Wilson line

Refactorizing the U-Soft function

The U-Soft function S depends on two scales : Mx(1-z), Mw.

Hence further factorization is needed to separate these scales.



Collinear-soft: $2M_\chi(1-z)(1,\tilde{\lambda}^2,\tilde{\lambda})$

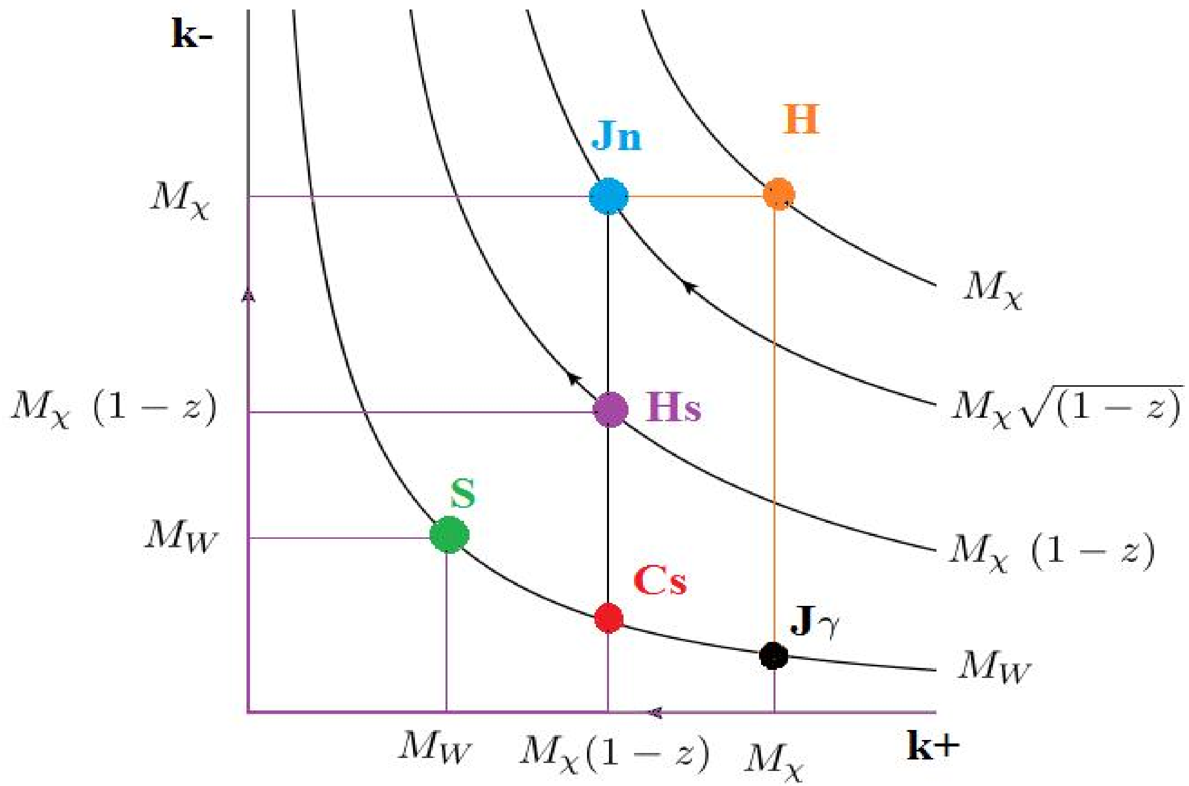
soft: $2M_\chi(1-z)(\tilde{\lambda},\tilde{\lambda},\tilde{\lambda})$

$$\tilde{\lambda} = M_W/(2M_\chi(1-z))$$

$$S'^{\,aba'b'}_{ij} = H_{S,ijkl}\left(C^{\{A\}}_{S,k}S^{\{B\}}_l\right)^{aba'b'}$$

$$(\mathrm{C+S})^{aba'b'}_{12} = \left[Y^{ce}_n Y^{B'e}_{\bar{n}}\delta(q^+ - \mathcal{P}^+)|X_{CS}\rangle\langle X_{CS}|Y^{c'g'}_n Y^{A'g'}_{\bar{n}}\right]\left[S^{3c}_n S^{3c'}_n S^{a'A'}_v S^{b'B'}_v\right]\delta^{ab}$$

