

LANL FIESTA: Fission School & Workshop
SEPTEMBER 8–12, 2014, ELDORADO HOTEL,
SANTA FE, NM, USA

**Fission Barriers and Fission-Fragment Yields
in the Region $170 < A \leq 330$**

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Collaborators on this and other projects:

W. D. Myers, J. Randrup(LBL), H. Sagawa (Aizu), S. Yoshida (Hosei), T. Ichikawa(YITP), A. J. Sierk(LANL), A. Iwamoto (JAEA), S. Aberg (Lund), R. Bengtsson (Lund), S. Gupta (IIT, Ropar), and many experimental groups (e. g. K.-L. Kratz (Mainz), H. Schatz (MSU), A. Andreyev (York), K. Nishio (JAEA) ...).

More details about fission (PRC 79 064304, PRC 84 034613, PRC 88 064606), other projects (beta-decay, masses), associated ASCII data files, interactive access to data (type in Z, A and get specific data, contour maps) and figures are at

<http://t2.lanl.gov/nis/molleretal/>

Potential Energy of Deformation

We use the macroscopic-microscopic method introduced by Swiatecki and Strutinsky:

$$E_{\text{pot}}(\text{shape}) = E_{\text{macr}}(\text{shape}) + E_{\text{micr}}(\text{shape}) \quad (1)$$

The macroscopic term is calculated in a liquid-drop type model (for a specific deformed shape).

The microscopic correction is determined in the following steps

1. A shape is prescribed
2. A single-particle potential with this shape is generated. A spin-orbit term is included.
3. The Schrödinger equation is solved for this deformed potential and single-particle levels and wave-functions are obtained
4. The shell correction is calculated by use of Strutinsky's method.
5. The pairing correction is calculated in the BCS or Lipkin-Nogami method.

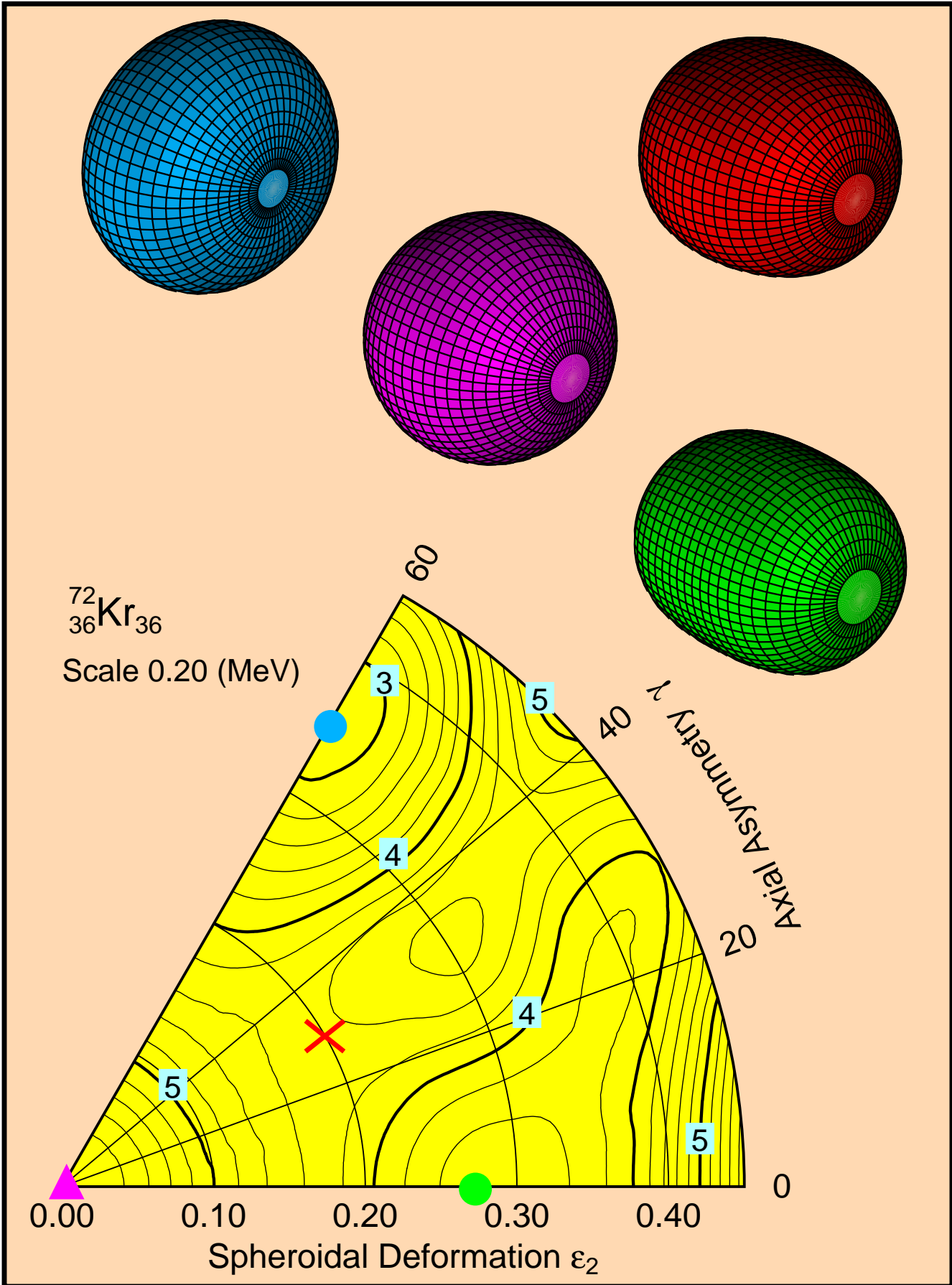
Shape Parameterizations

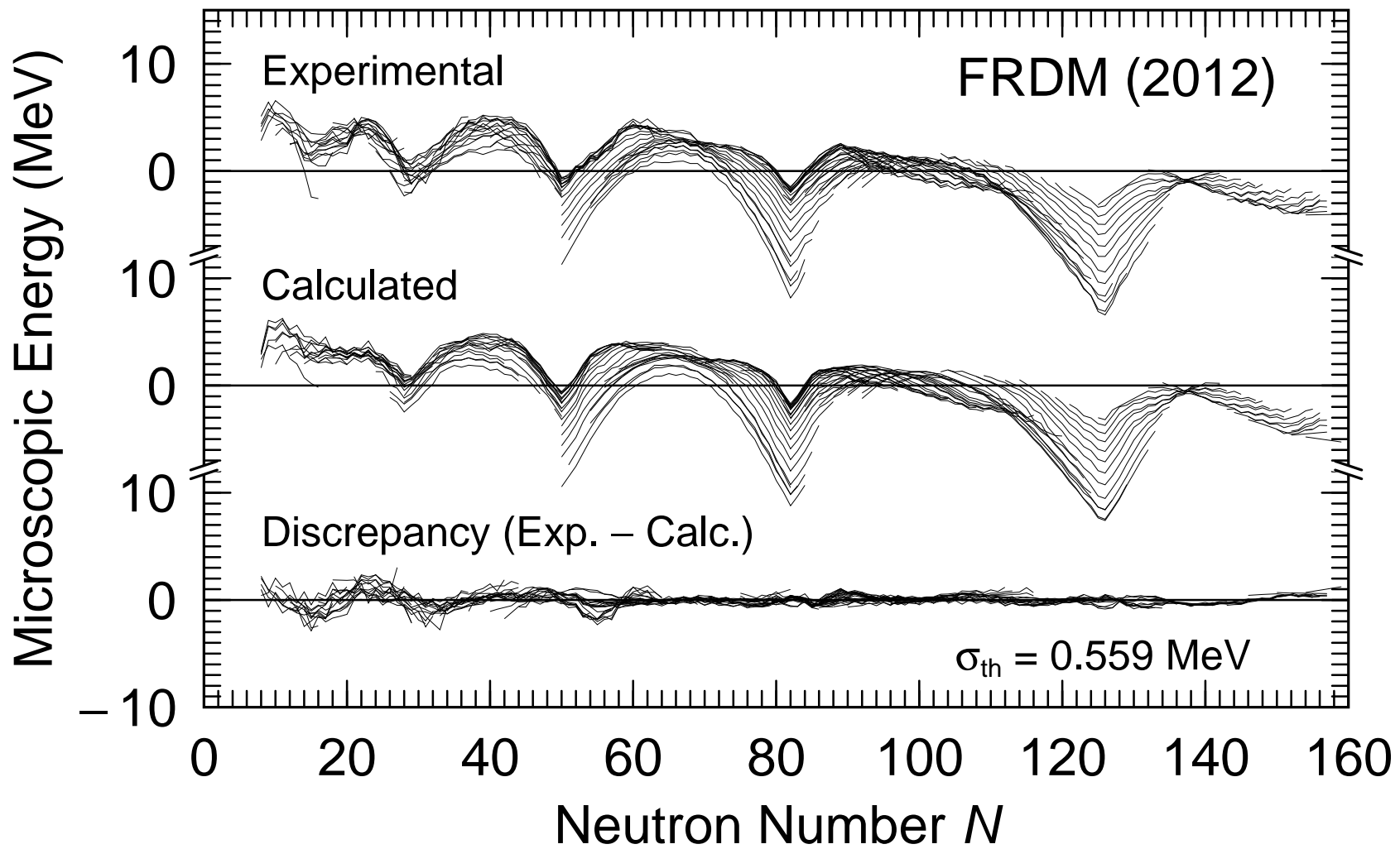
For small distortions we use multipole expansions, for example the β parameterization:

$$r(\theta, \phi) = R_0 \left(1 + \sum_{l=1}^{\infty} \sum_{m=-l}^l \beta_{lm} Y_l^m \right)$$

For large deformations near the outer saddle in the actinide region or beyond we use the three-quadratic-surface parameterization:

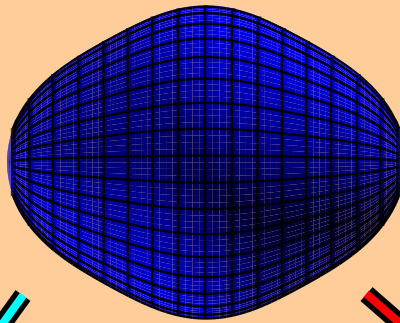
$$\rho(z)^2 = \begin{cases} a_1^2 - \frac{a_1^2}{c_1^2} (z - l_1)^2, & l_1 - c_1 \leq z \leq z_1 \\ a_2^2 - \frac{a_2^2}{c_2^2} (z - l_2)^2, & z_2 \leq z \leq l_2 + c_2 \\ a_3^2 - \frac{a_3^2}{c_3^2} (z - l_3)^2, & z_1 \leq z \leq z_2 \end{cases}$$



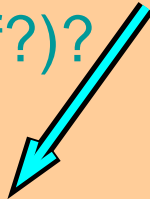


In fission, what are the shapes and related energies involved in the transition from a single ground-state shape to two separated fission fragments?

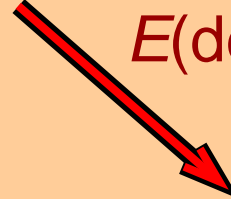
^{234}U



$E(\text{def}?)?$



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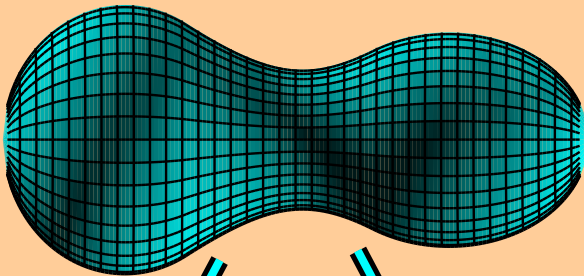


^{234}U : Asymmetric valley at $Q_2 = 76$

$\epsilon_{f1} = -0.1000$ $\epsilon_{f2} = 0.2500$ $M_H/M_L = 135.7/98.3$

^{234}U : Symmetric valley at $Q_2 = 76$

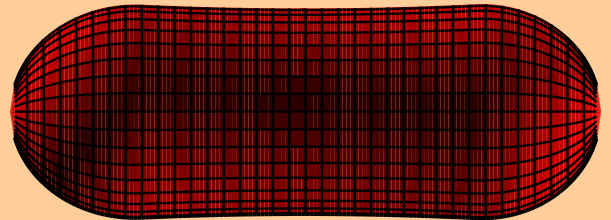
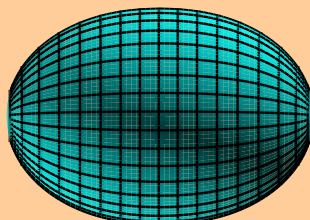
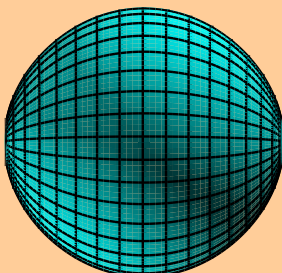
$\epsilon_{f1} = 0.1500$ $\epsilon_{f2} = 0.1000$ $M_H/M_L = 119.3/114.7$



^{136}I



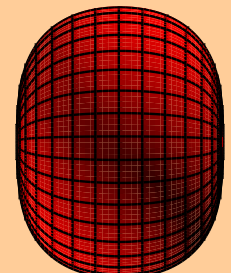
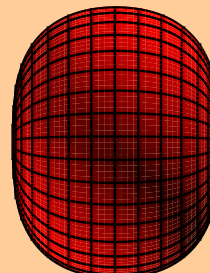
^{98}Y

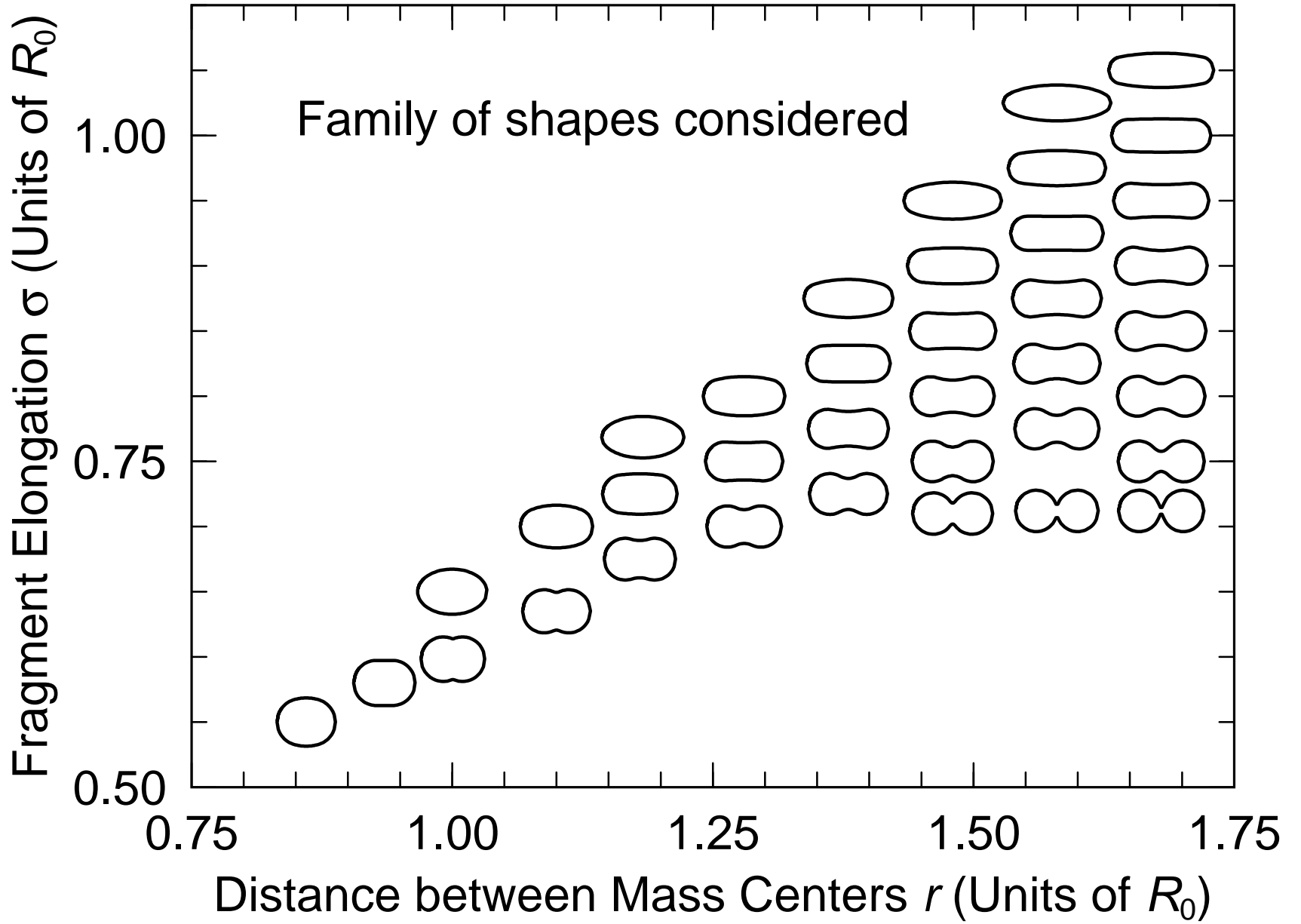


^{117}Pd

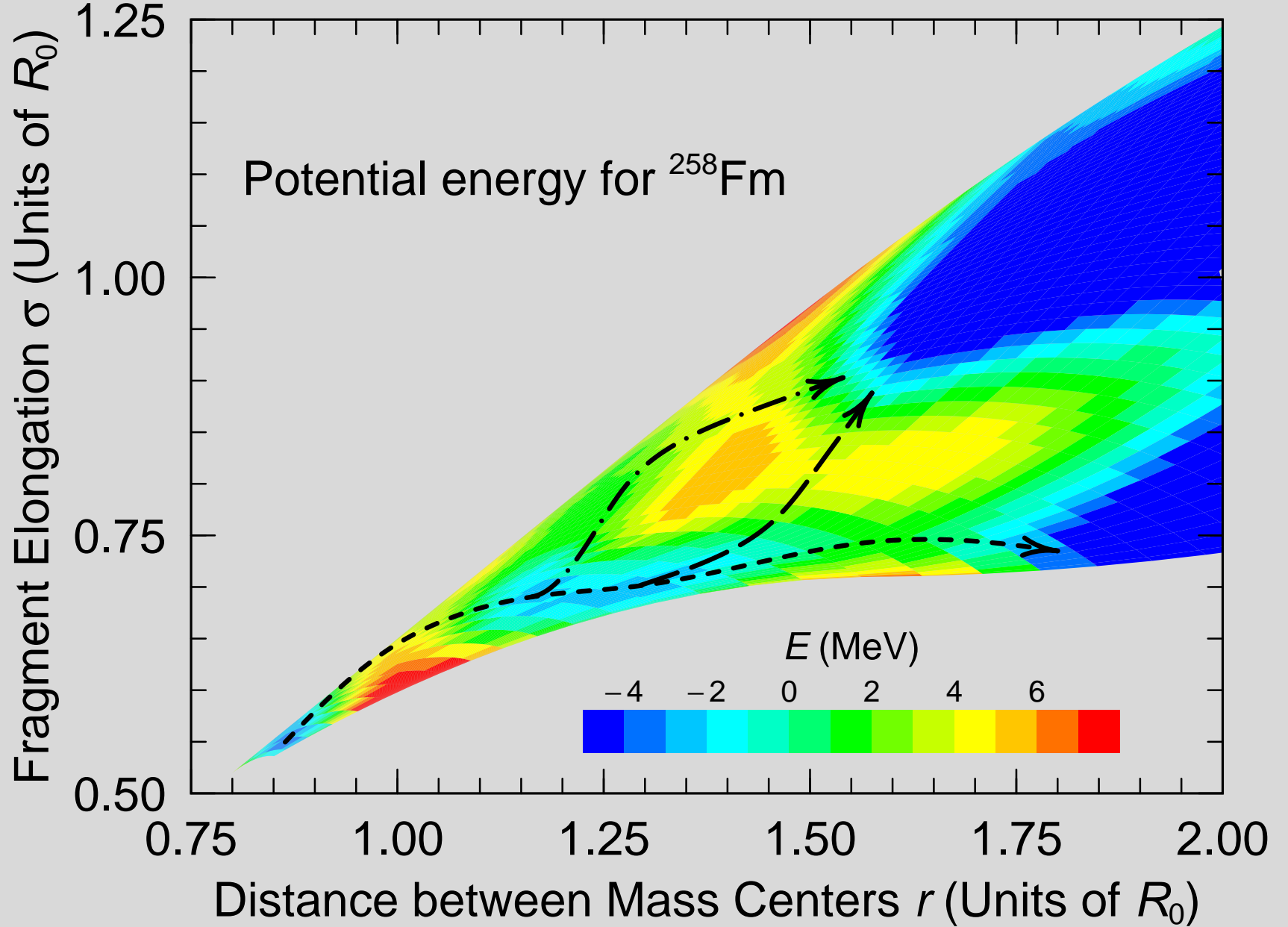


^{117}Pd

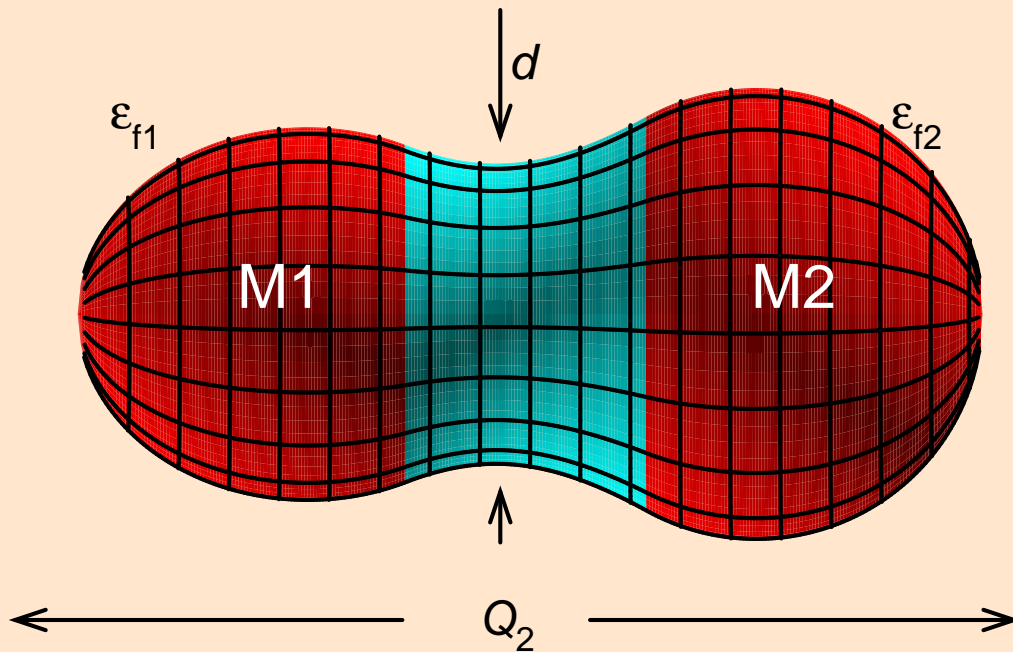




From: [Journ. Phys. G: Nucl. Part. Phys. 20 \(1994\) 1681](#)