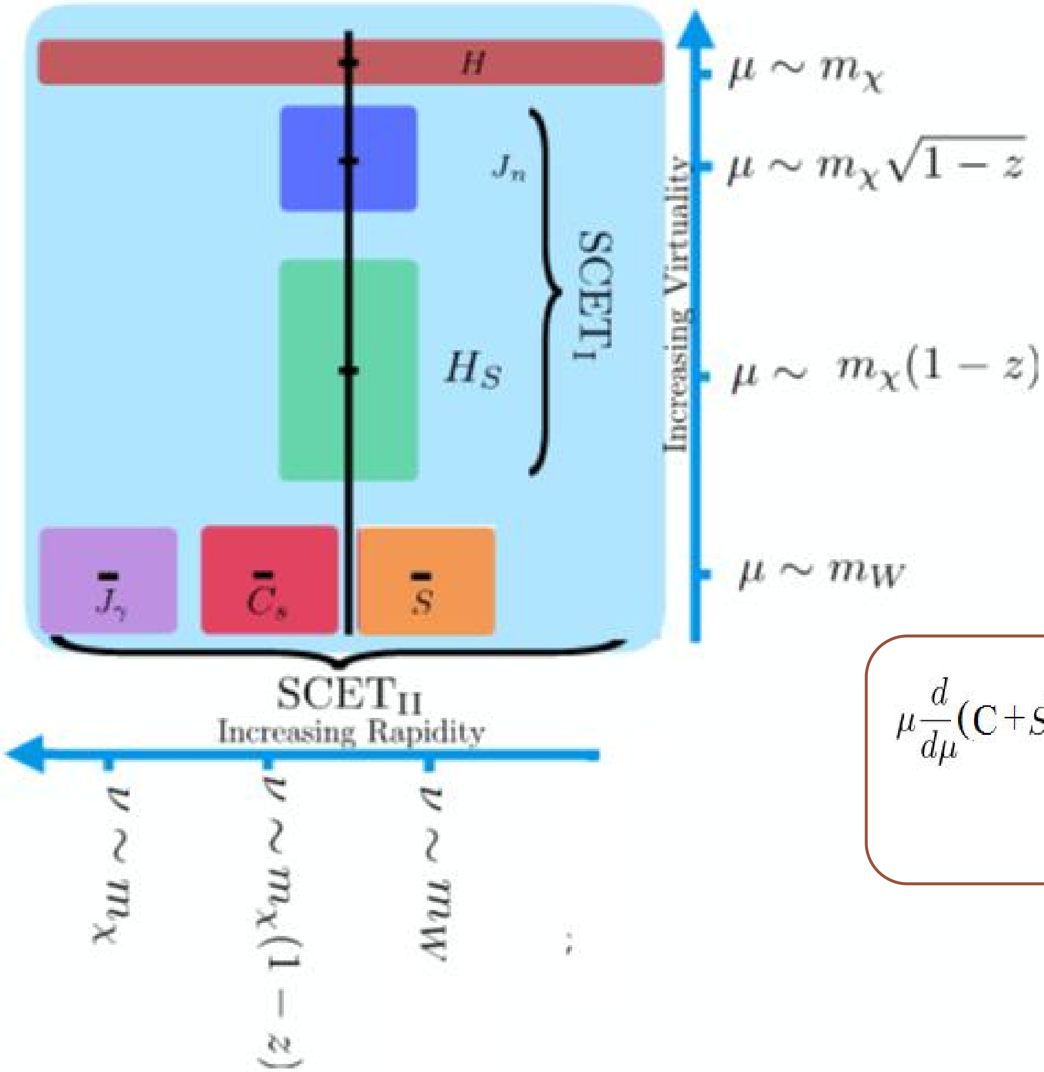
Factorization



Hyperbolas of invariant masses in k^+- k^- plane

$$\frac{d\hat{\sigma}}{dz} = H(M_\chi,\mu)\, J_\gamma(m_W,\mu,\nu)\, S(m_W,\mu,\nu)\times$$
$$J_{\bar{n}}(M_\chi,(1-z),\mu)\otimes H_S(M_\chi,(1-z),\mu)\otimes C_S(M_\chi,(1-z),m_W,\mu,\nu)\,.$$