Five Essential Fission Shape Coordinates

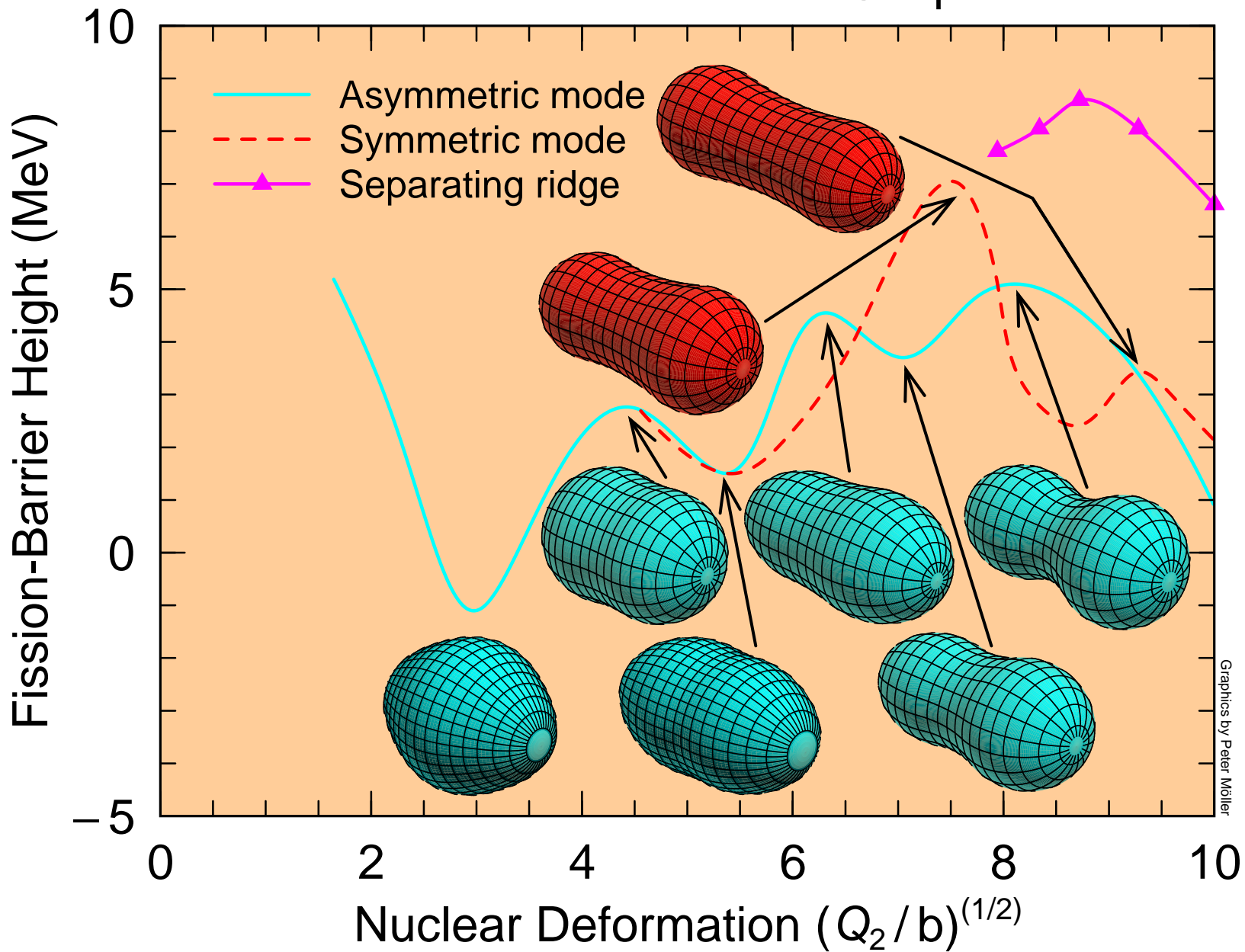


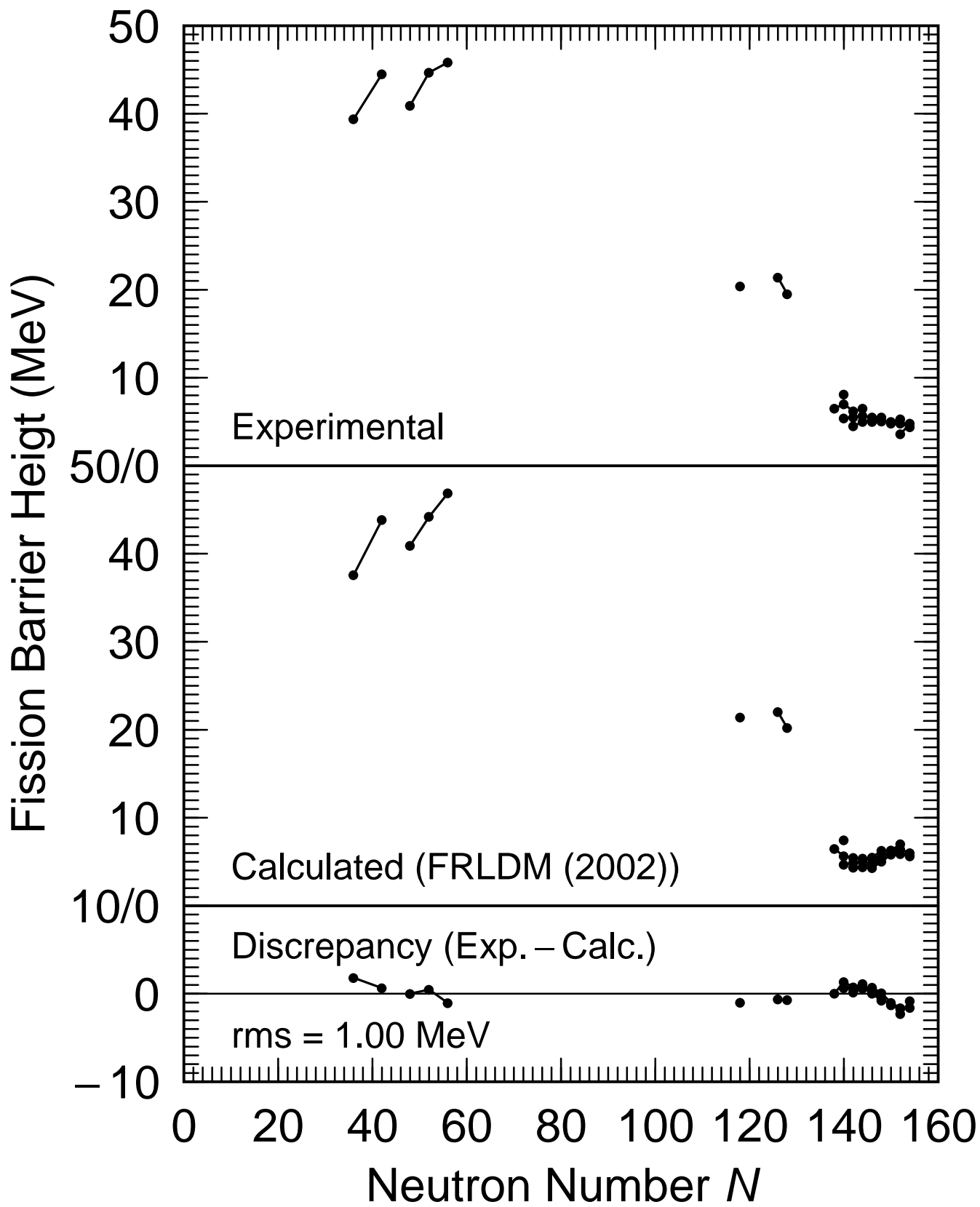
45	$Q_2 \sim$ Elongation (fission direction)
⊗	
15	$d \sim$ Neck
⊗	
15	$\epsilon_{f1} \sim$ Left fragment deformation
⊗	
15	$\epsilon_{f2} \sim$ Right fragment deformation
⊗	
35	$\alpha_g \sim (M1-M2)/(M1+M2)$ Mass asymmetry

⇒ 5 315 625 grid points – 306 300 unphysical points

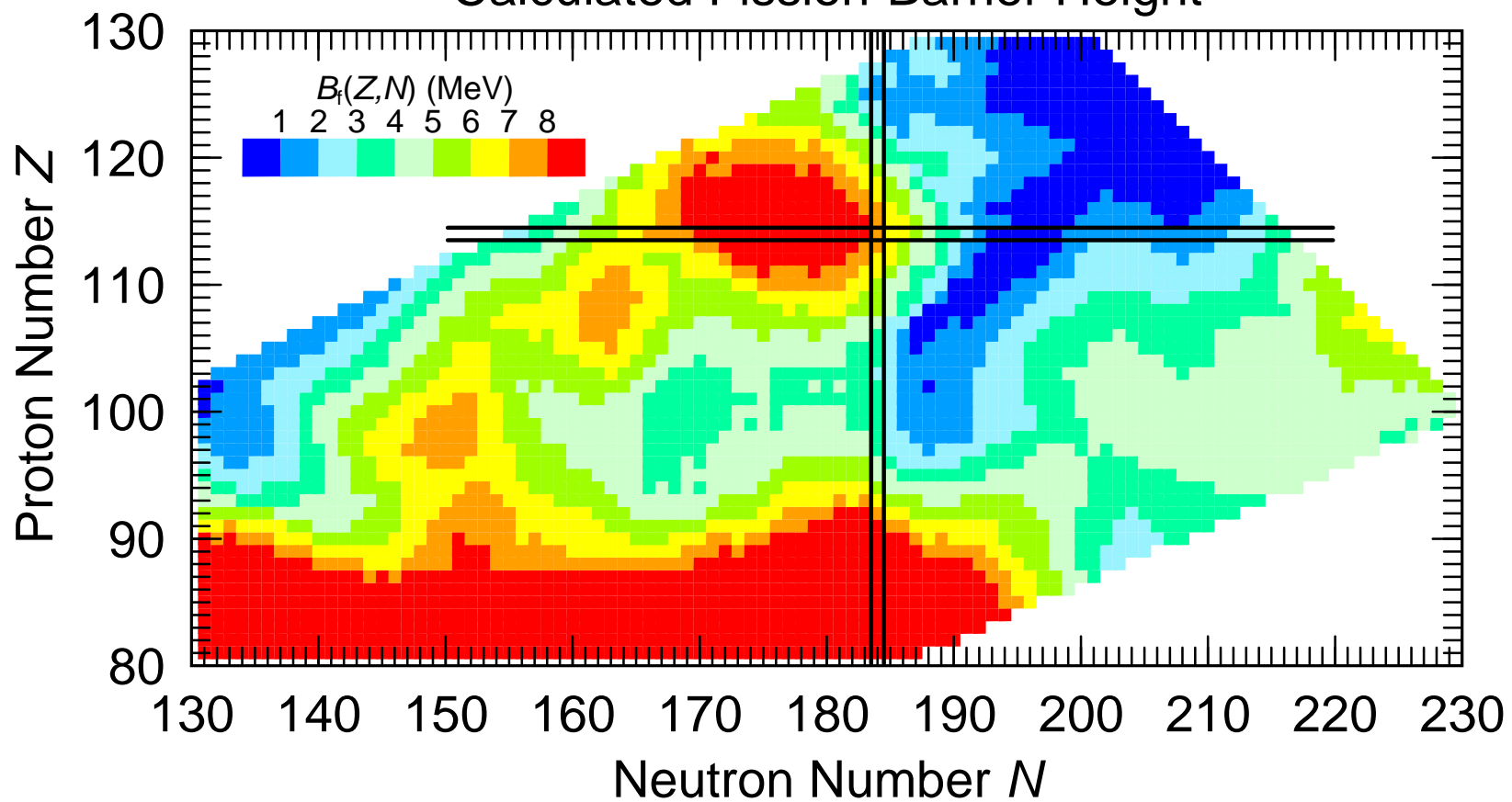
⇒ **5 009 325 physical grid points**

Fission Barrier and Associated Shapes for ^{232}Th

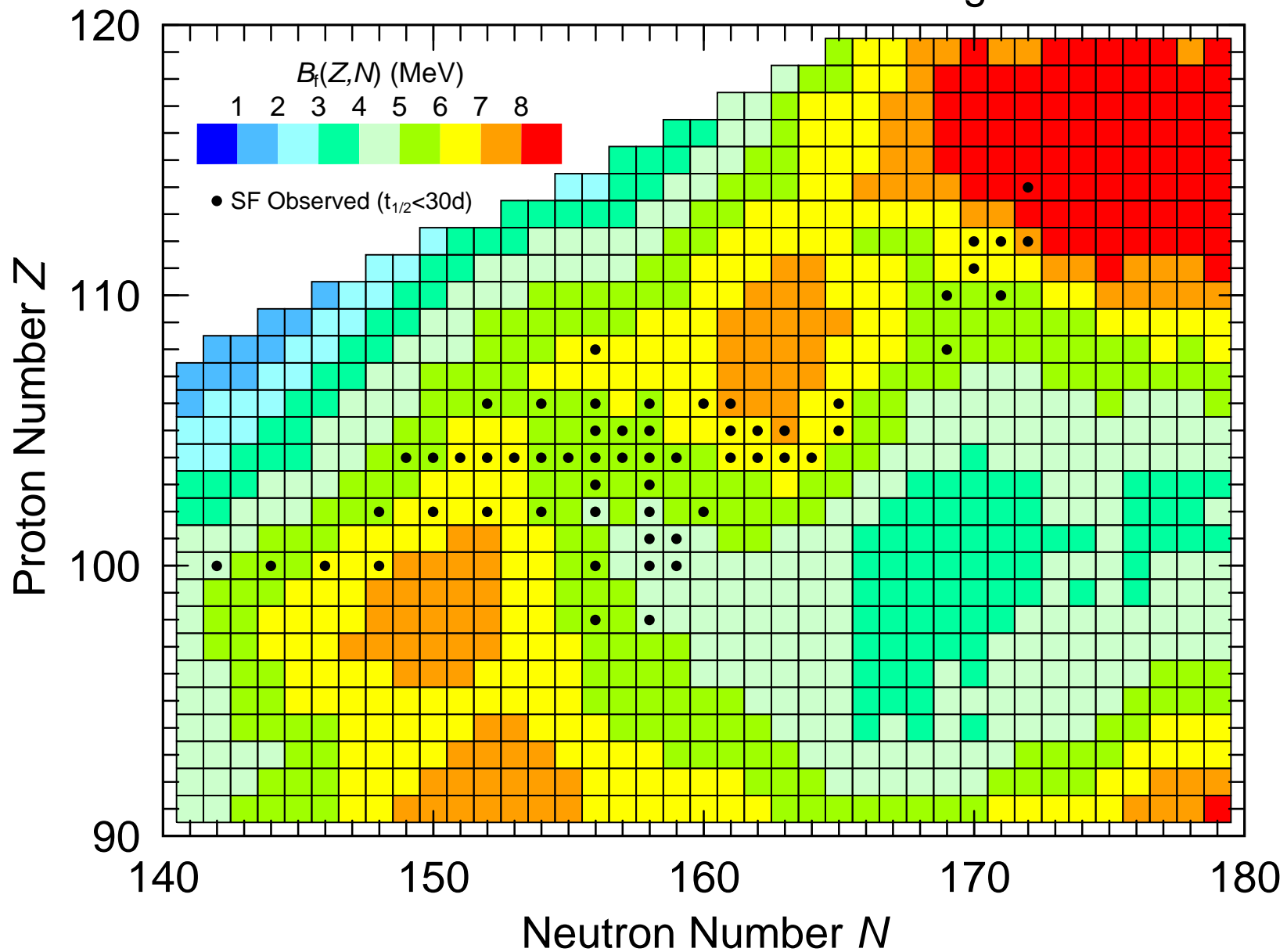


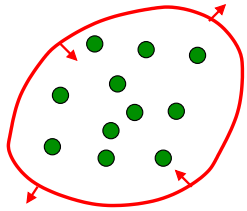


Calculated Fission-Barrier Height



Calculated Fission-Barrier Height





Brownian shape motion

Nuclear deformation energy: $E_{\text{def}}(i,j,k,l,m)$

Bias potential: $V_{\text{bias}}(i) = V_0 (Q_0/Q_2)^2$

Level density parameter: $a_A = A/(8 \text{ MeV})$

Temperature T : $E^* - E_{\text{def}} = a_A T^2$

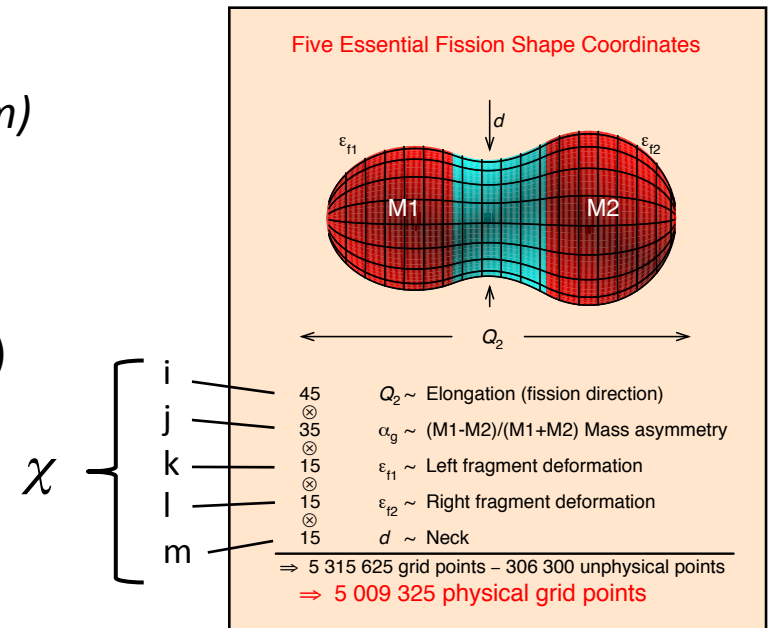
$$\Rightarrow V(\chi) = E_{\text{def}} + V_{\text{bias}}$$

Metropolis walk:

Change shape: $\chi \rightarrow \chi'$?

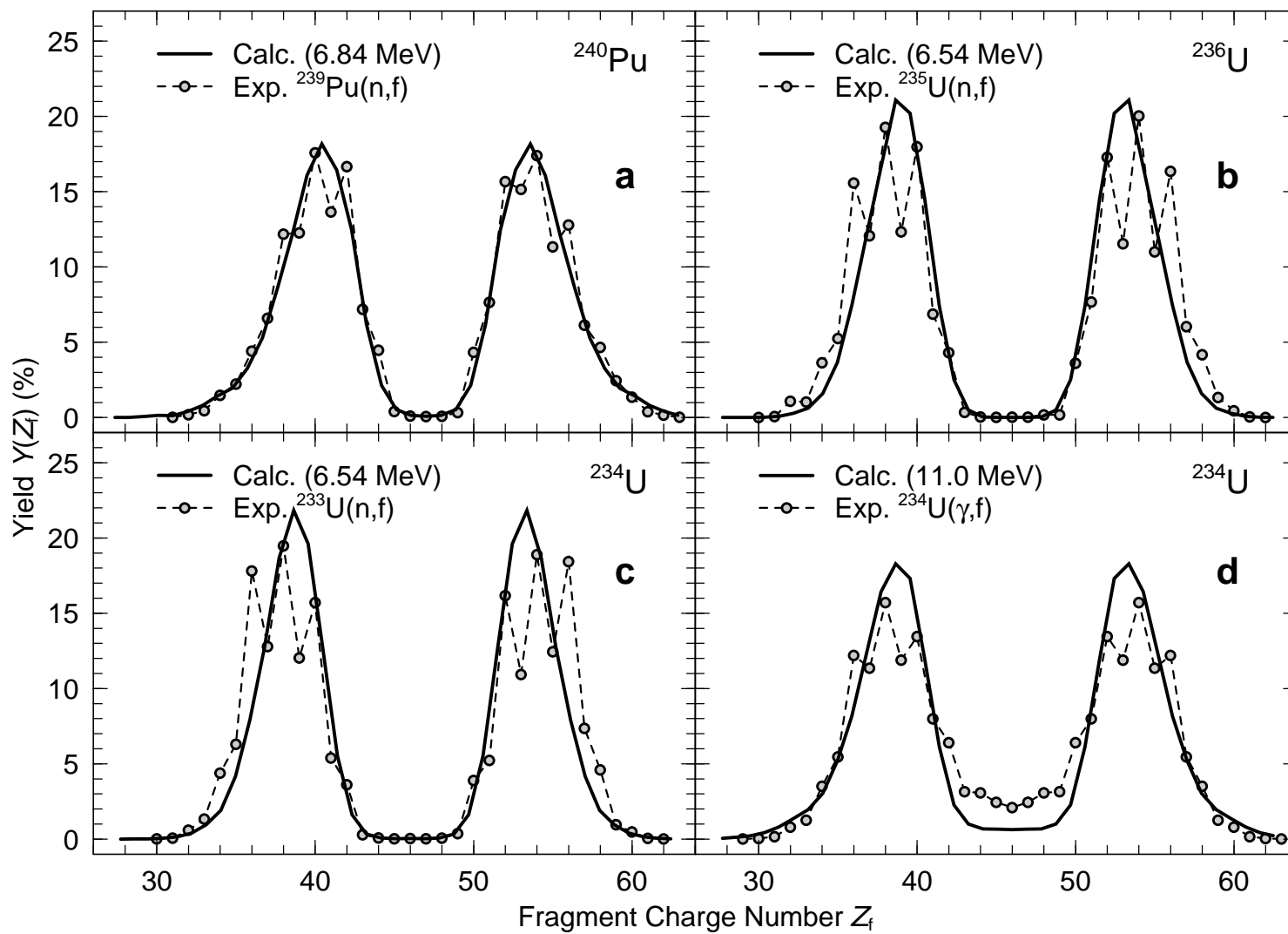
$$\begin{cases} V(\chi') < V(\chi): \text{ move with } P = 1 \\ V(\chi') > V(\chi): \text{ move with } P = \exp(-\Delta V/T) \end{cases}$$

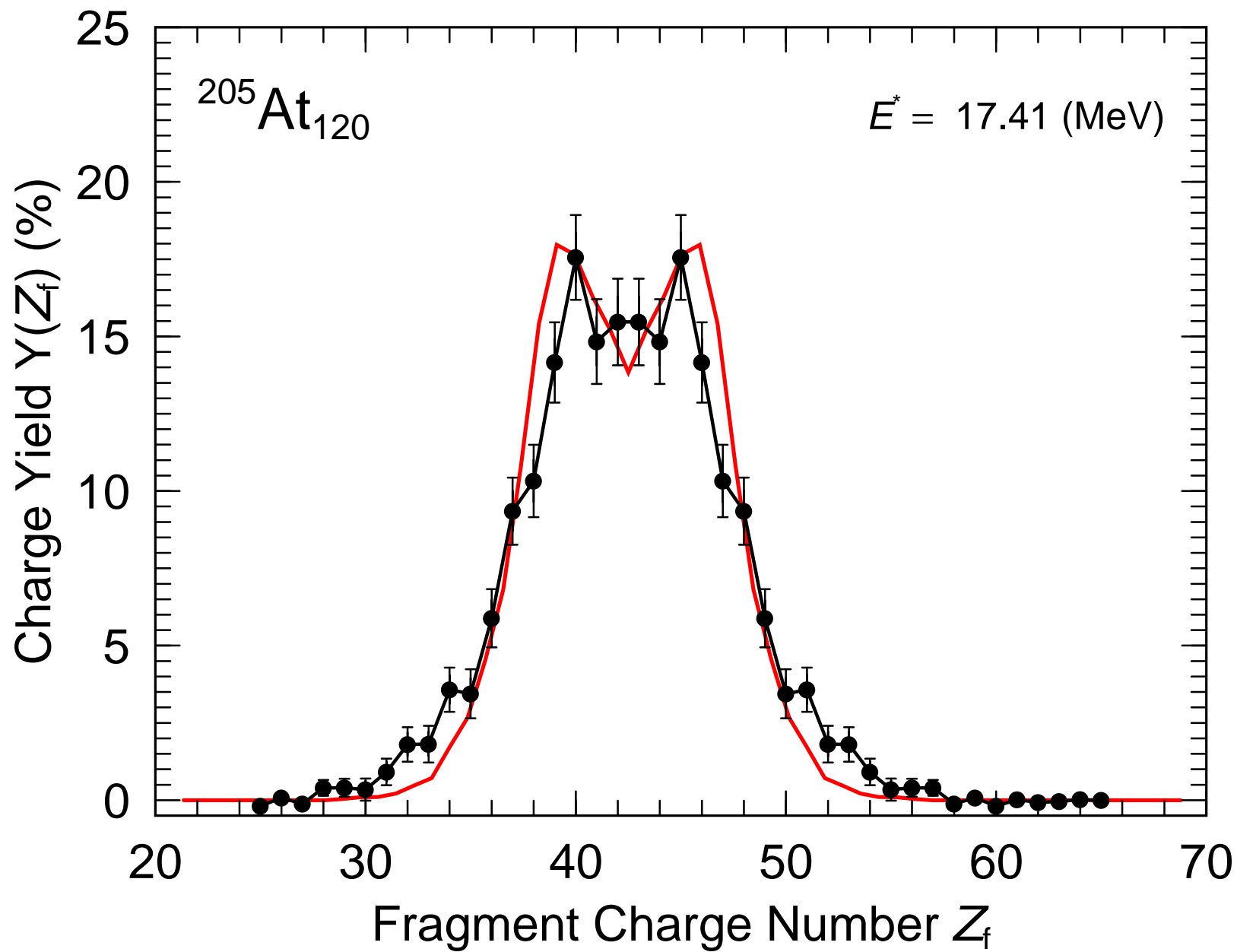
Scission: Critical neck radius $c_0 \approx 2.5 \text{ fm}$

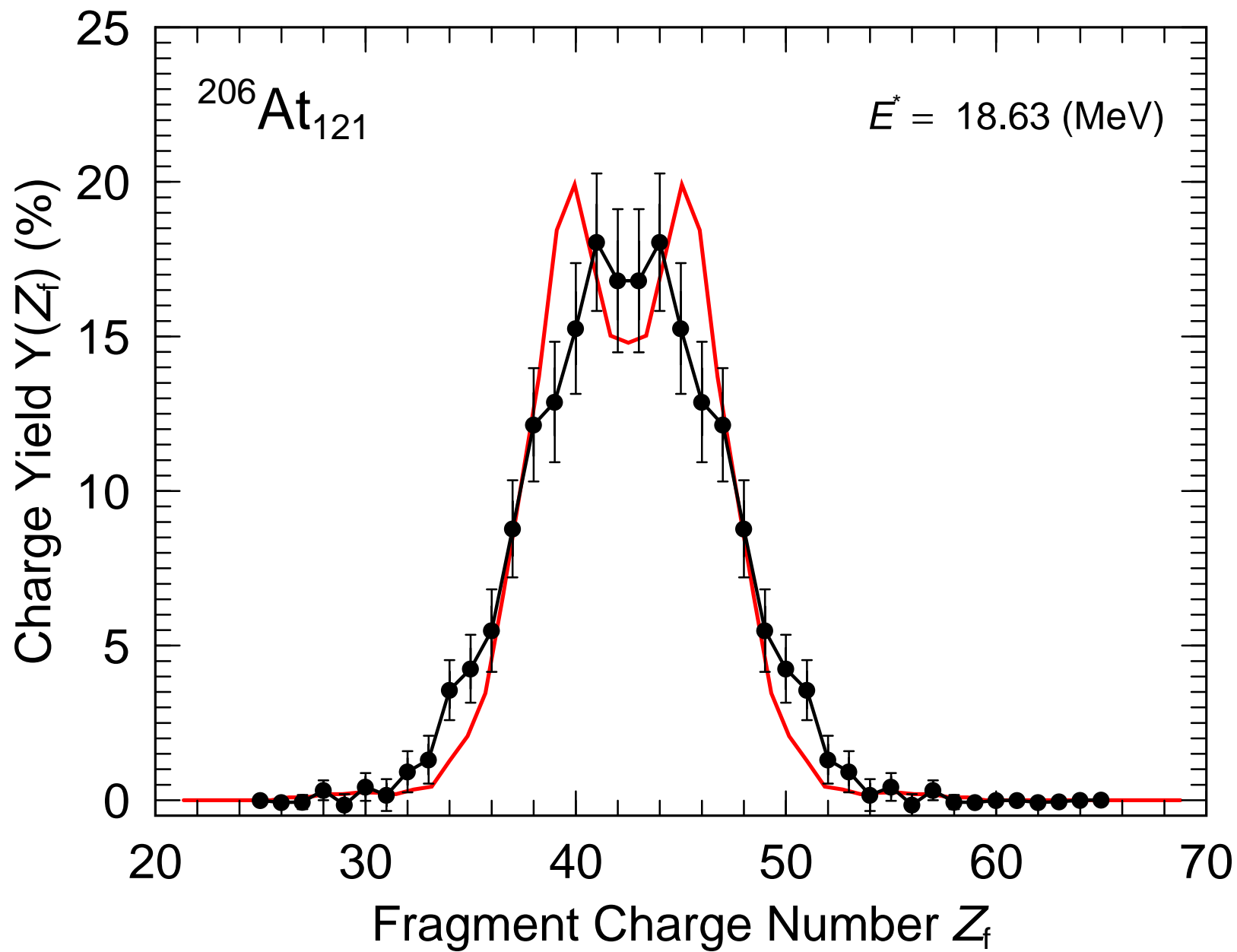


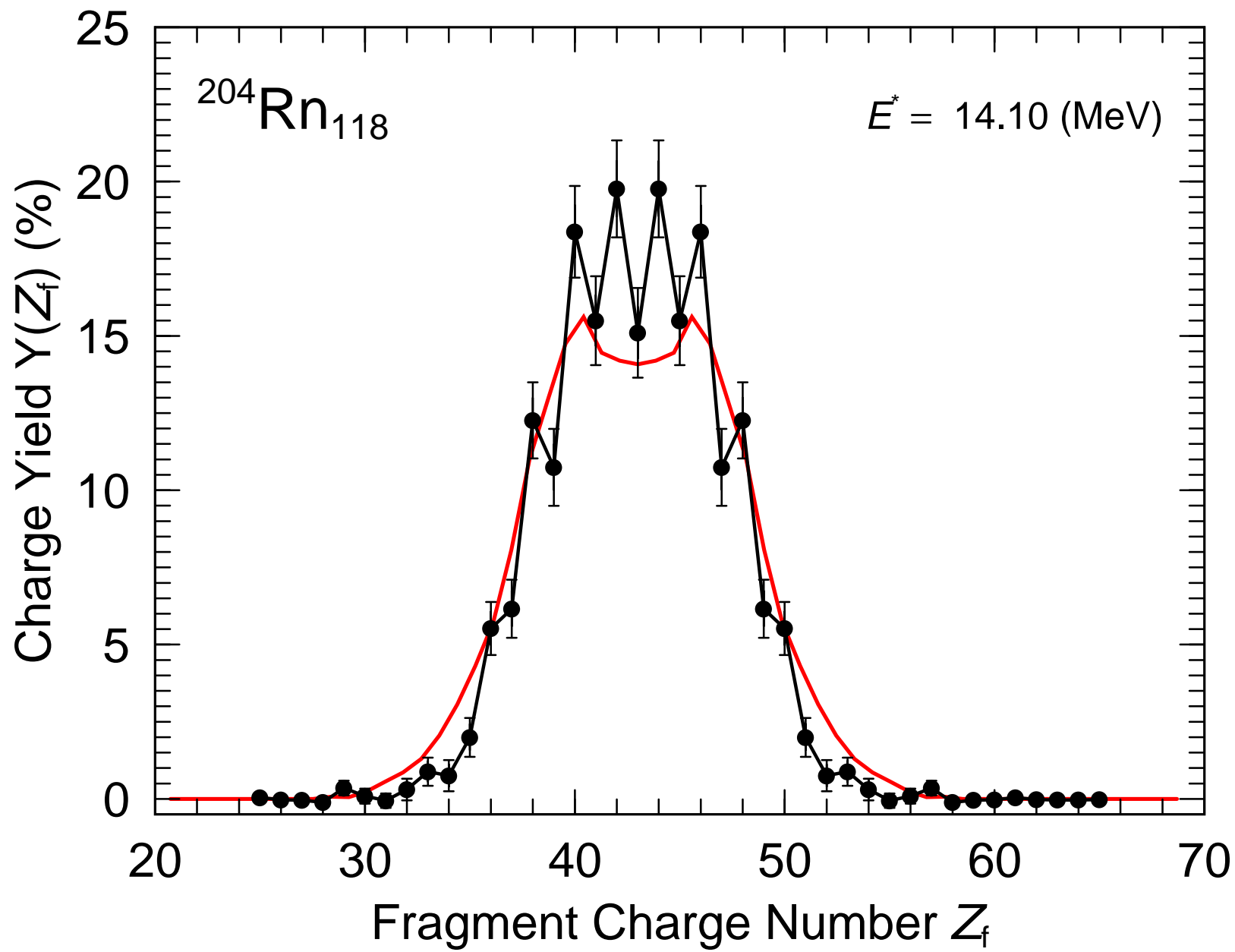
P. Möller *et al*, Nature 409 (2001) 785

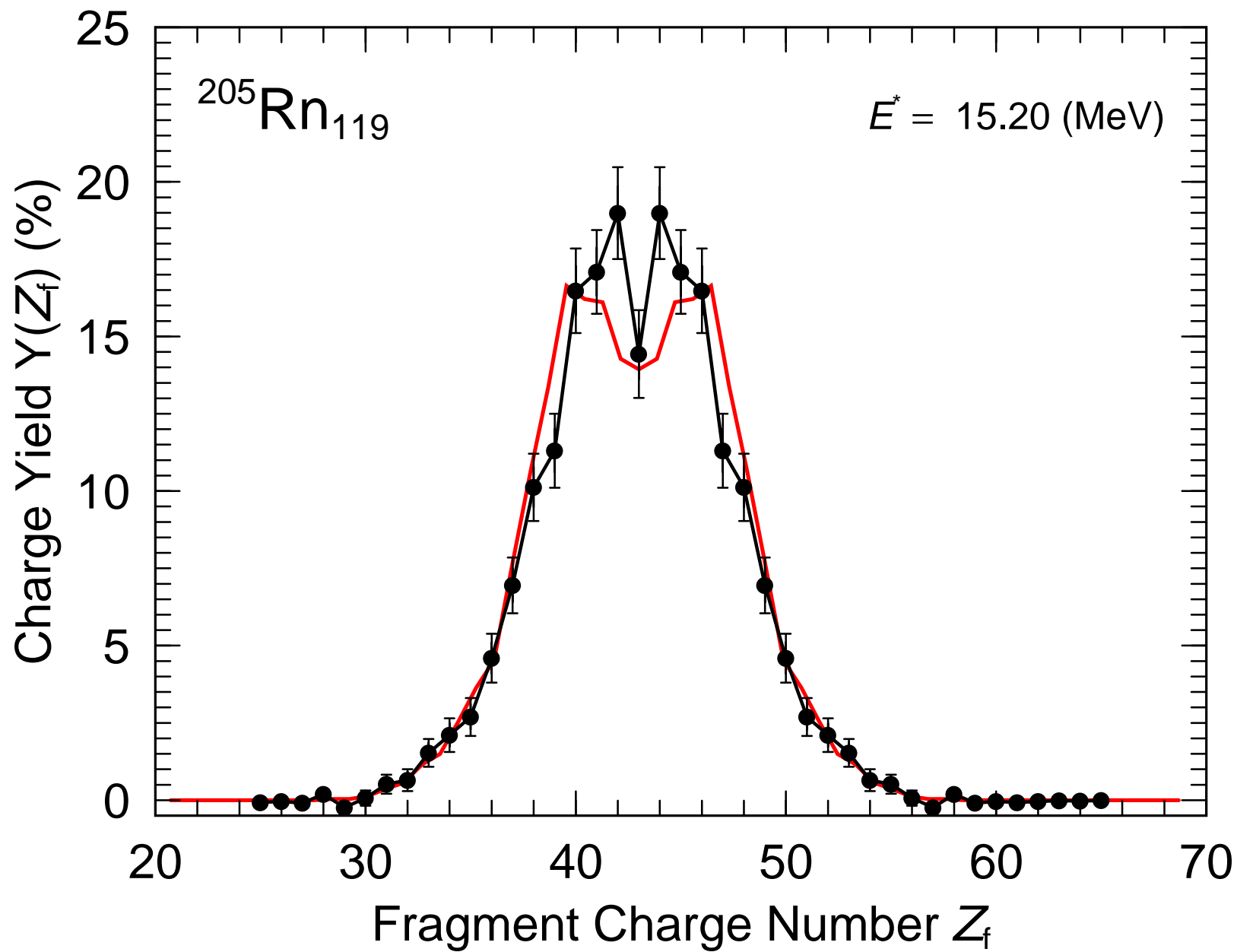
N. Metropolis *et al*, J Chem Phys 26 (1953) 1087