Anomalous dimensions



$$\gamma_\mu^H = -2C_2(W)\frac{\alpha_W}{\pi}\log\left(\frac{\mu^2}{(2M_\chi)^2 z}\right)$$

$$\gamma_\mu^n = 2C_2(W)\frac{\alpha_W}{\pi}\log\left(\frac{\mu^2 s e^{\gamma_E}}{2M_\chi}\right)$$

$$\gamma_\mu^\gamma = 2C_2(W)\frac{\alpha_W}{\pi}\log\left(\frac{\nu}{2E_\gamma}\right)$$

$$\gamma_\mu^{H_S} = -6\frac{\alpha_W}{\pi}\log(\mu s)$$

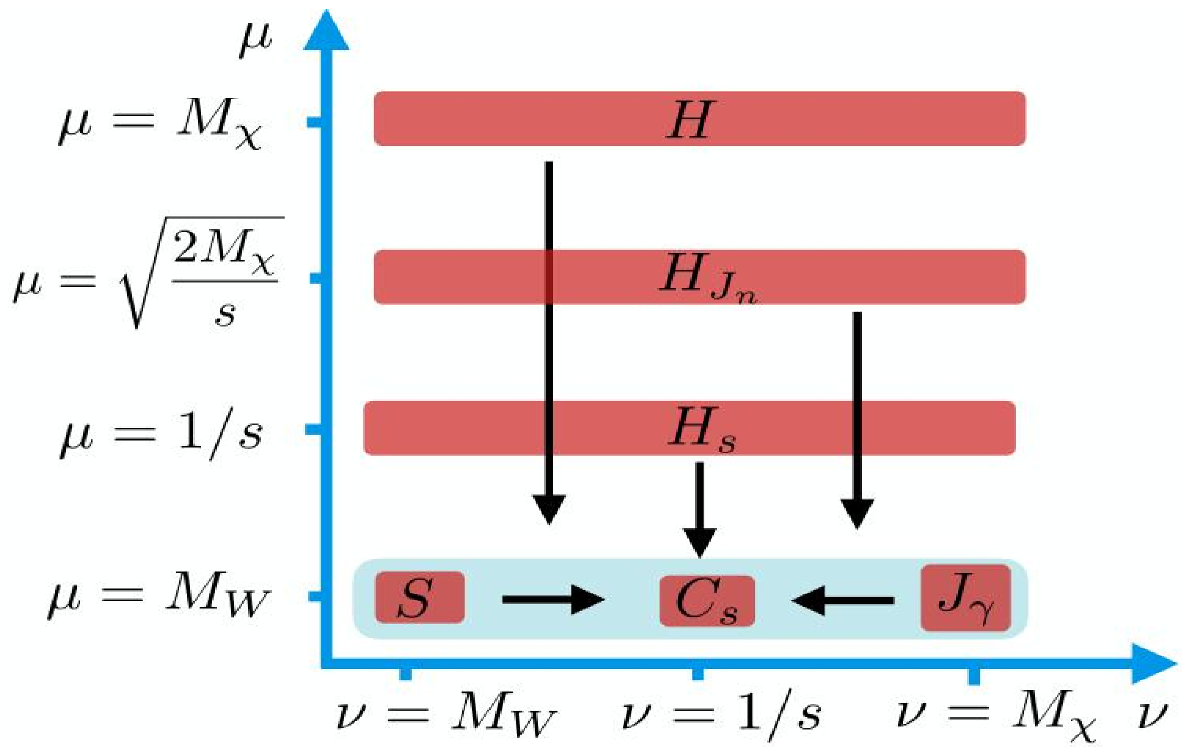
$$\mu\frac{d}{d\mu}(C+S)_{12}^{aba'b'} = \left(-2C_2(W)\frac{\alpha_W}{\pi}\log(\nu s) + 3C_2(W)\frac{\alpha_W}{\pi}\log(\mu s)\right)(C+S)_{12}^{aba'b'} - C_2(W)\frac{\alpha_W}{\pi}\log(\mu s)(C+S)_{11}^{aba'b'}$$

$$\gamma_\nu^\gamma = 2C_2(W)\frac{\alpha_W}{\pi}\log\left(\frac{\mu}{M}\right)$$

$$\gamma_\nu^{(C+S)} = -2C_2(W)\frac{\alpha_W}{\pi}\log\left(\frac{\mu}{M}\right)$$

- 2Mx(1-z) ~ 1/s

Resummation at LL



Resummation path in the μ-ν plane

$$\frac{d\sigma}{dz} = \frac{\pi \alpha_W^2 \sin^2\theta_W}{M_\chi v} e^{-2C_2(W)\frac{\alpha_W}{\pi}\log^2\left(\frac{2M_\chi}{M}\right)} \mathcal{L}^{-1}\Bigg\{ e^{C_2(W)\frac{\alpha_W}{2\pi}\log^2\left(\frac{M^2 s}{2M_\chi}\right)} \times \left\{ \frac{4}{3}|s_{00}|^2 \tilde{f}_- + 2|s_{0\pm}|^2 \tilde{f}_+ + \frac{2\sqrt{2}}{3}\left(s_{00}s_{0\pm}^* + c.c\right)\tilde{f}_- \right\} \Bigg\}$$

$$\tilde{f}_\pm = 1 \pm e^{-\frac{3}{2}C_2(W)\frac{\alpha_W}{\pi}\log^2(Ms)}$$

Resummed cross-section

$$\frac{d\sigma}{dz} = \frac{\pi\alpha_W^2 \sin^2\theta_W}{2M_\chi^2 v} e^{\left[-2C_2(W)\frac{\alpha_W}{\pi}\log^2\left(\frac{2M_\chi}{M}\right)\right]} \left\{ \boxed{(F_0 + F_1)\delta(1-z)} \right.$$

$$+ \left(C_2(W)\frac{\alpha_W}{\pi}\log\left(\frac{4M_\chi^2(1-z)}{M^2}\right) \frac{e^{\left[C_2(W)\frac{\alpha_W}{2\pi}\log^2\left(\frac{M^2}{4M_\chi^2(1-z)}\right)\right]}}{1-z} \right)_+ F_0$$

$$+ \left[\left(C_2(W)\frac{\alpha_W}{\pi}\log\left(\frac{4M_\chi^2(1-z)}{M^2}\right) + 3C_2(W)\frac{\alpha_W}{\pi}\log\left(\frac{M}{2M_\chi(1-z)}\right) \right) \right.$$

$$\left. \left. \times \left(\frac{e^{\left[-\frac{3}{2}C_2(W)\frac{\alpha_W}{\pi}\log^2\left(\frac{M}{2M_\chi(1-z)}\right) + C_2(W)\frac{\alpha_W}{2\pi}\log^2\left(\frac{M^2}{4M_\chi^2(1-z)}\right)\right]}}{1-z} \right) \right]_+ F_1 \right\}$$

$$F_1 = \left(-\frac{4}{3}|s_{00}|^2 + 2|s_{0\pm}|^2 - \frac{2\sqrt{2}}{3}\left(s_{00}s_{0\pm}^* + c.c\right)\right).$$

$$F_0 = \left(\frac{4}{3}|s_{00}|^2 + 2|s_{0\pm}|^2 + \frac{2\sqrt{2}}{3}\left(s_{00}s_{0\pm}^* + c.c\right)\right).$$

Sommerfeld factors

$$\int_0^\infty dq \left(\frac{\ln\left(\frac{q}{U}\right)}{q}\right)_+ F(q) = \int_U^\infty dq \left(\frac{\ln\left(\frac{q}{U}\right)}{q}\right) F(q)$$

Plus functions

Ressummed cross section

- We can identify three distinct logarithms :

$$\frac{\alpha_W}{\pi} \log^2\left(\frac{2M_\chi}{M}\right)$$

Sudakov double logs
from virtual corrections

$$\frac{\alpha_W}{\pi}\left(\frac{\log\left(\frac{4M_\chi^2(1-z)}{M^2}\right)}{1-z}\right)_+$$