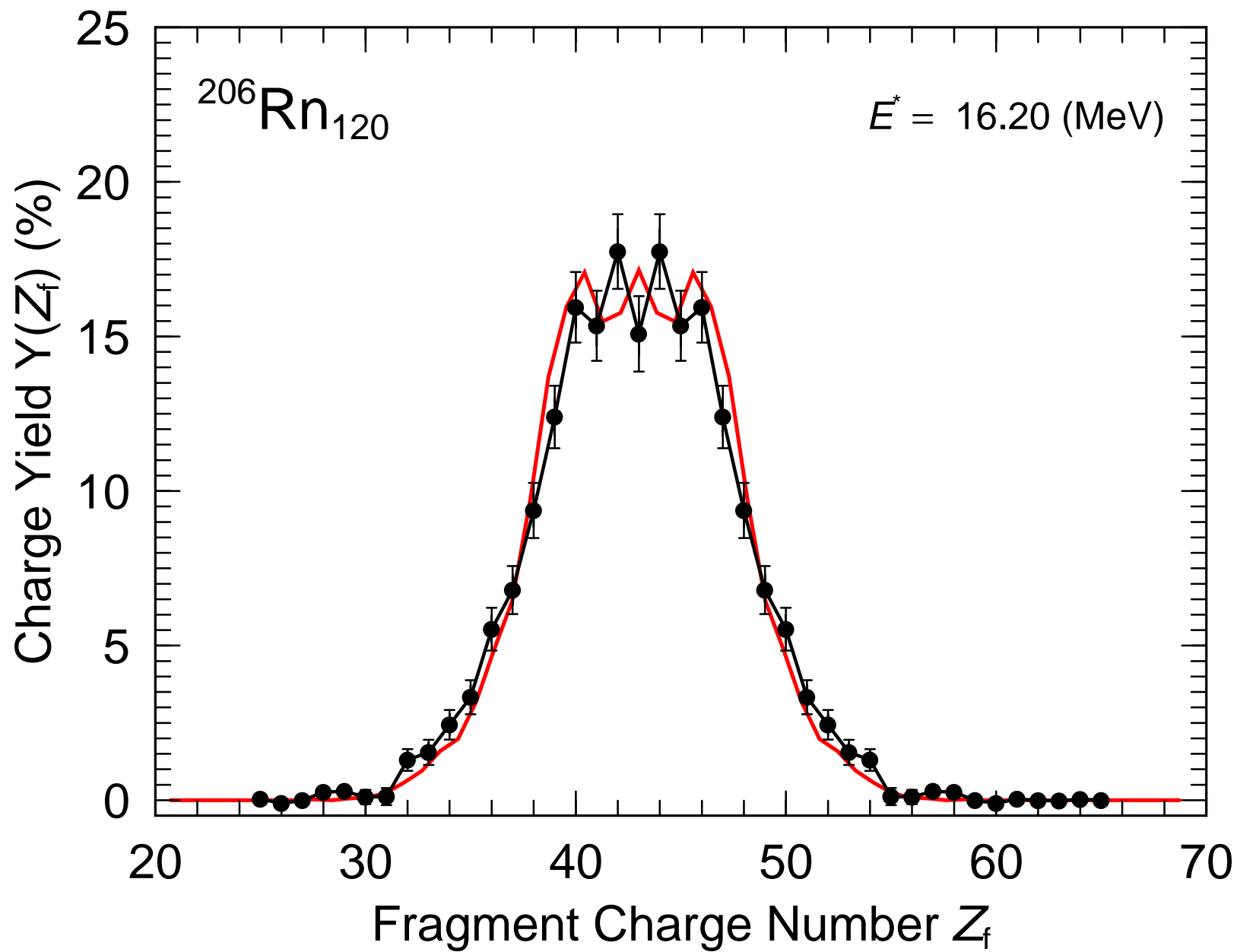


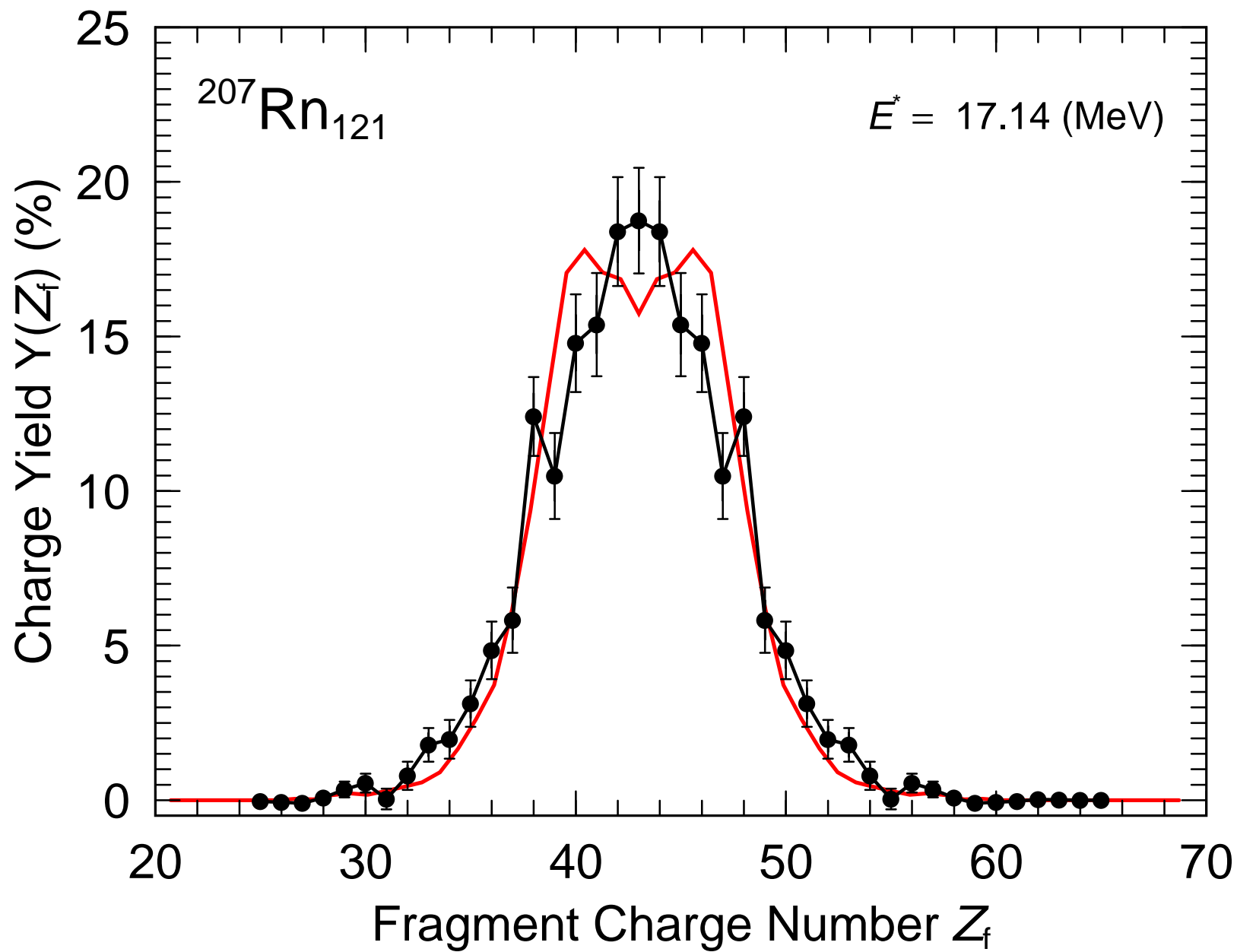


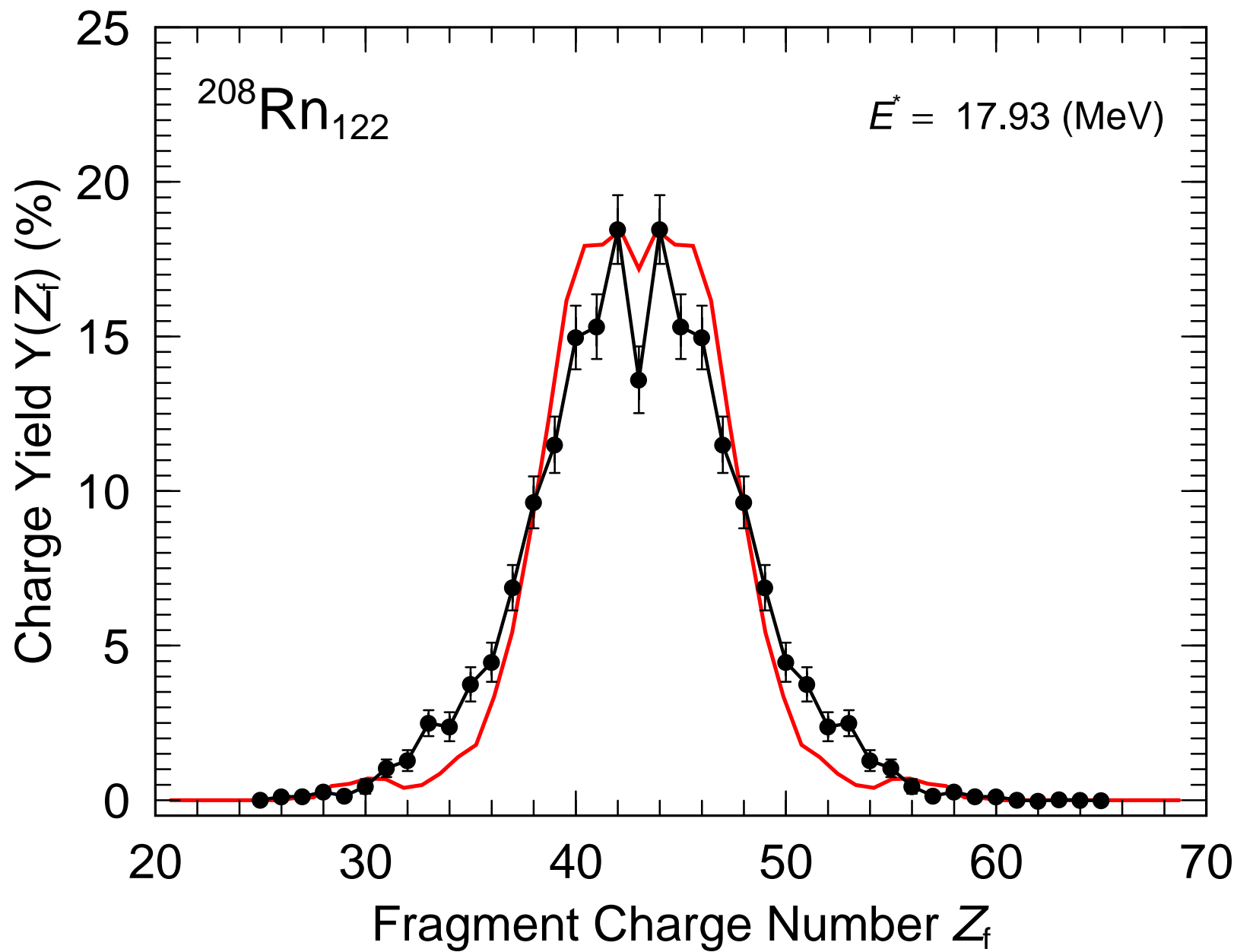


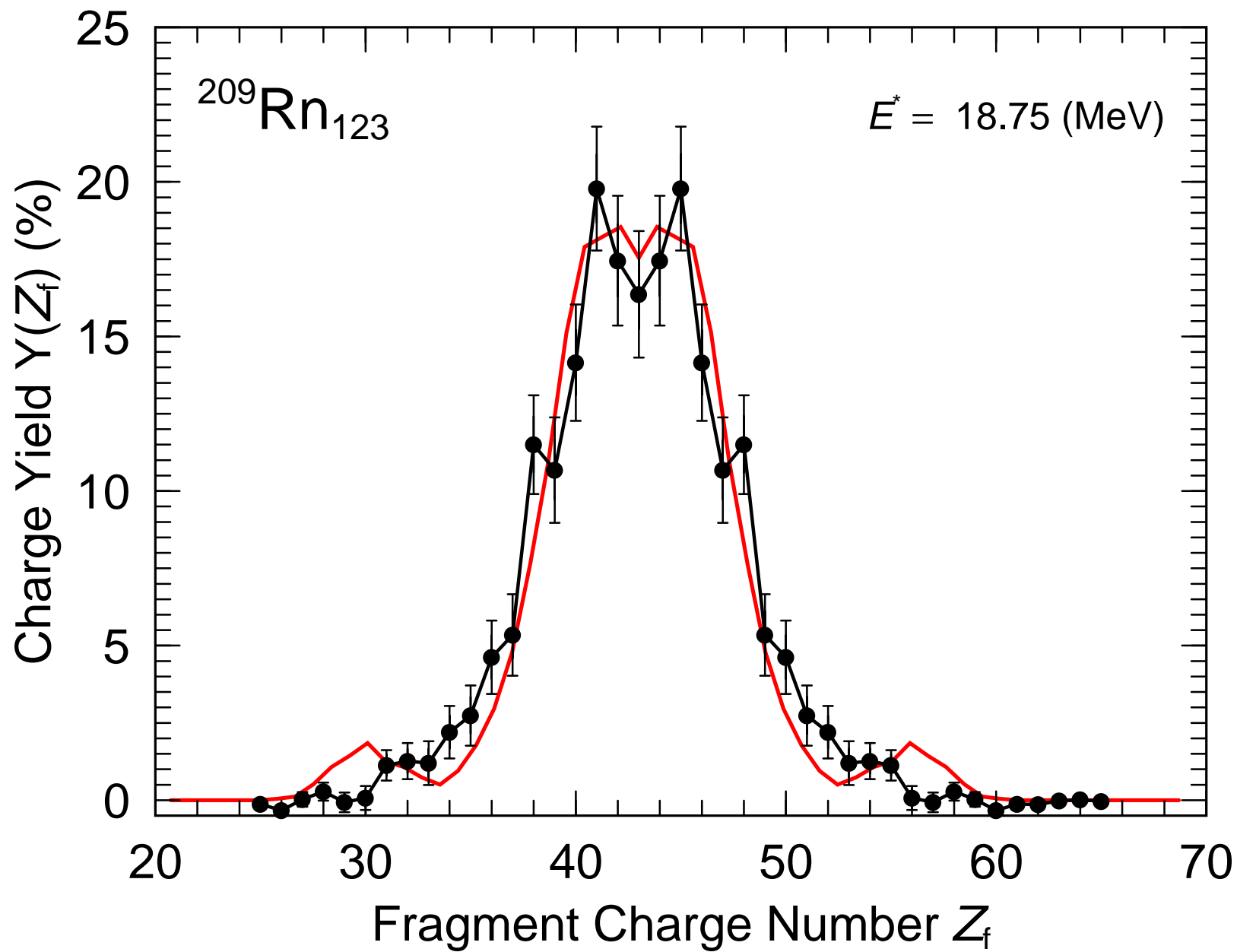


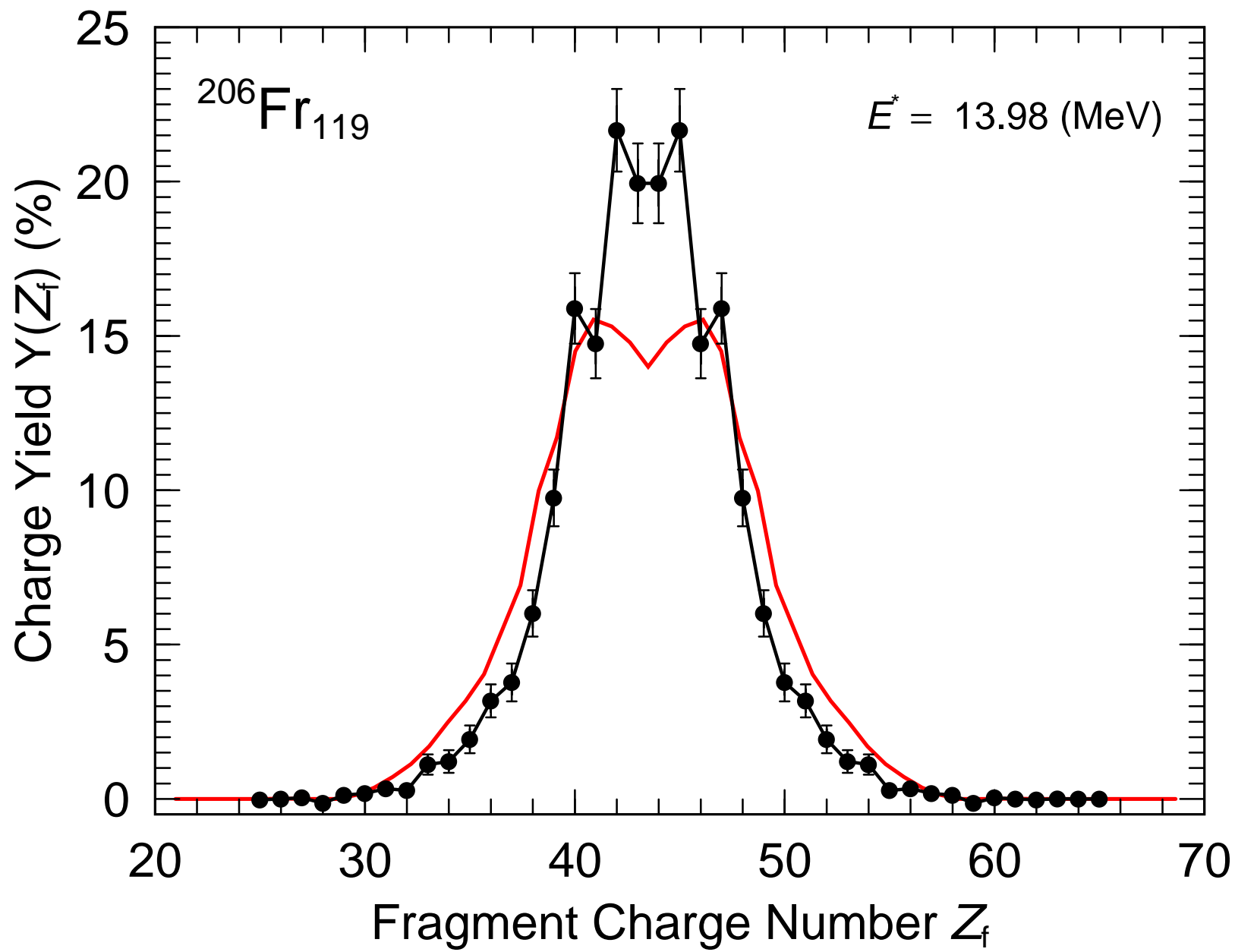


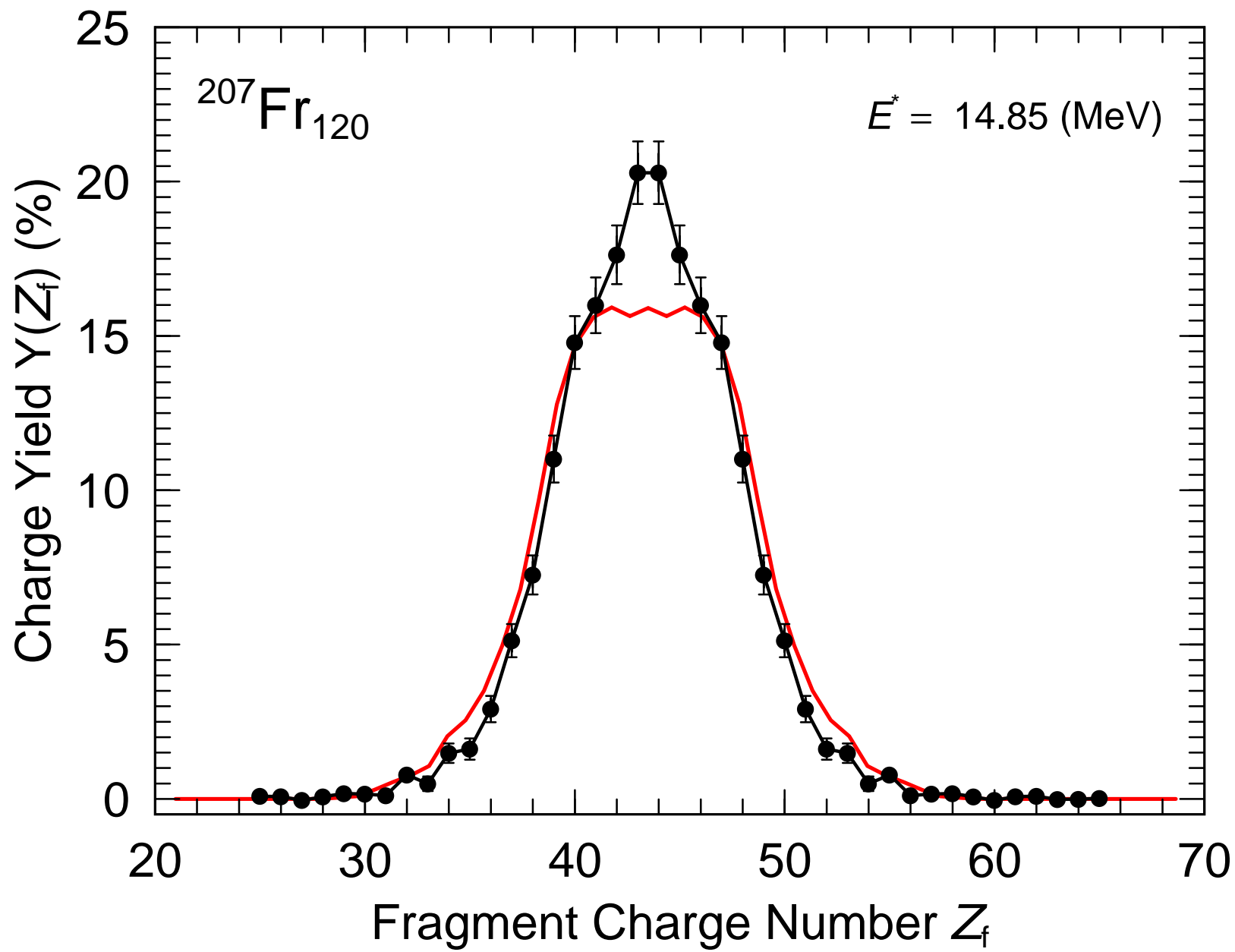


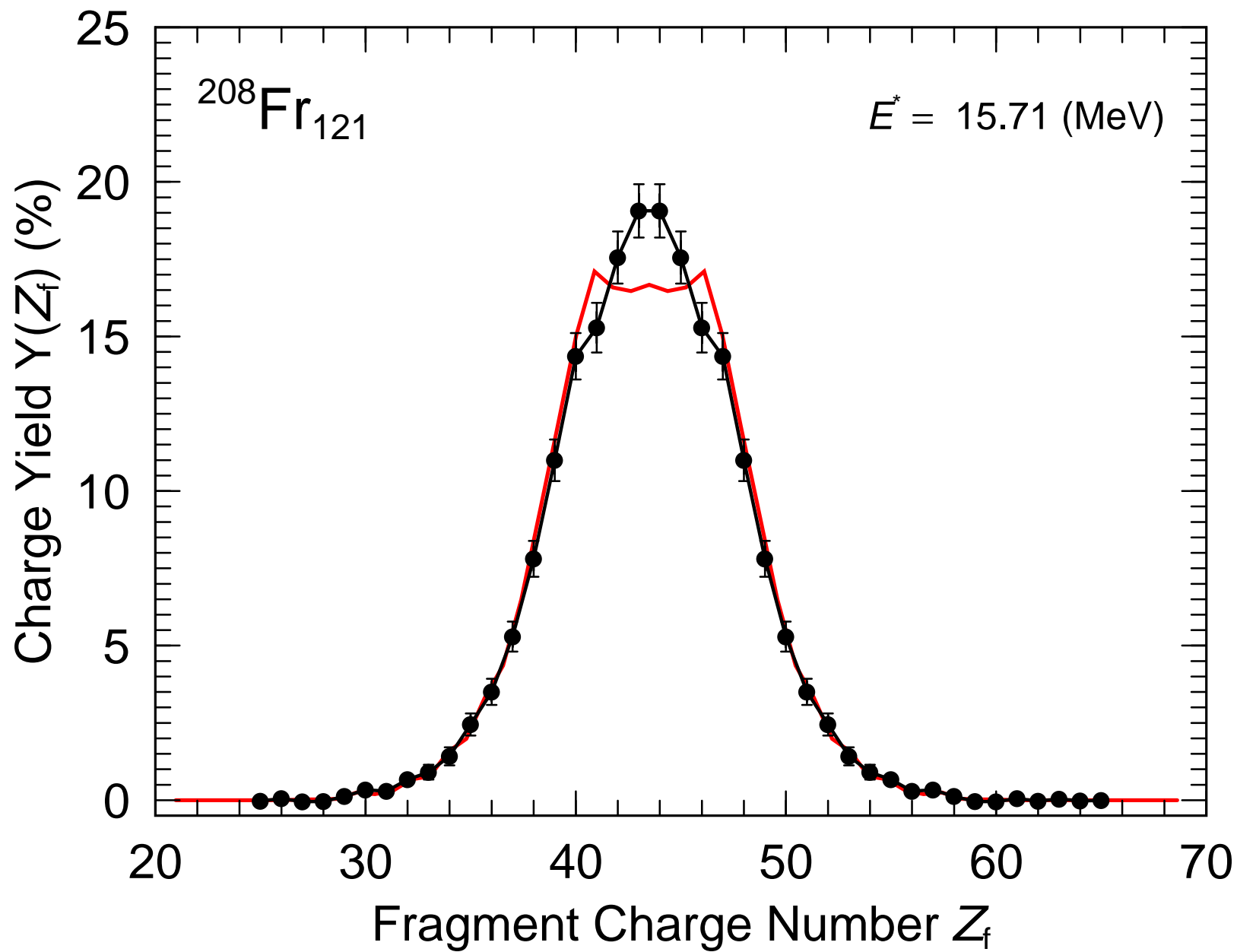