Plus function with a threshold

$$z_2 = 1 - M^2/(4M_\chi^2)$$

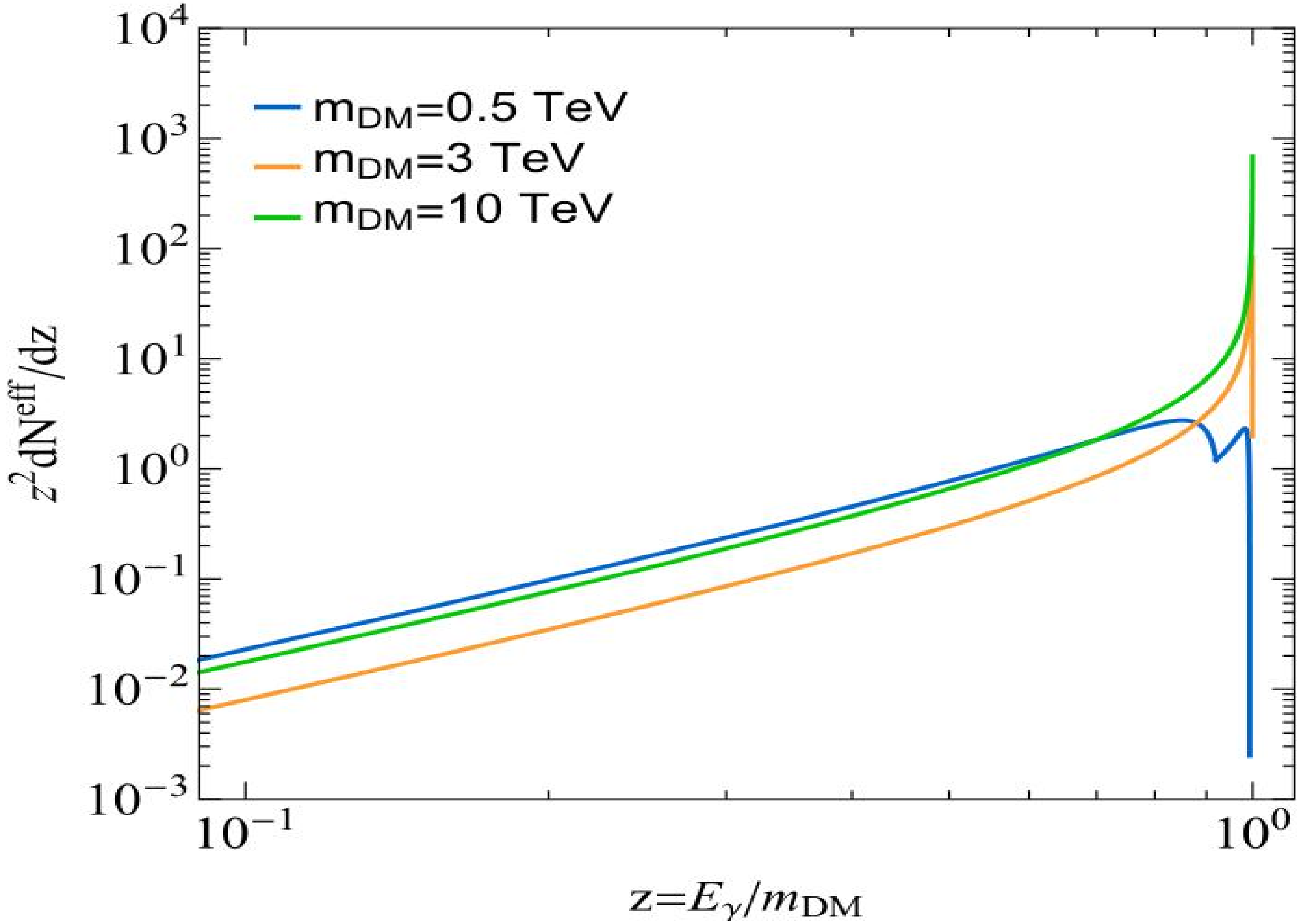
$$\frac{\alpha_W}{\pi}\left(\frac{\log\left(\frac{M}{2M_\chi(1-z)}\right)}{1-z}\right)_+$$

Plus function with a threshold

$$z_1 = 1 - M/(2M_\chi)$$

Economics

Differential Spectra



Transition to inclusive and exclusive cases

- Exclusive limit

$$\int_{1-M^2/(4M_\chi^2)}^{1} dz \frac{d\sigma}{dz} = 2\frac{\pi\alpha_W^2 \sin^2\theta_W}{M_\chi^2 v} e^{\left[-2C_2(W)\frac{\alpha_W}{\pi}\log^2\left(\frac{2M_\chi}{M}\right)\right]} |s_{0\pm}|^2 .$$

G. Ovanyesan, T. Slatyer, I. Stewart

- Inclusive limit

$$\int_0^1 dz \frac{d\sigma}{dz} = \frac{\pi\alpha_W^2 \sin^2\theta_W}{2M_\chi^2 v} \left\{ \frac{4}{3}|s_{00}|^2 f_- + 2|s_{0\pm}|^2 f_+ + \frac{2\sqrt{2}}{3}\left(s_{00}s_{0\pm}^* + c.c\right) f_- \right\}$$