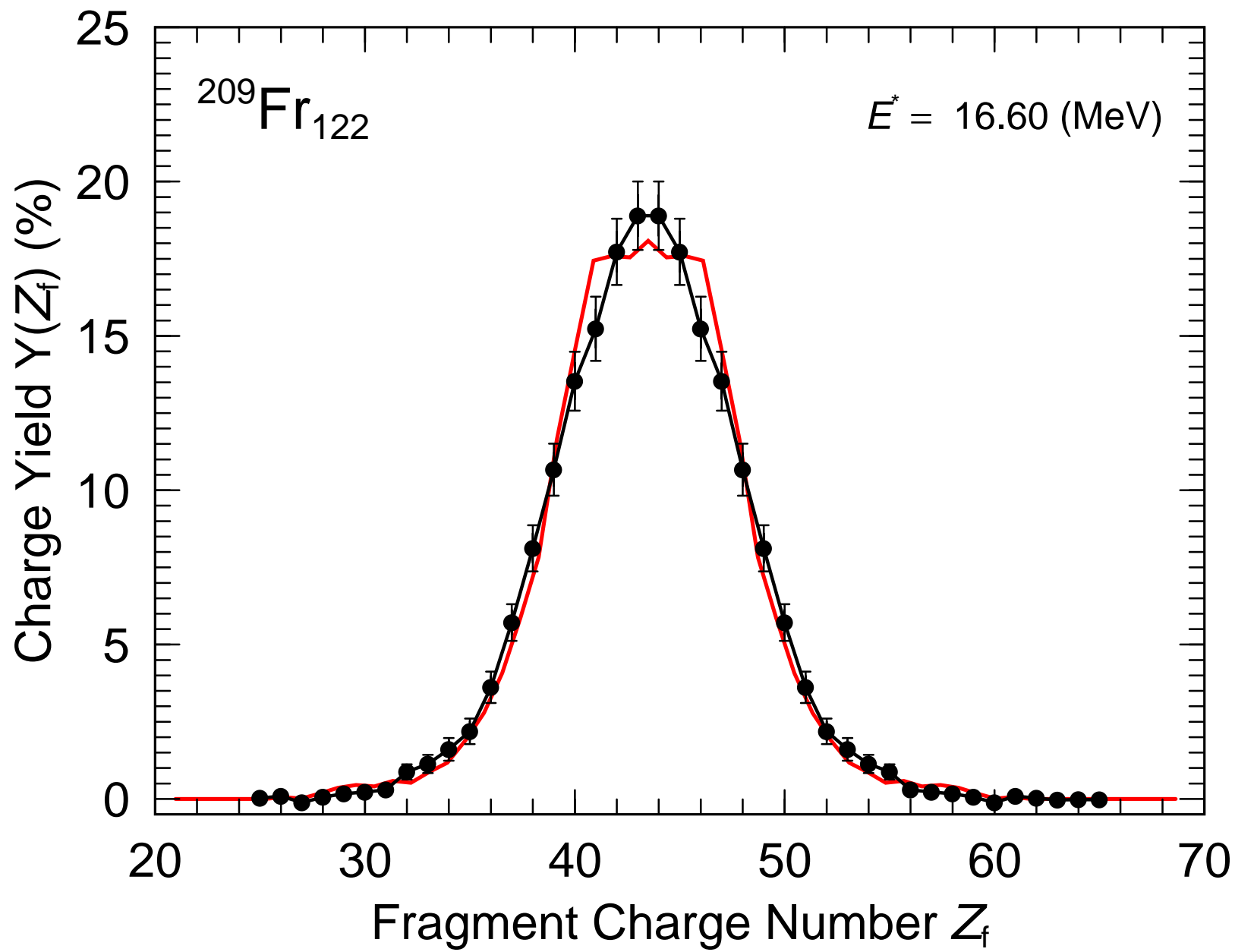


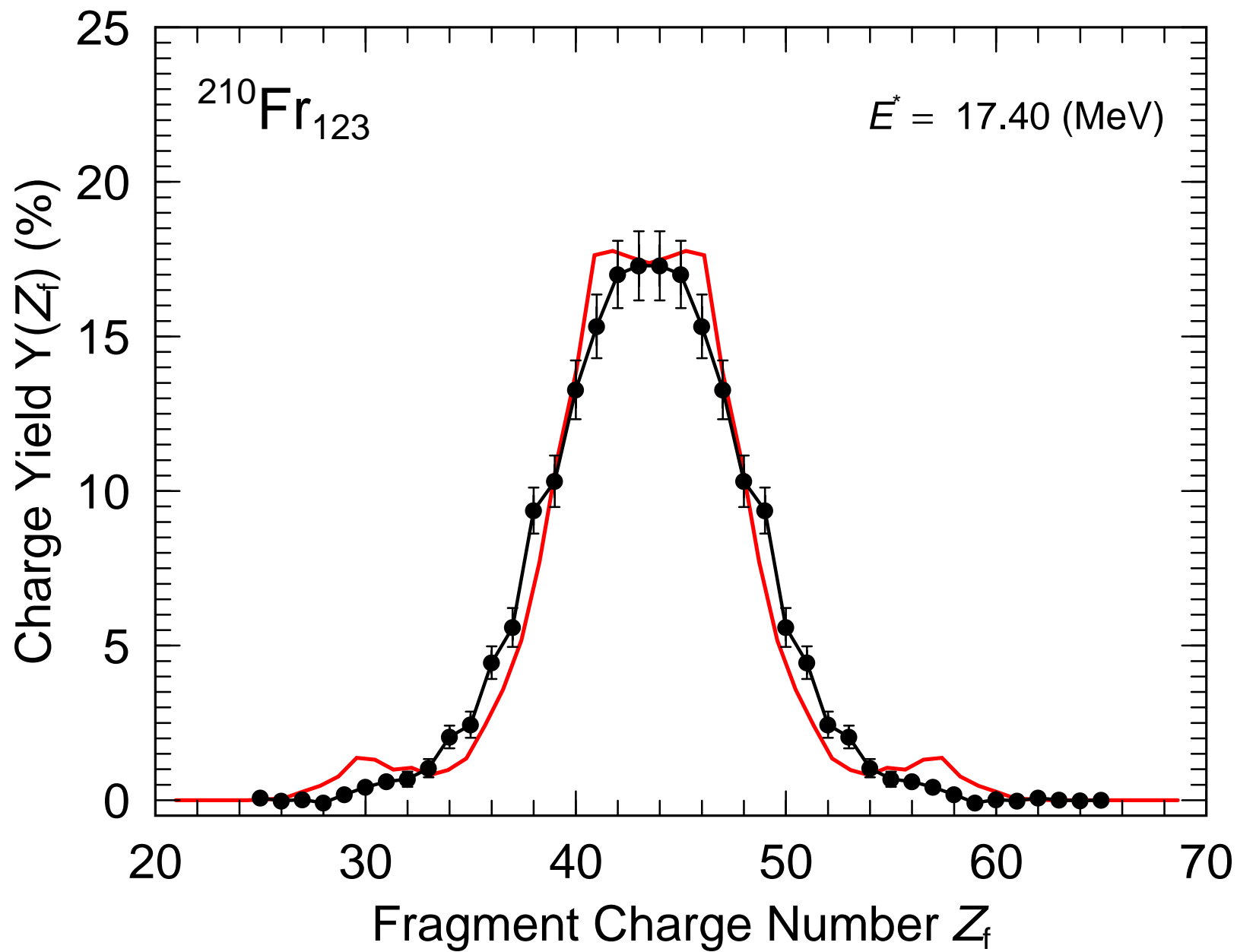


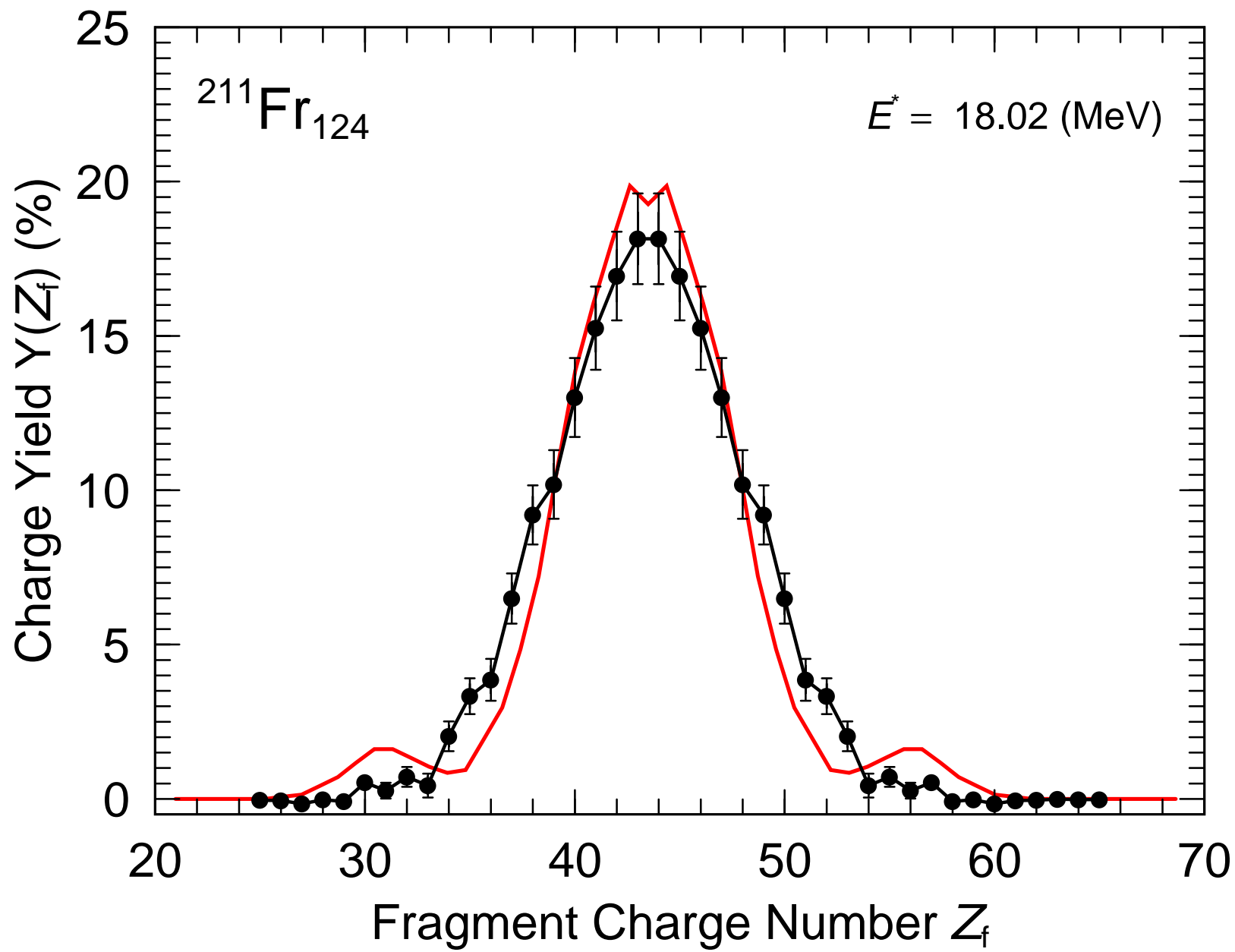


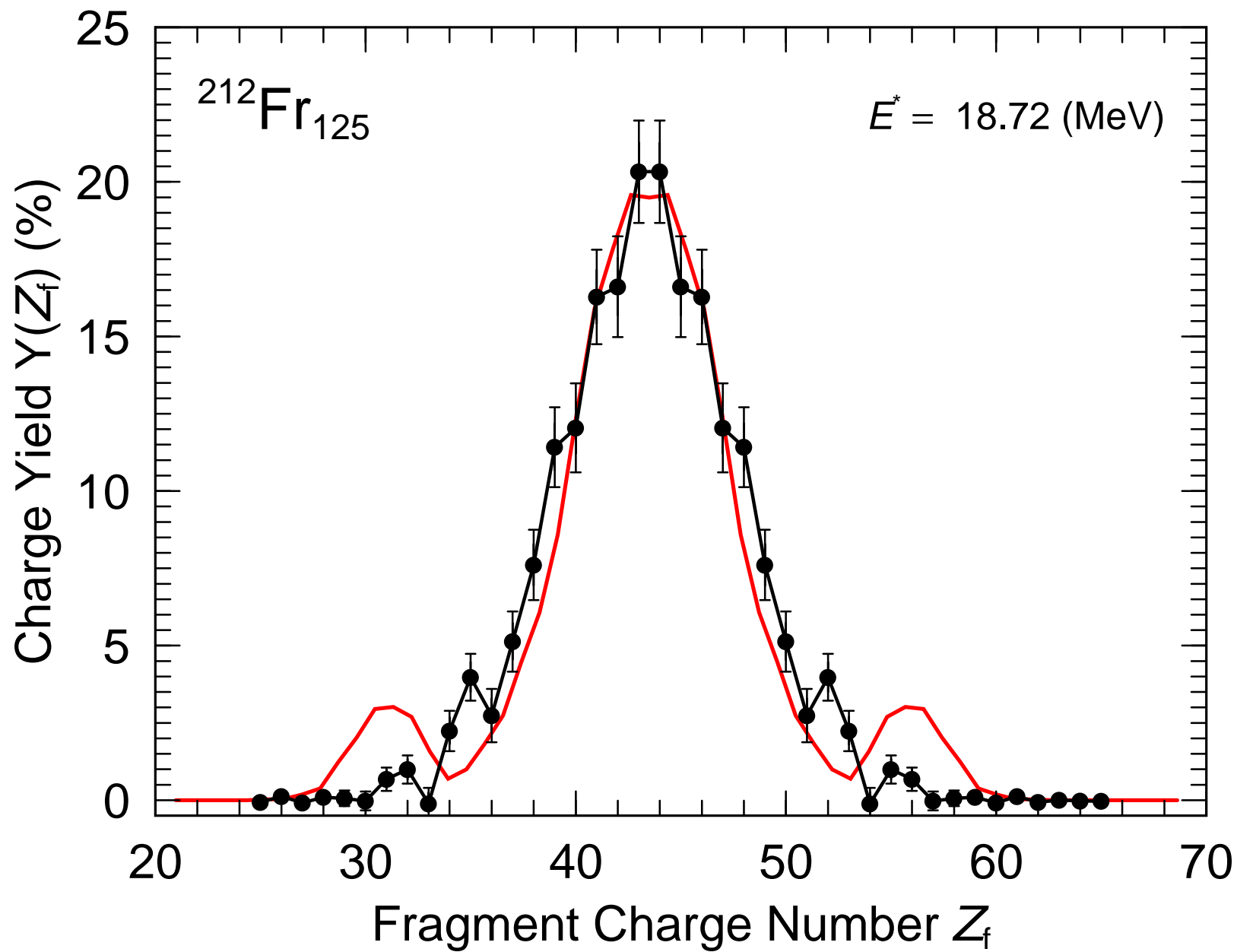


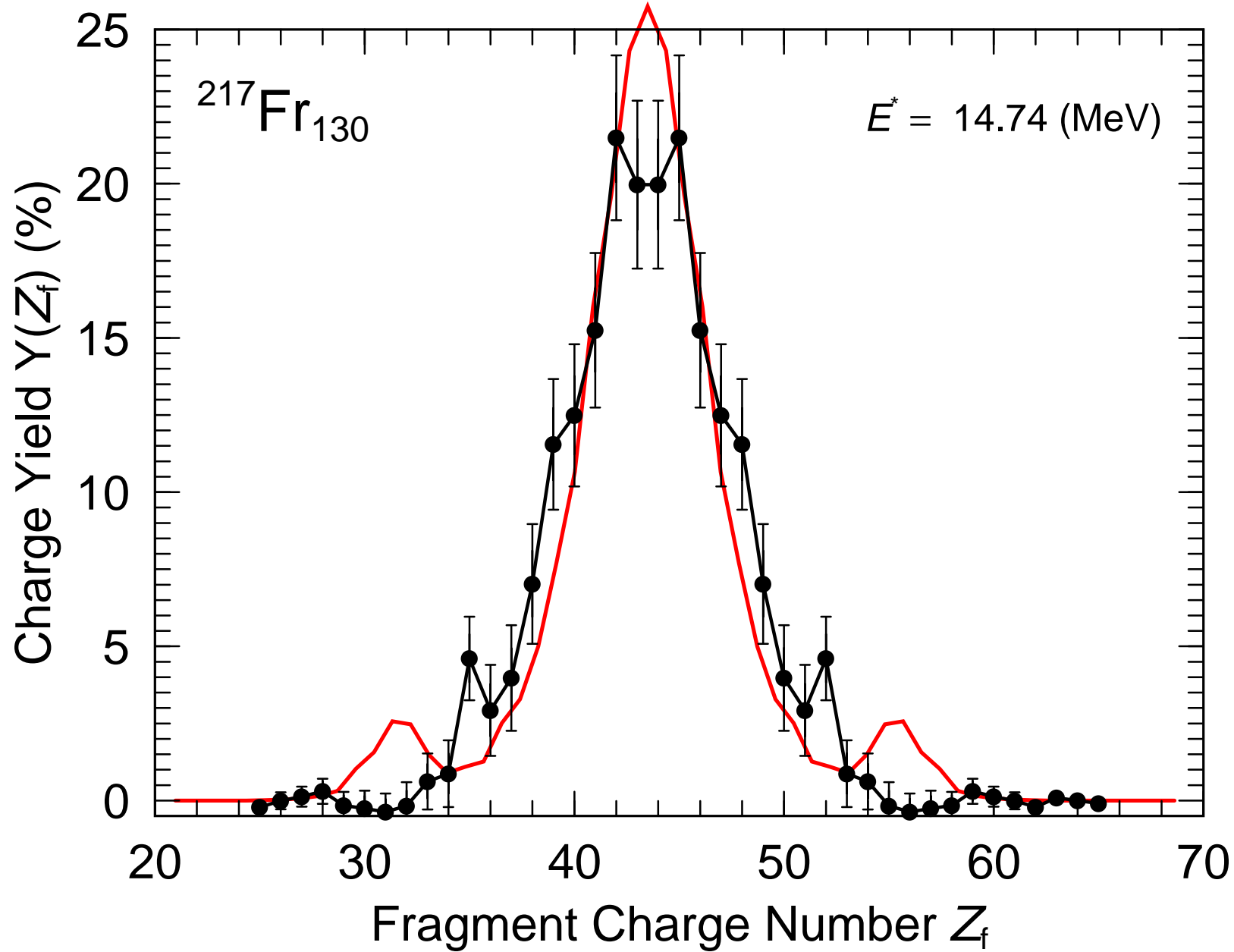


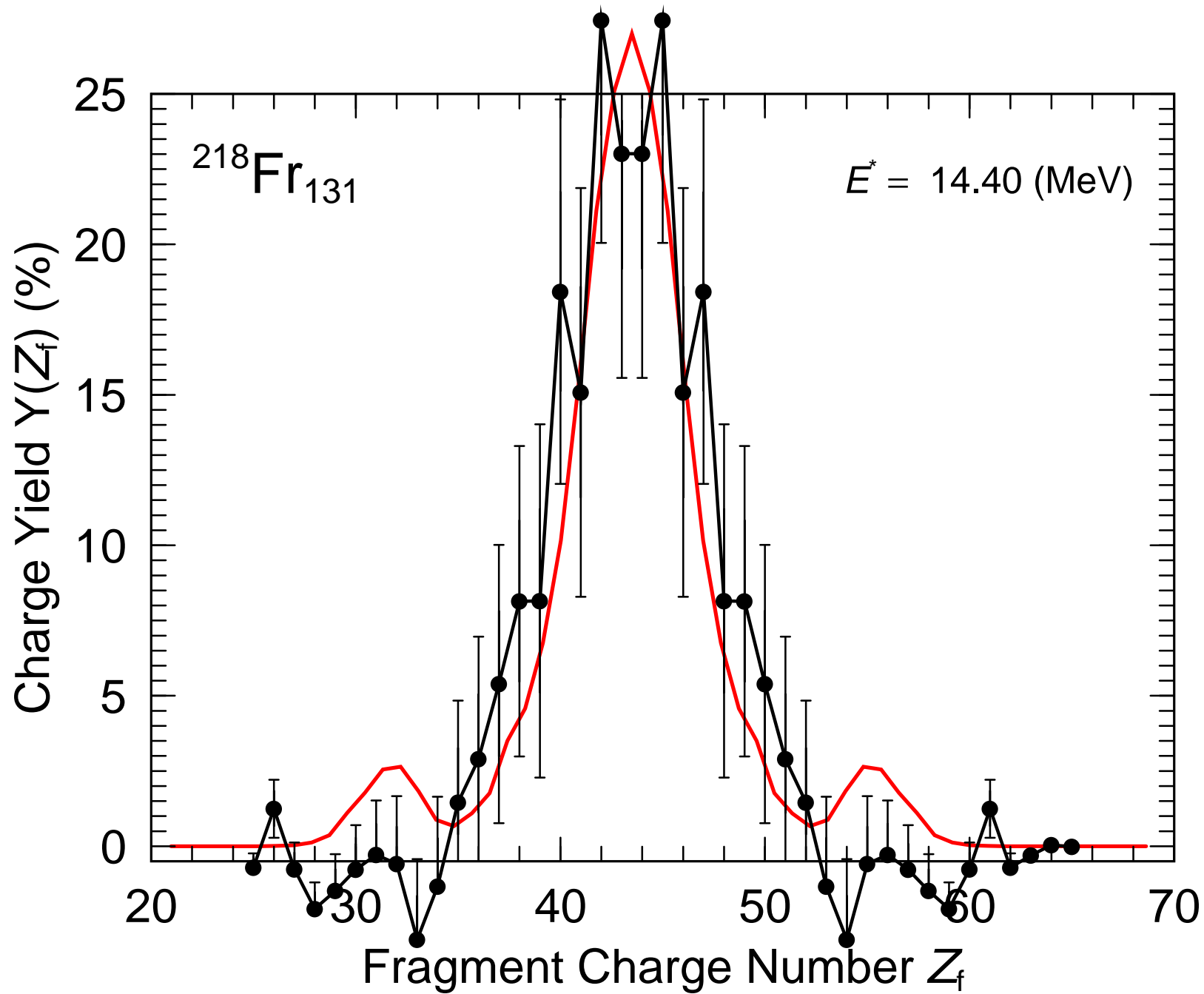


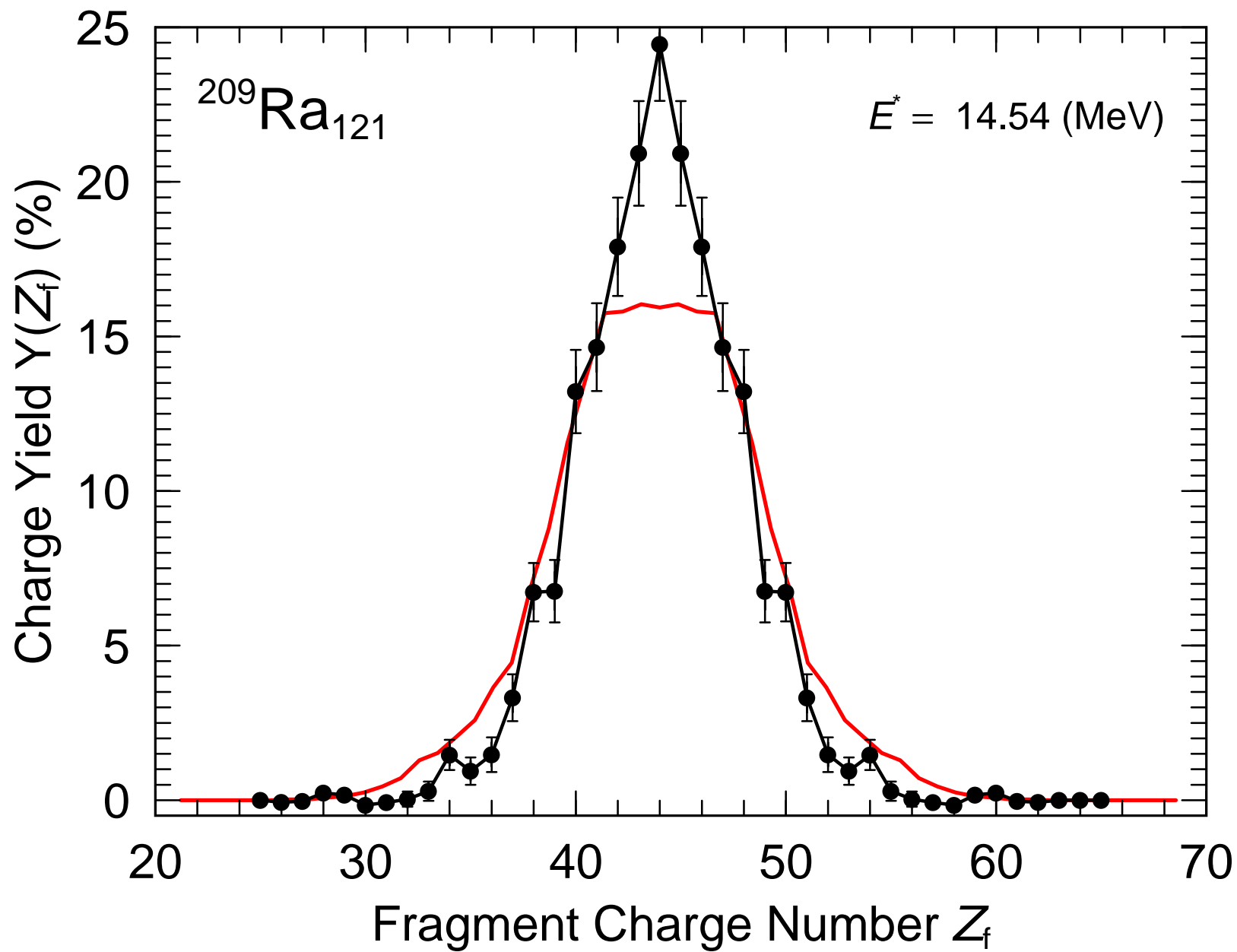