M. Baumgart, I. Rothstein, V. Vaidya

Modified constraints

- HESS line flux

$$\Phi_\gamma^{\mathrm{HL}} = \frac{(\sigma v)_{\gamma\gamma+1/2\gamma Z} J}{m_{DM}^2} \int_{bin} dE\delta(E - m_{DM}) = \frac{(\sigma v)_{\gamma\gamma+1/2\gamma Z} J}{m_{DM}^2}$$

- HESS true flux

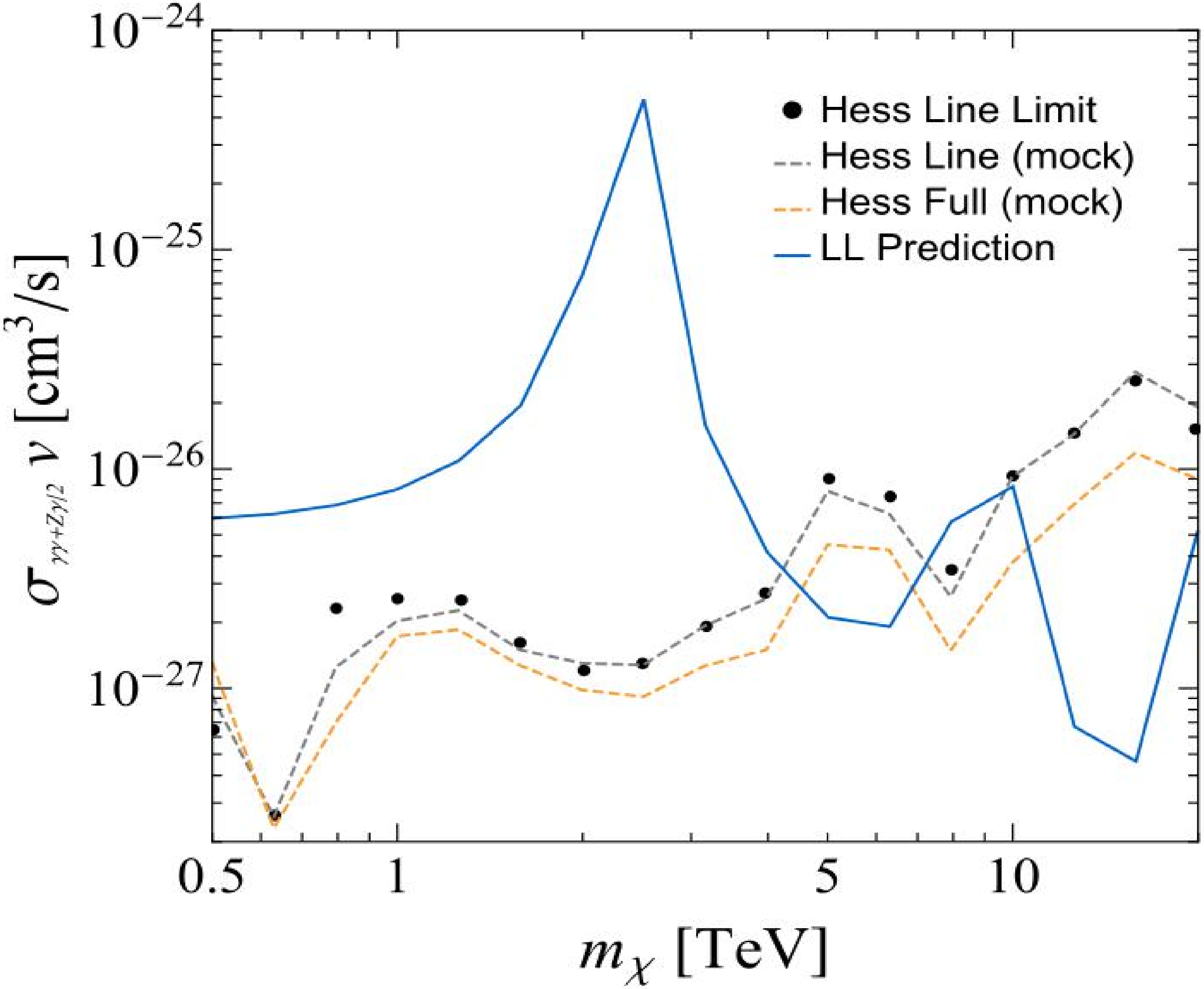
$$\Phi_\gamma^{\mathrm{HR}} = \frac{(\sigma v)_{\gamma\gamma+1/2\gamma Z} J}{m_{DM}^2} \left[1 + \frac{1}{\alpha} \int_{\mathrm{bin}} dE \,\mathrm{continuum} \right]$$