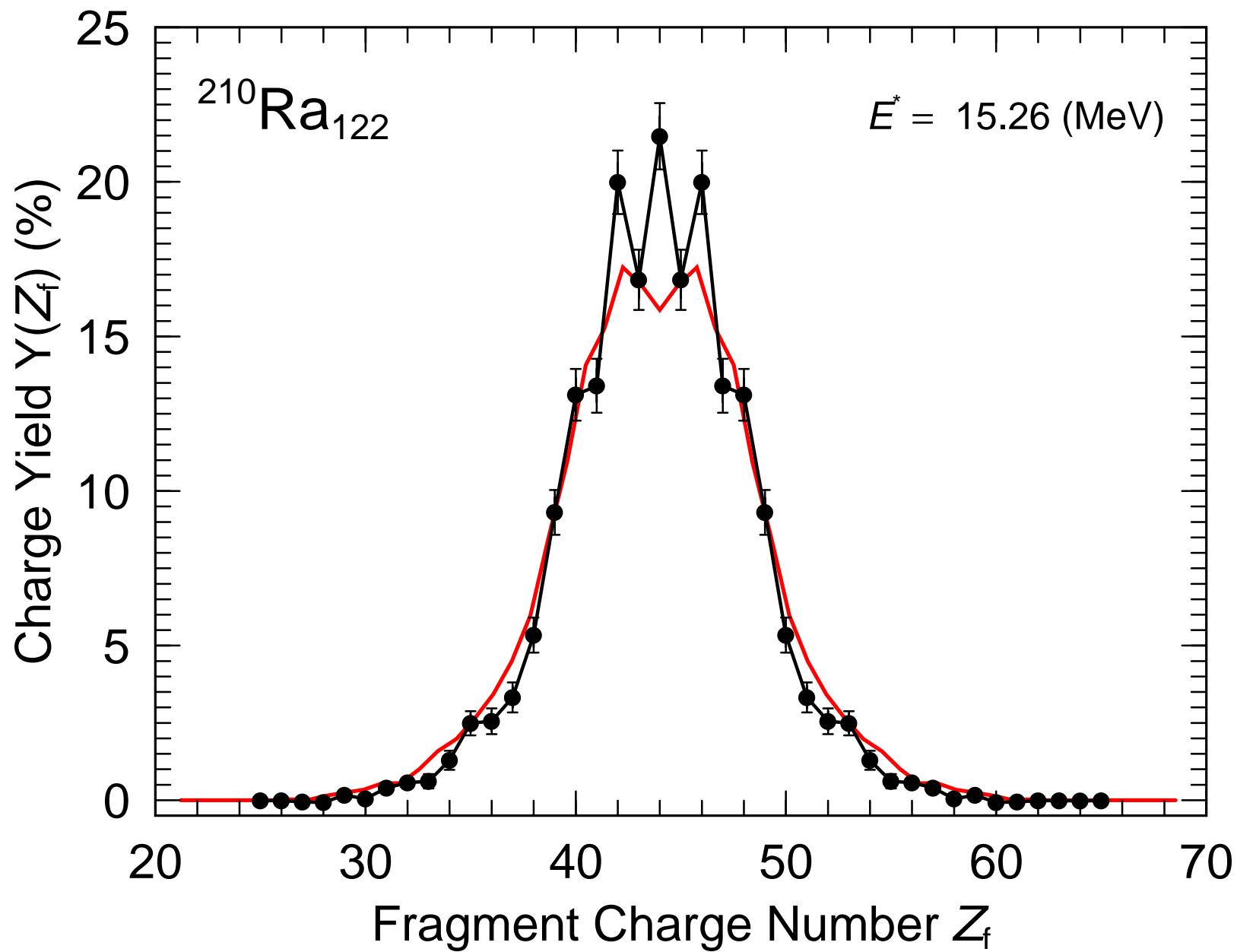


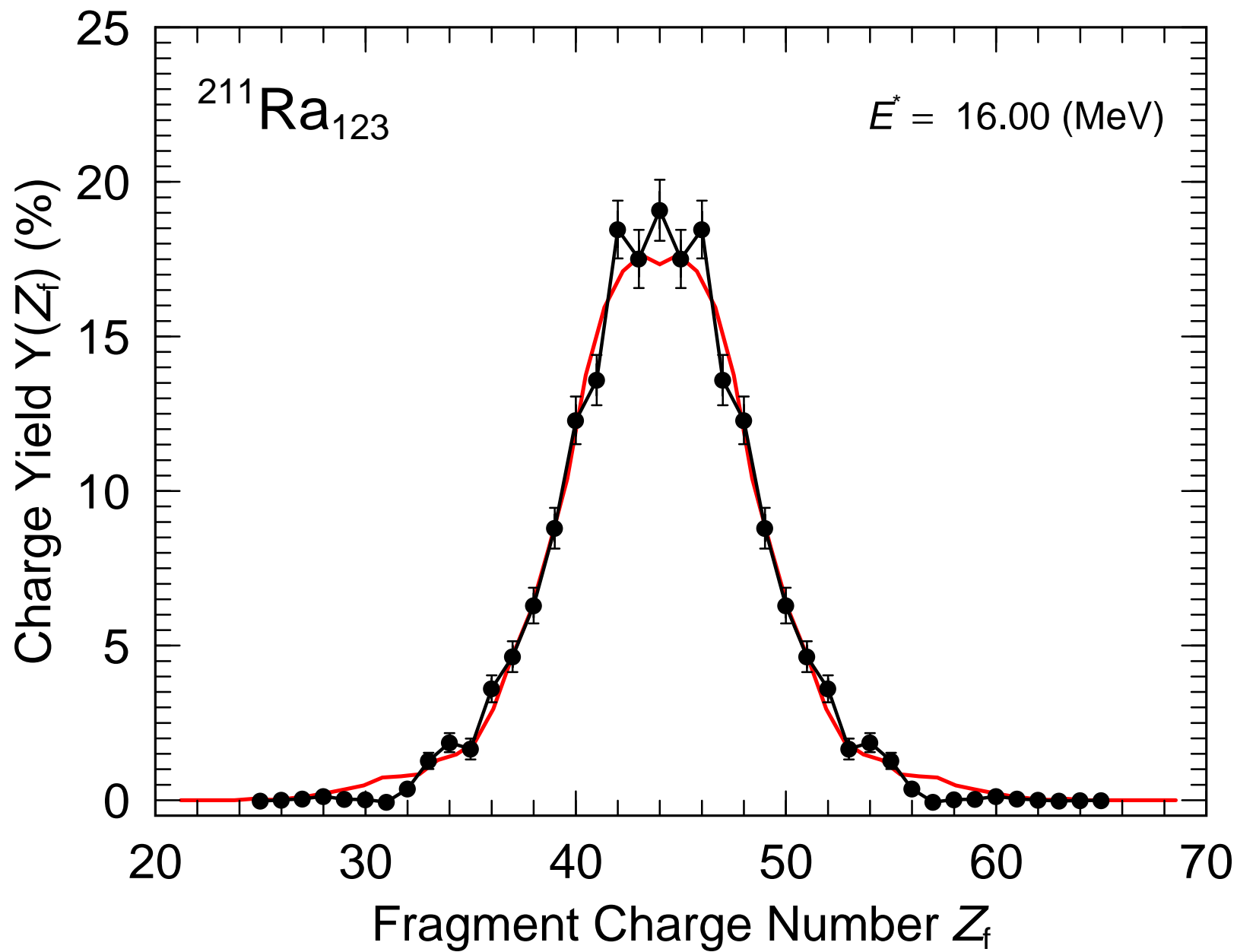


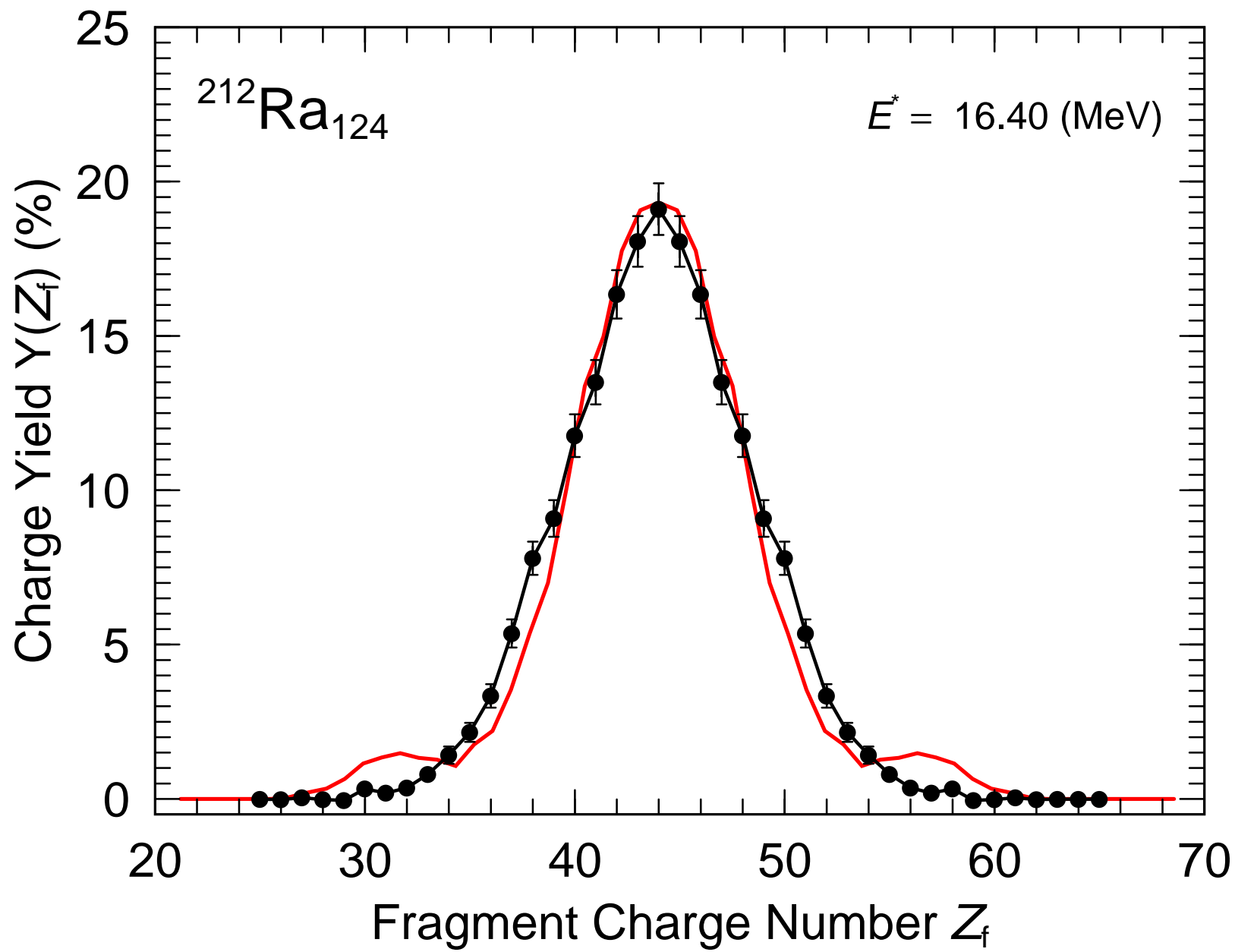


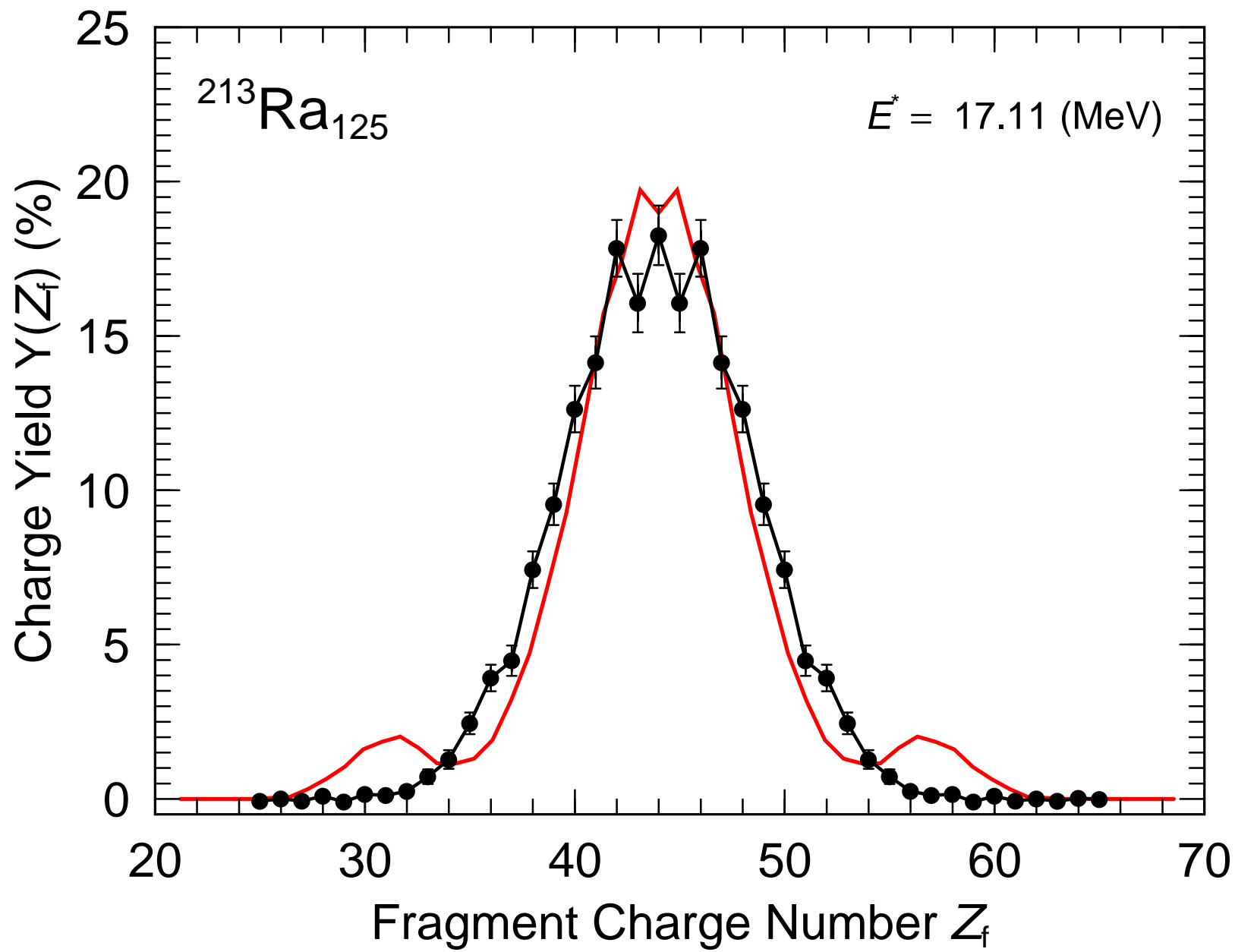


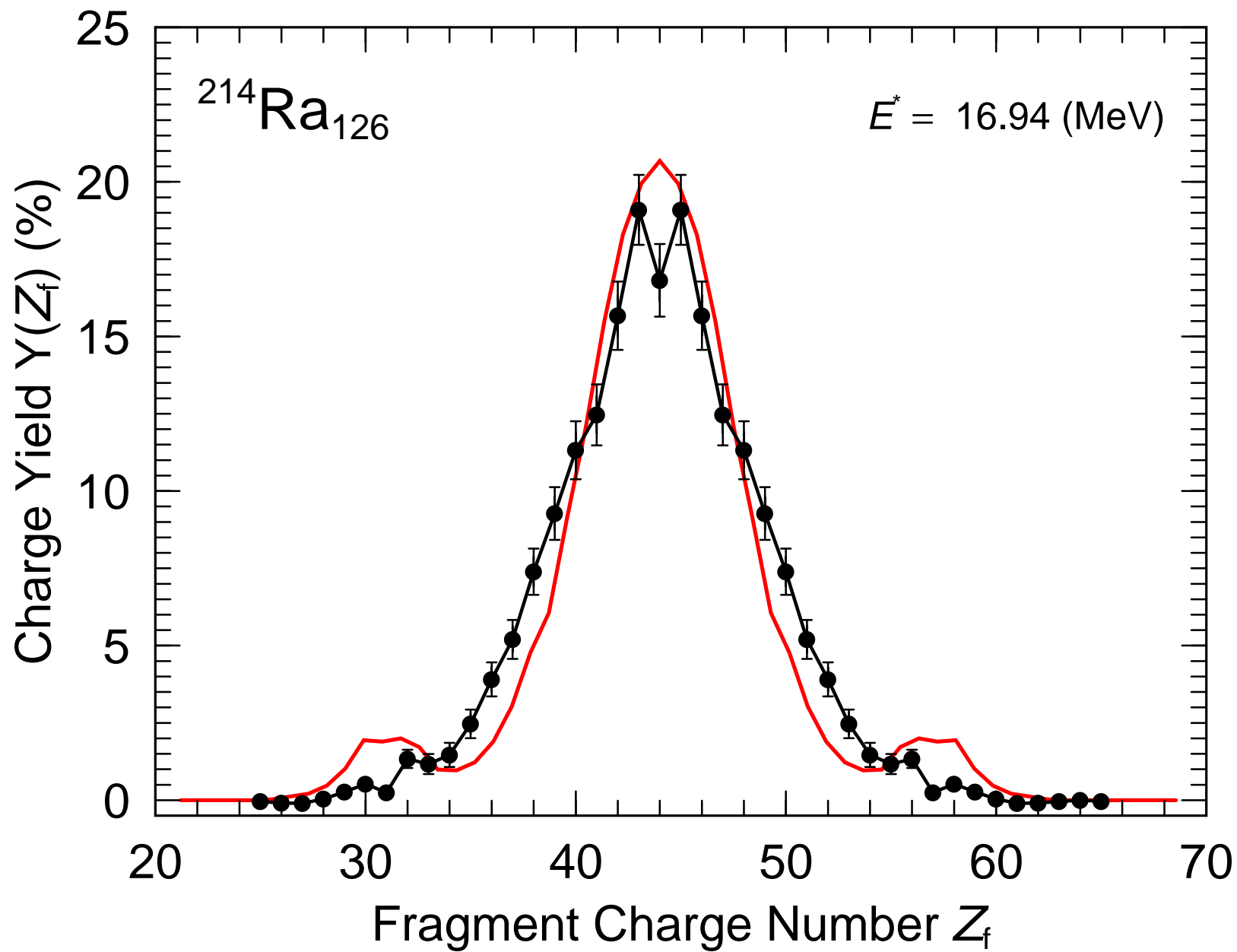


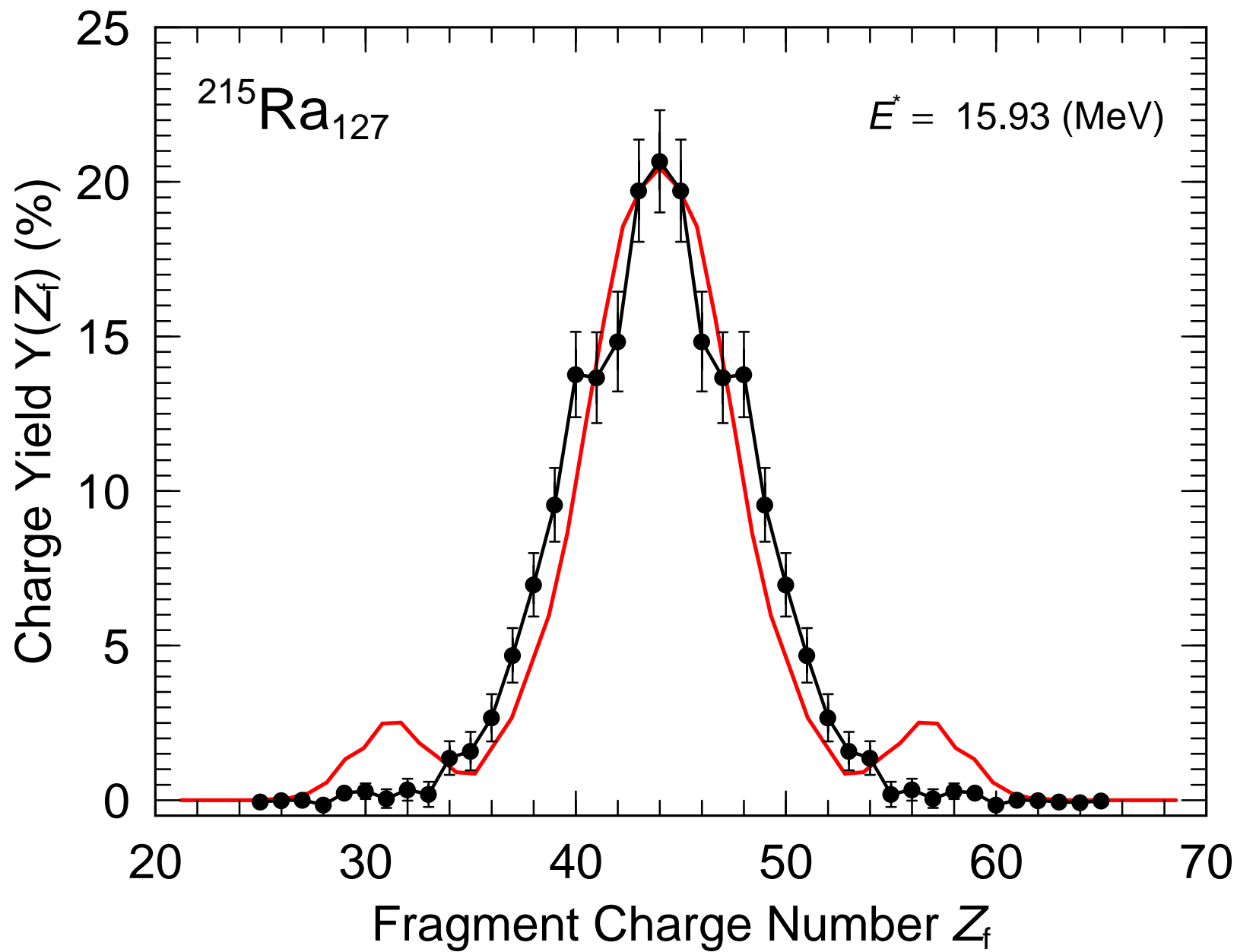


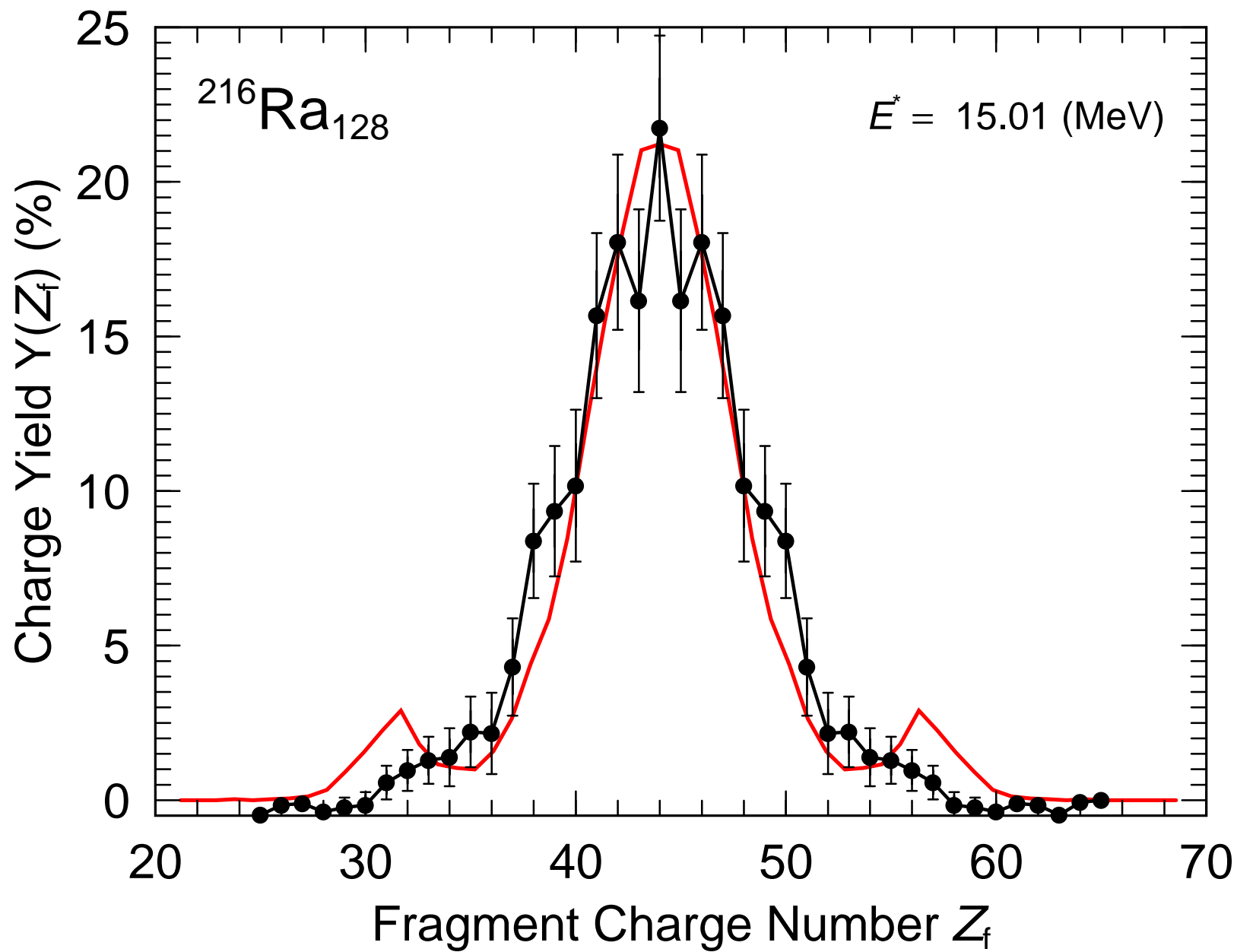


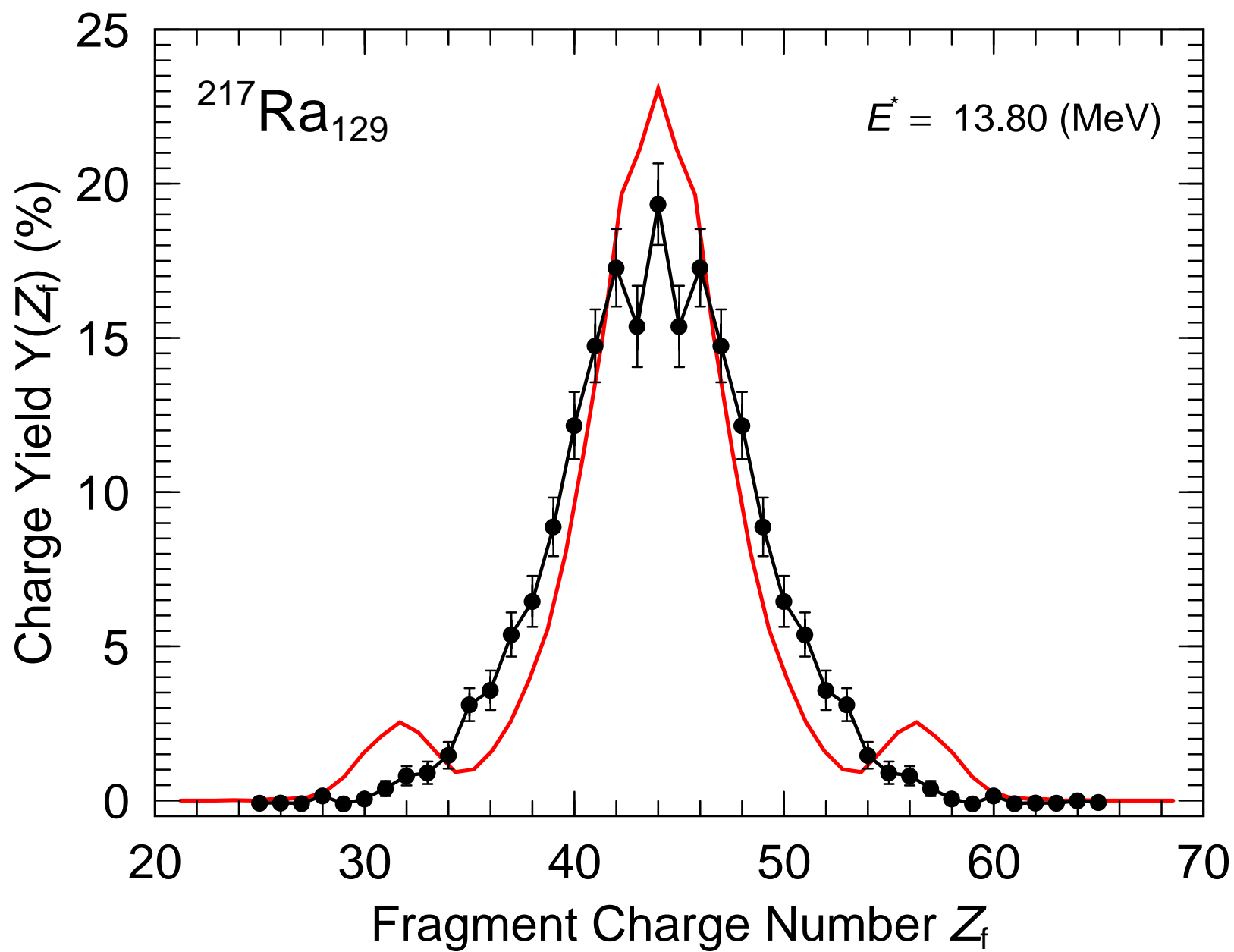


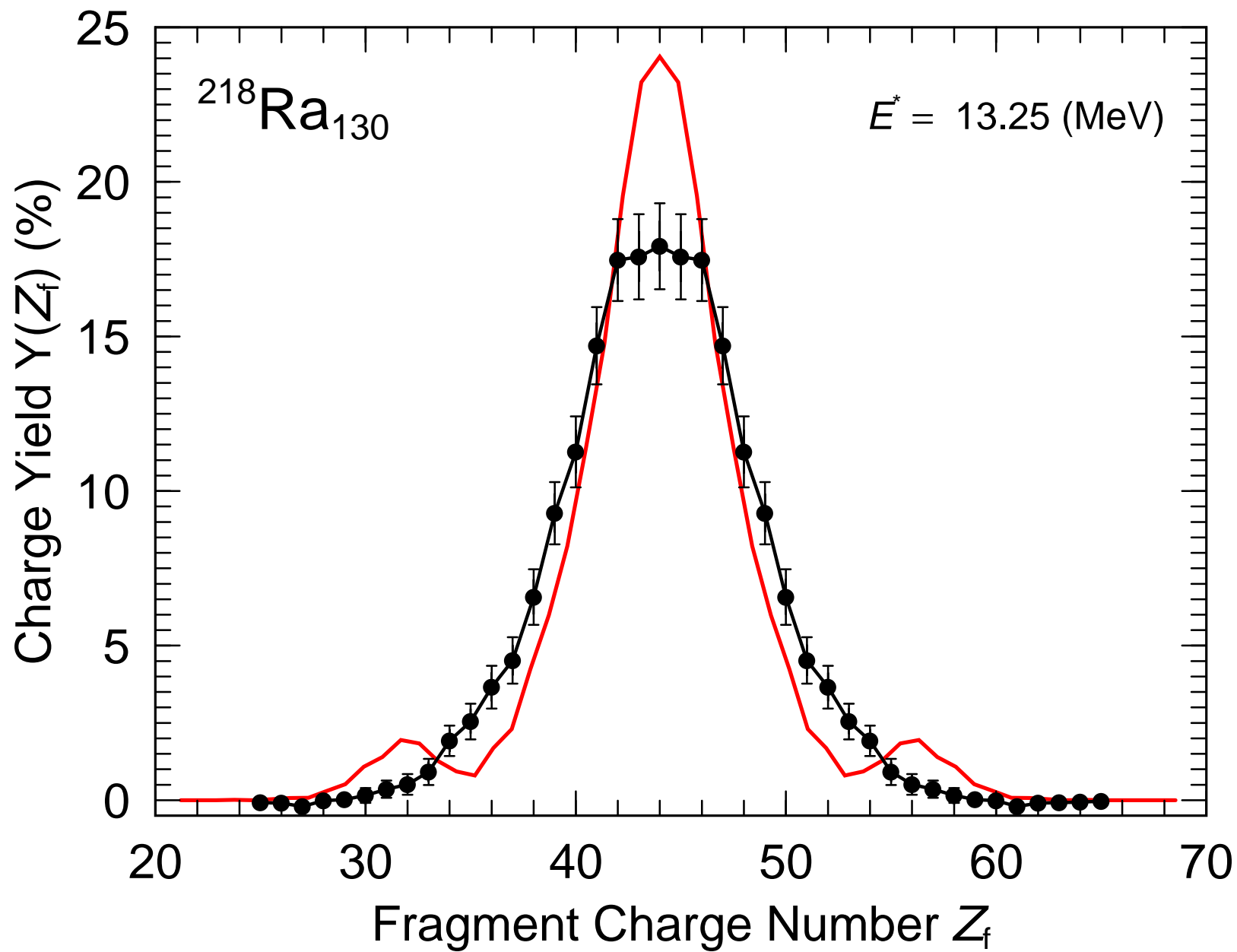


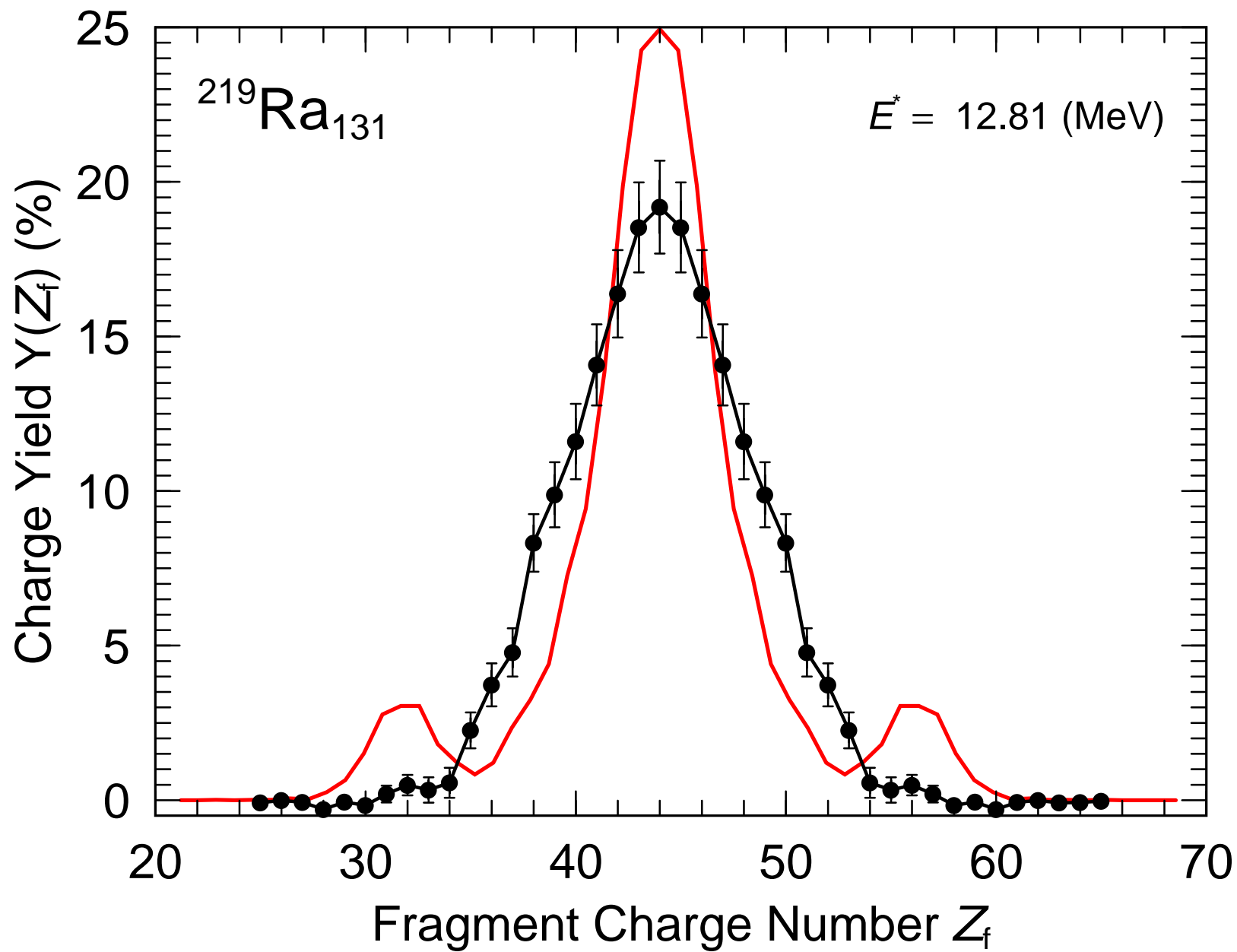


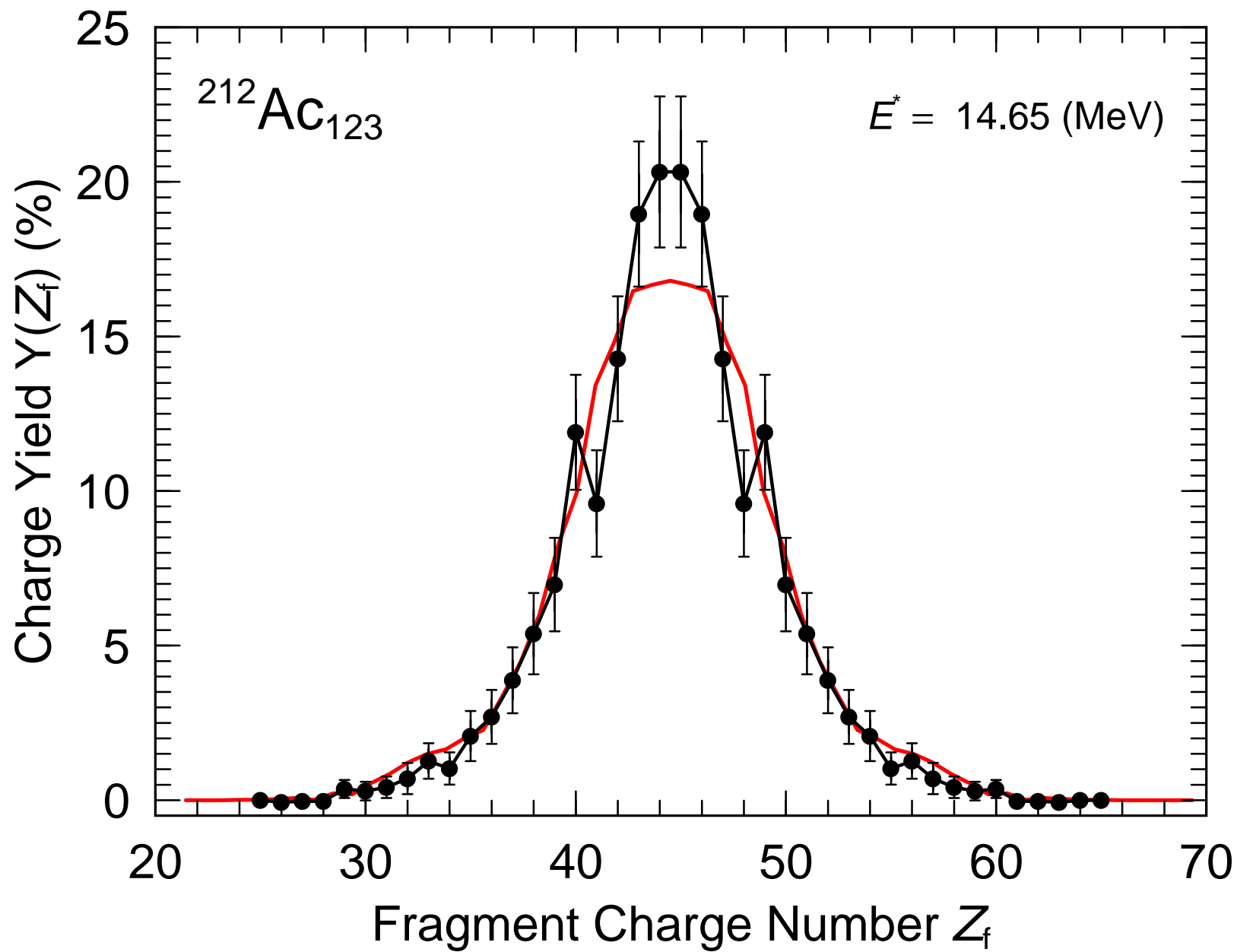


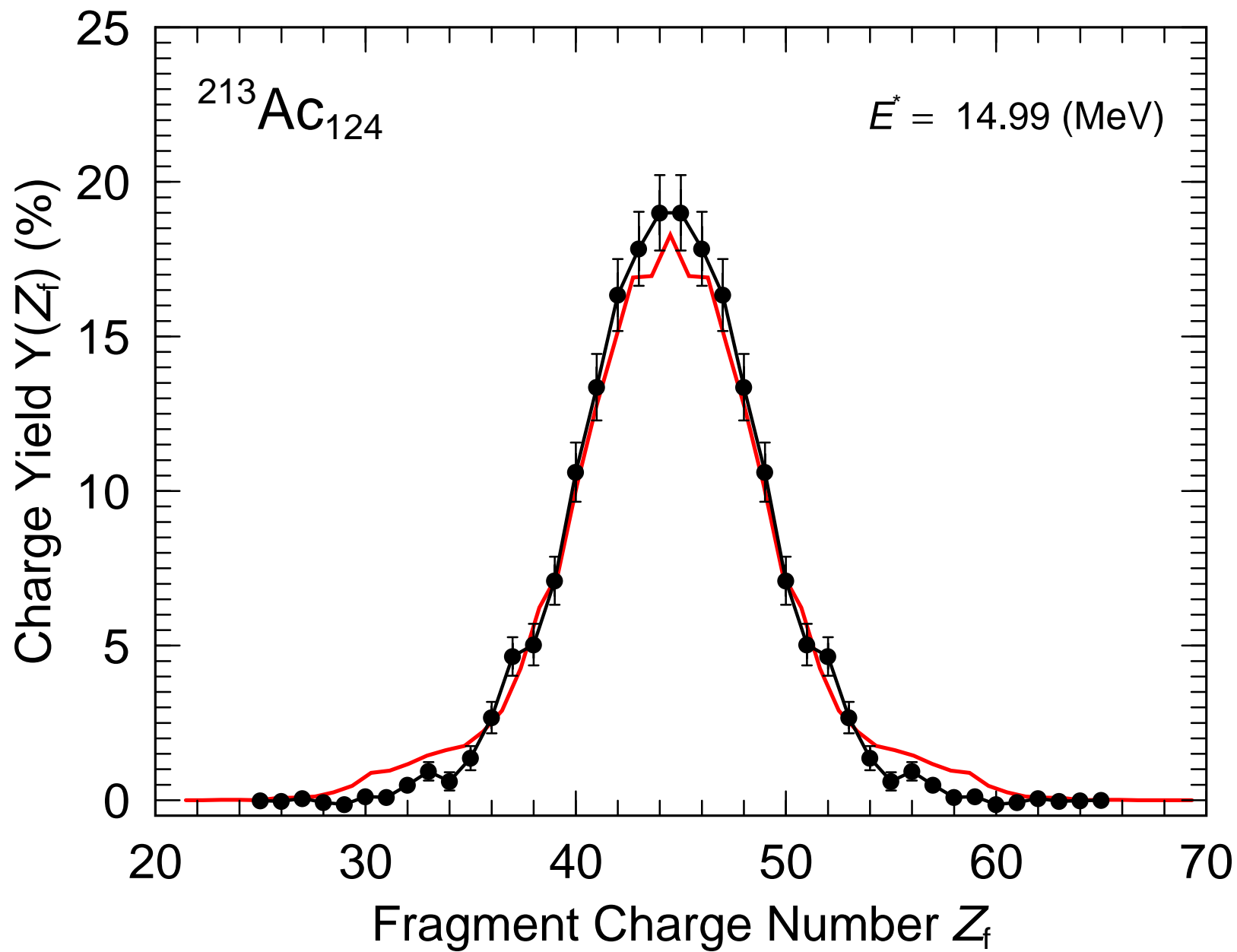


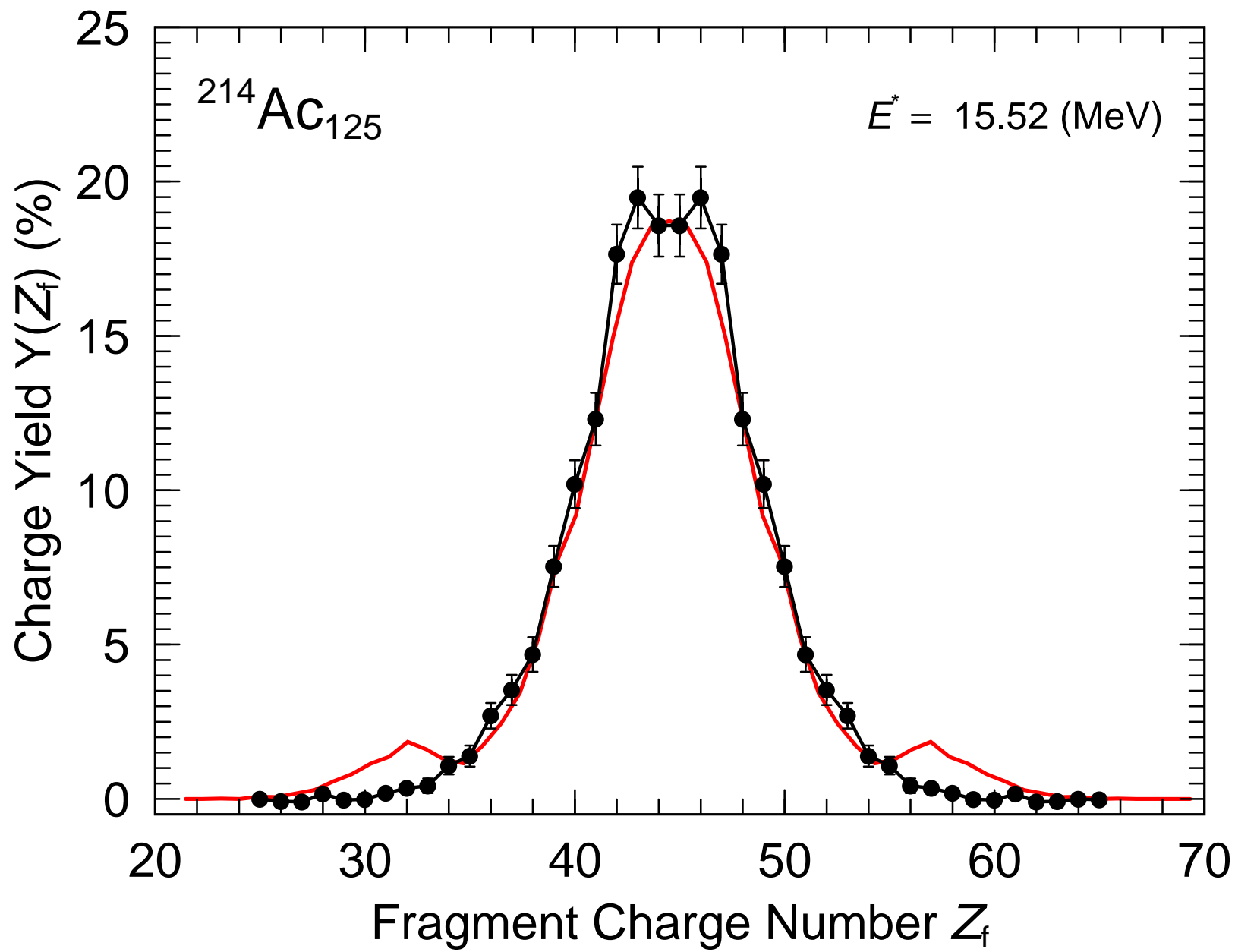


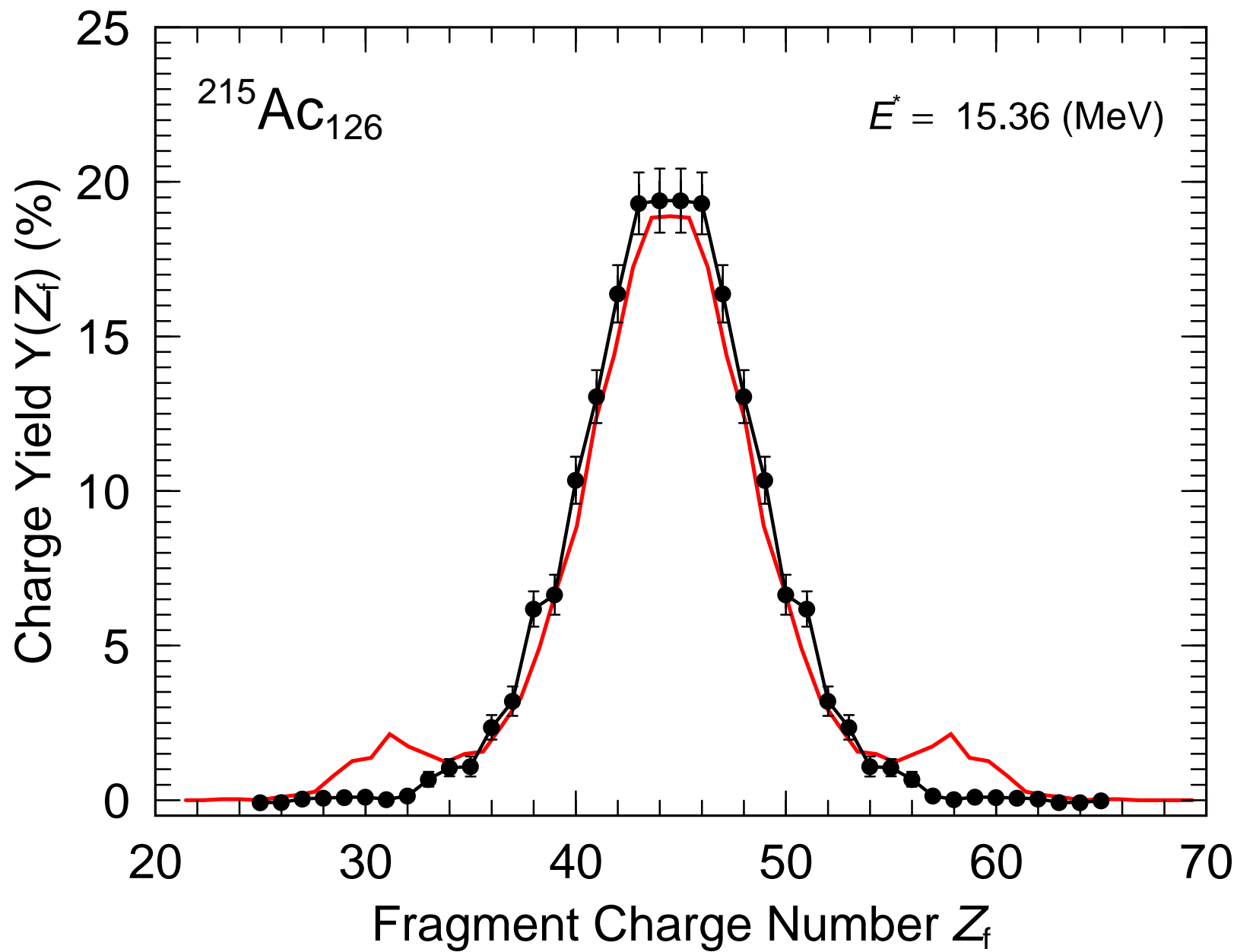