Set limits on line + continuum

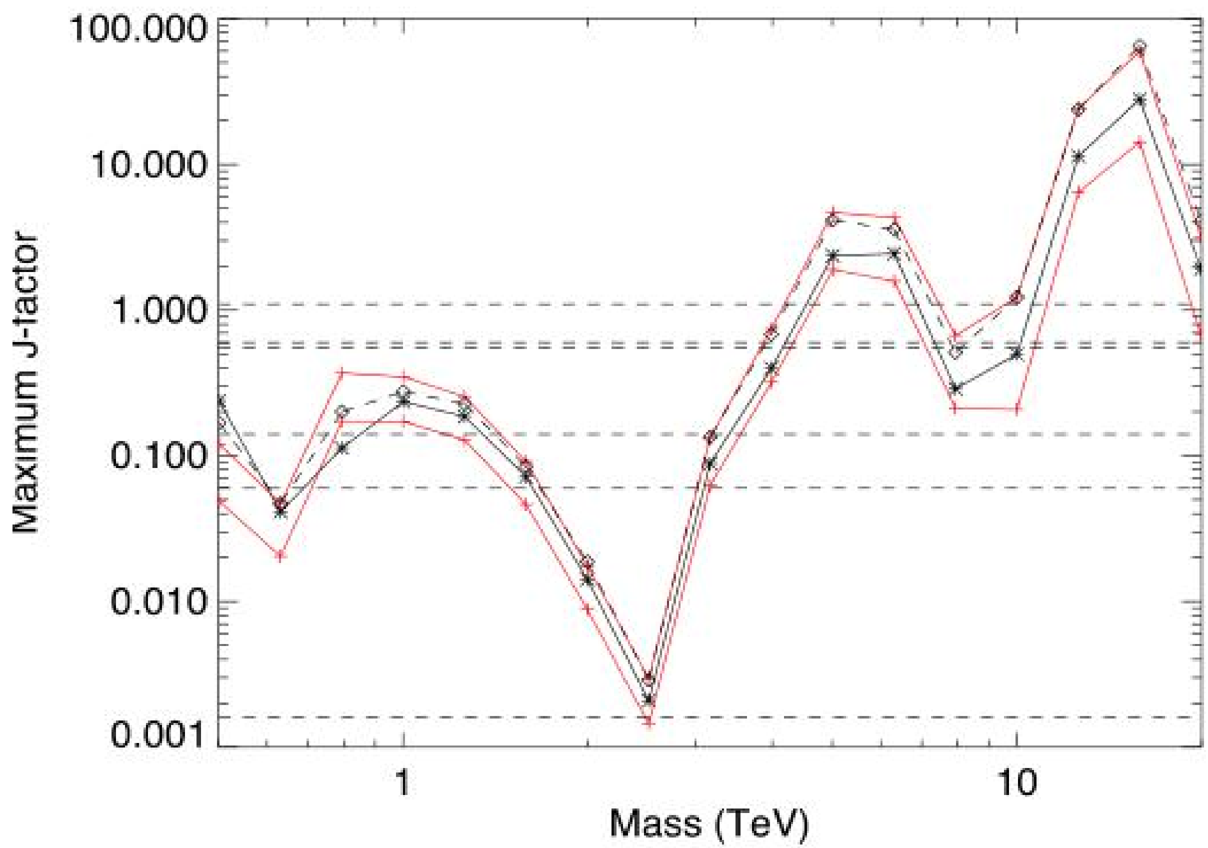
or

strenghten limits on the line(exlusive) cross section

Exclusion plot for WINO



Limits on J factor



Summary and Outlook

- Most accurate calculation of resummed WIMP photon spectrum
- Supercedes all earlier calculations
- Demonstrates the need to include continuum contribution in photon flux for DM data analysis
- Updated constraints on Heavy DM
- EFT developed can be easily used for other heavy DM models as well as for general collider physics
- Apply these techniques for the Higgsino
- Collaborate with existing and upcoming experiments for refining constraints