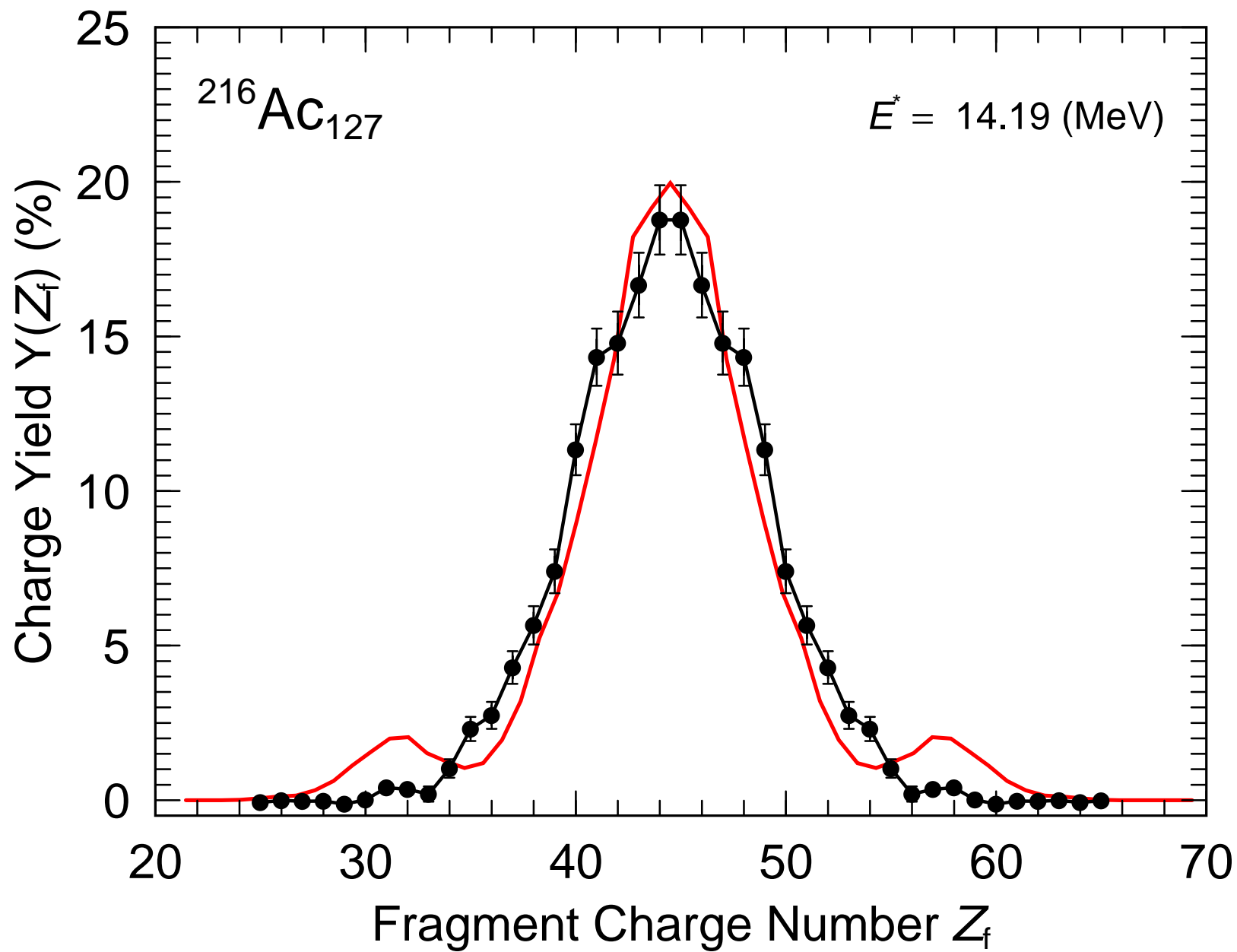


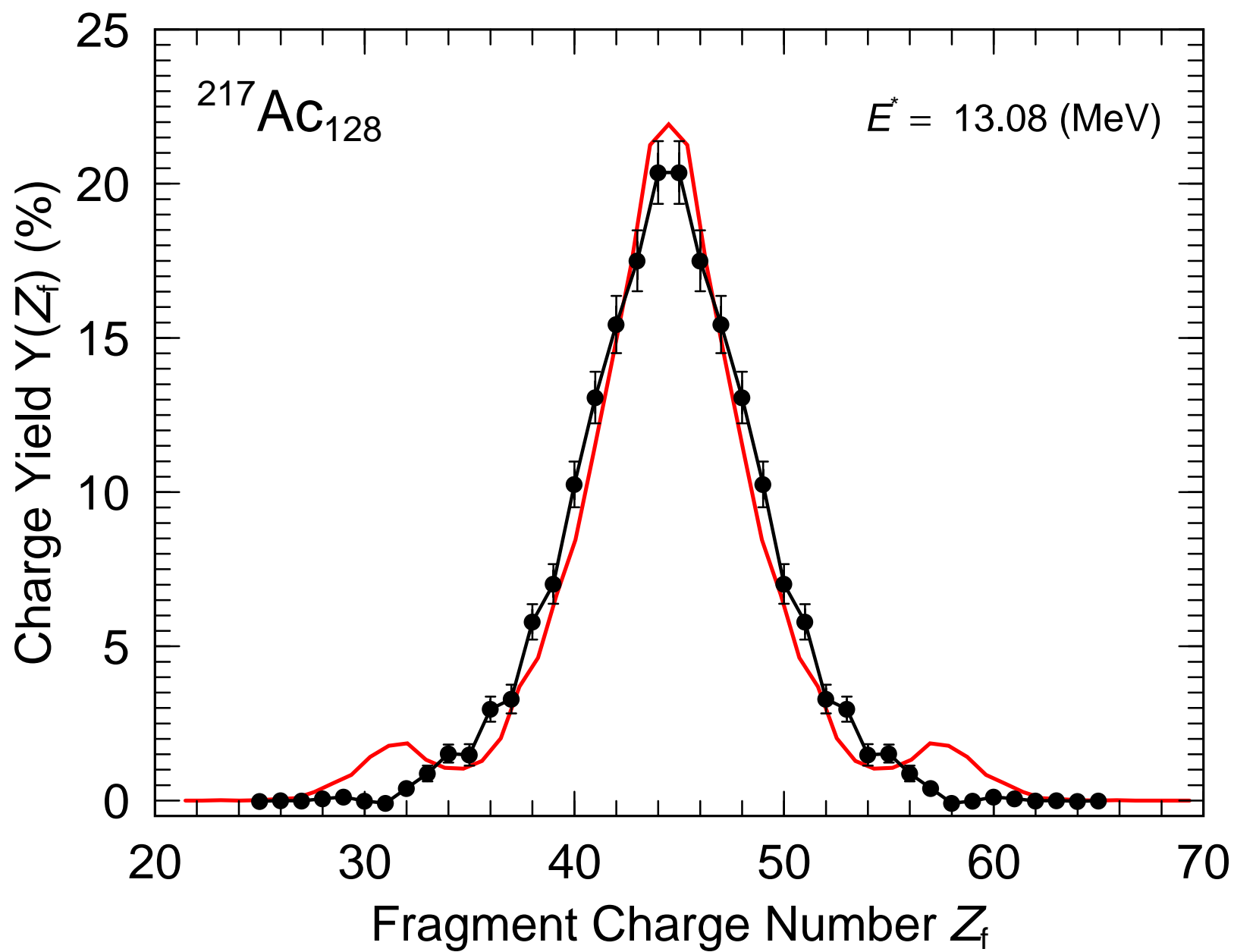


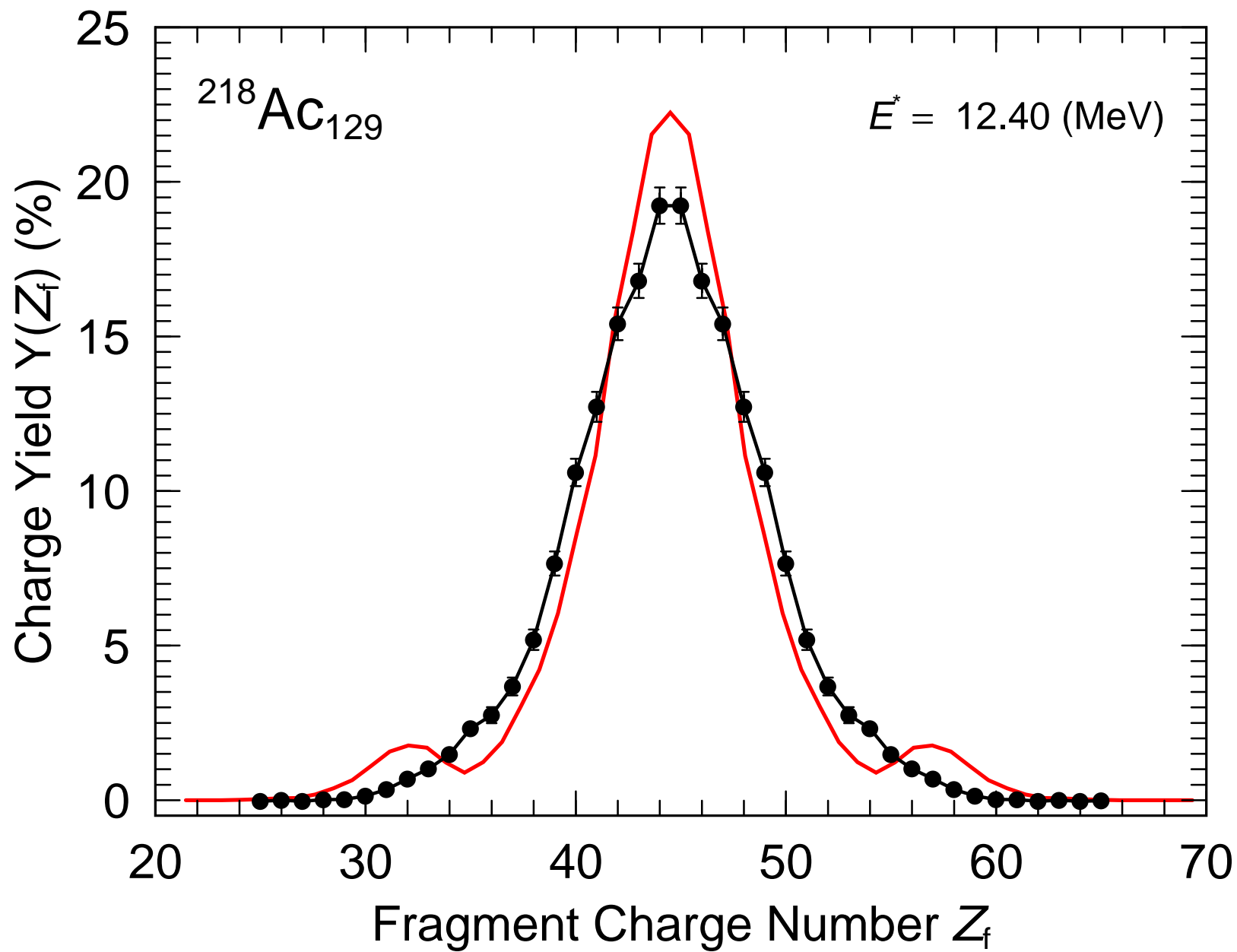


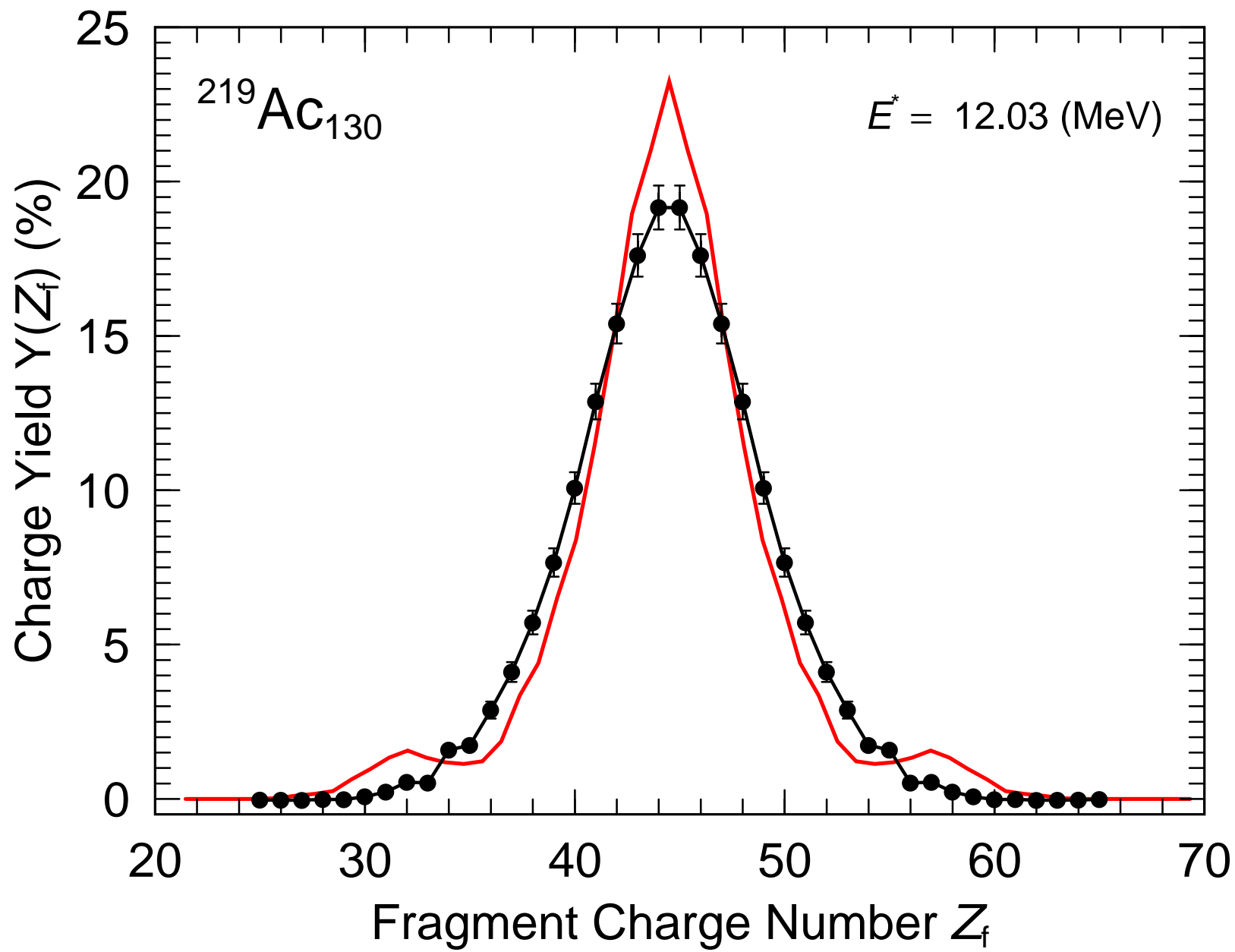


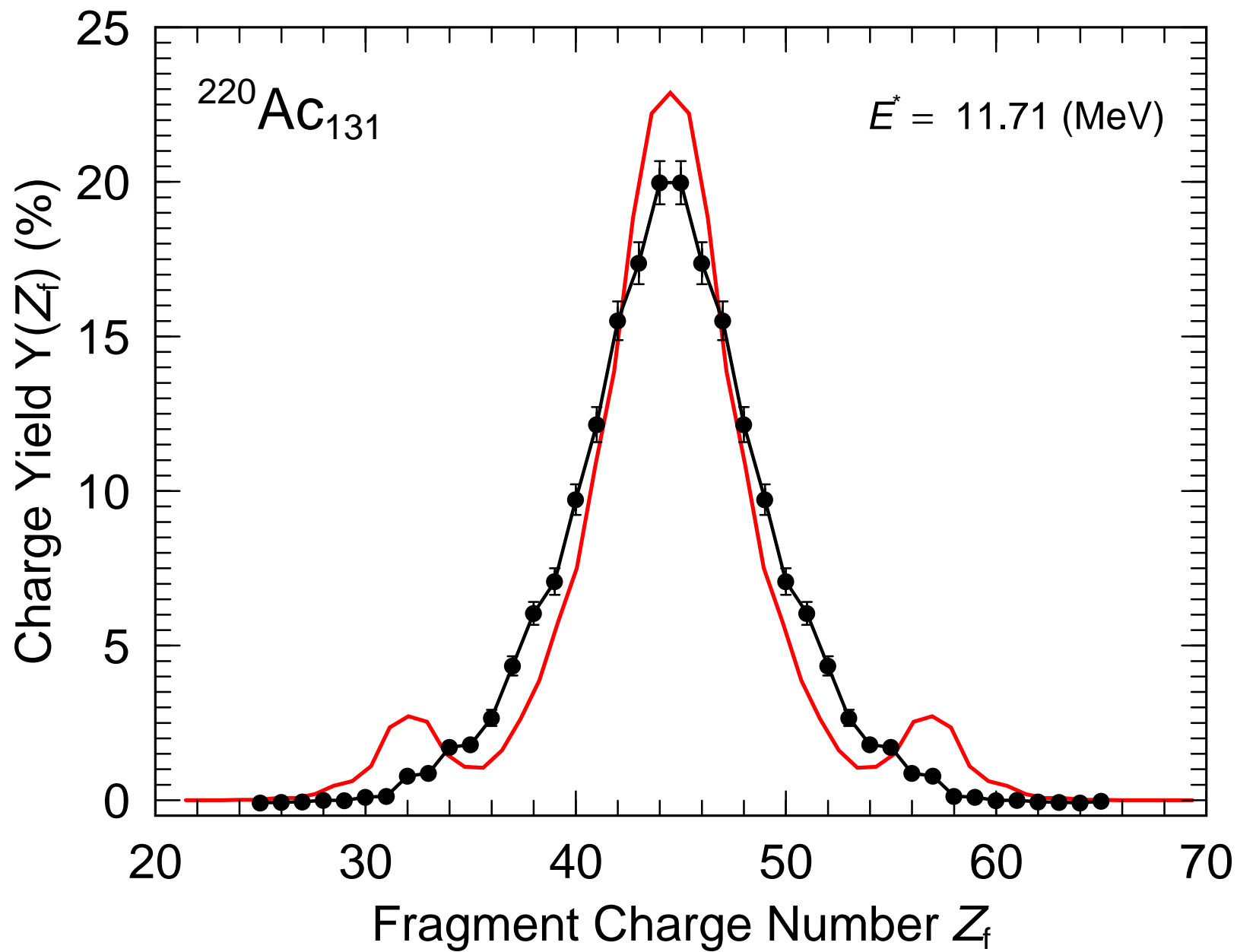


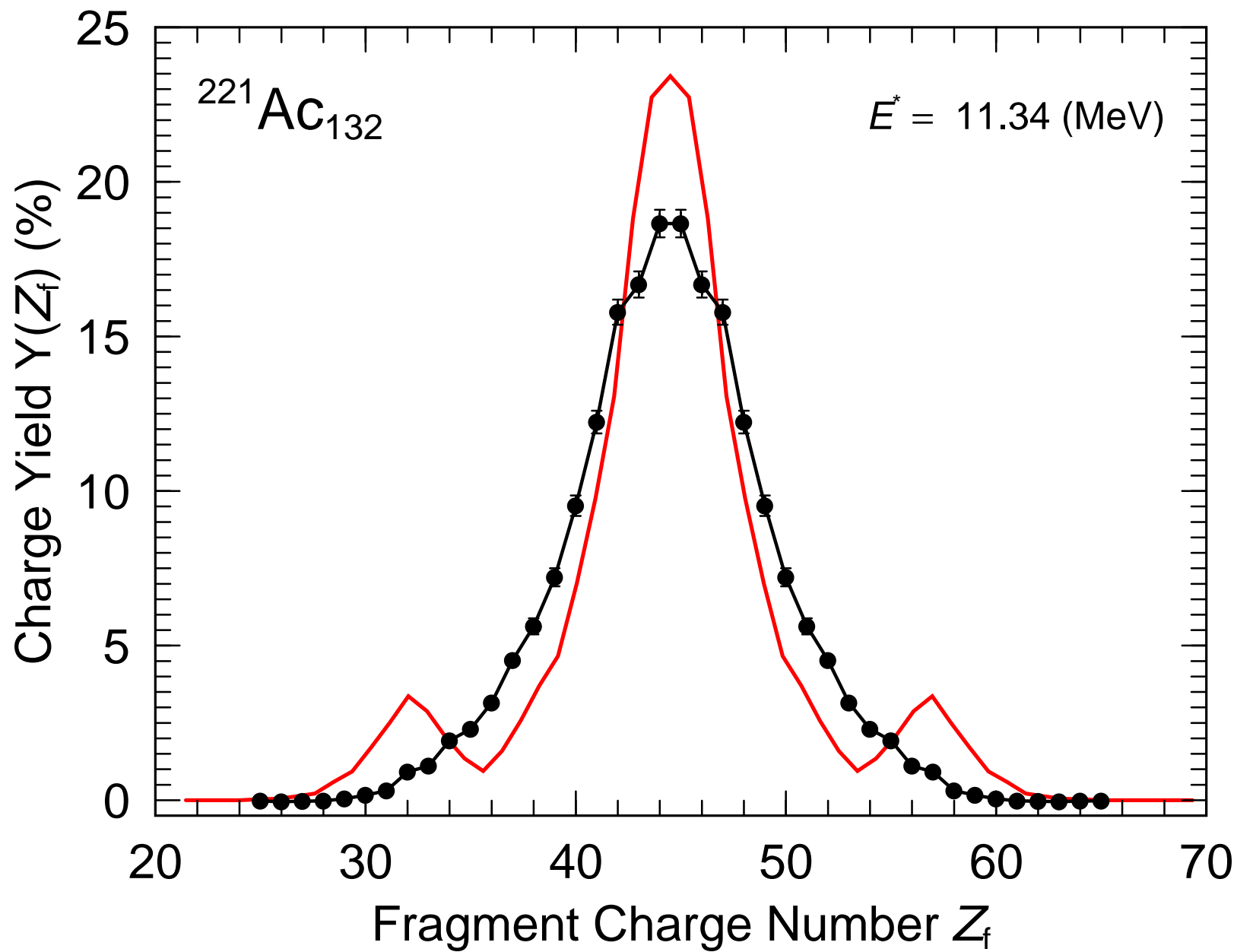


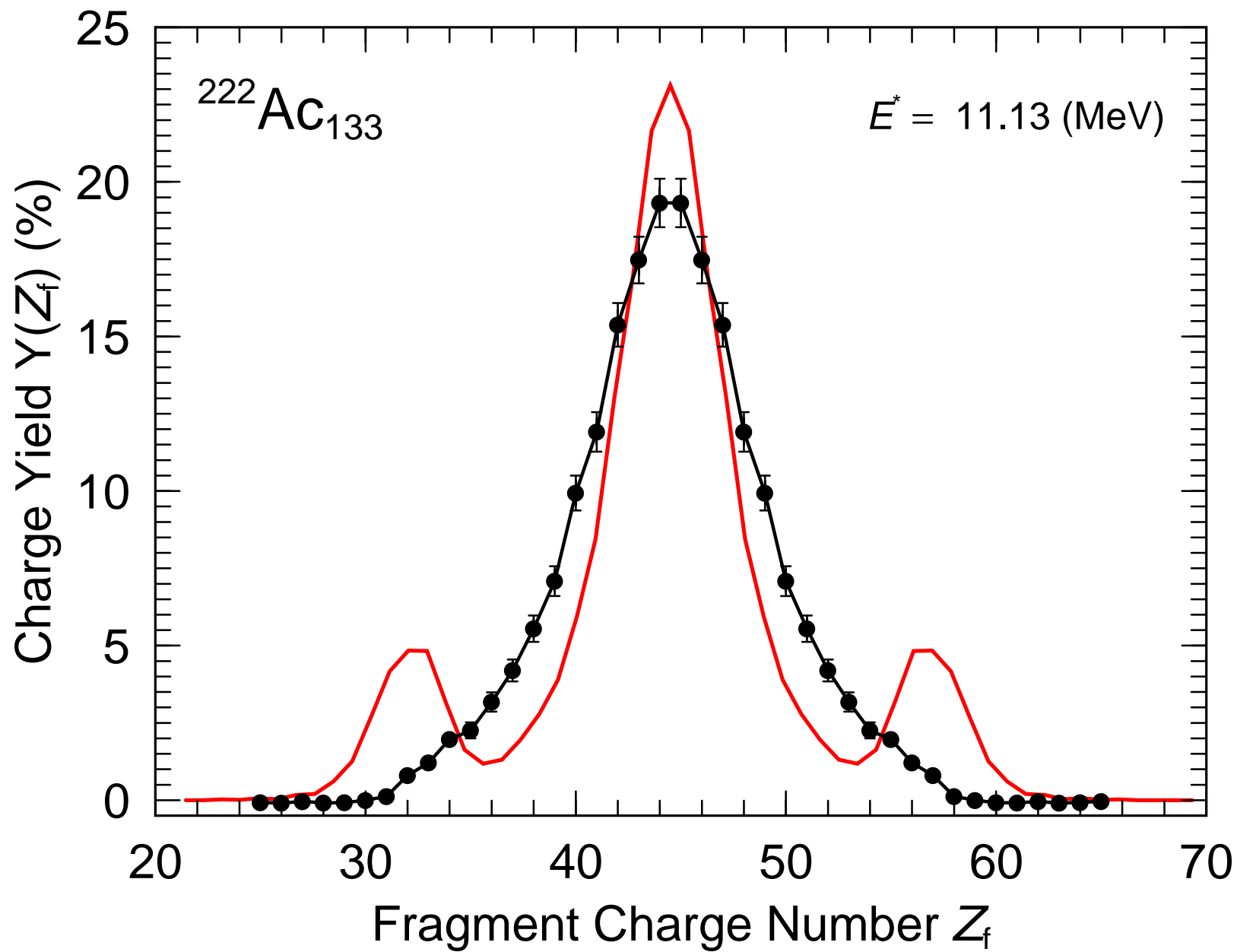


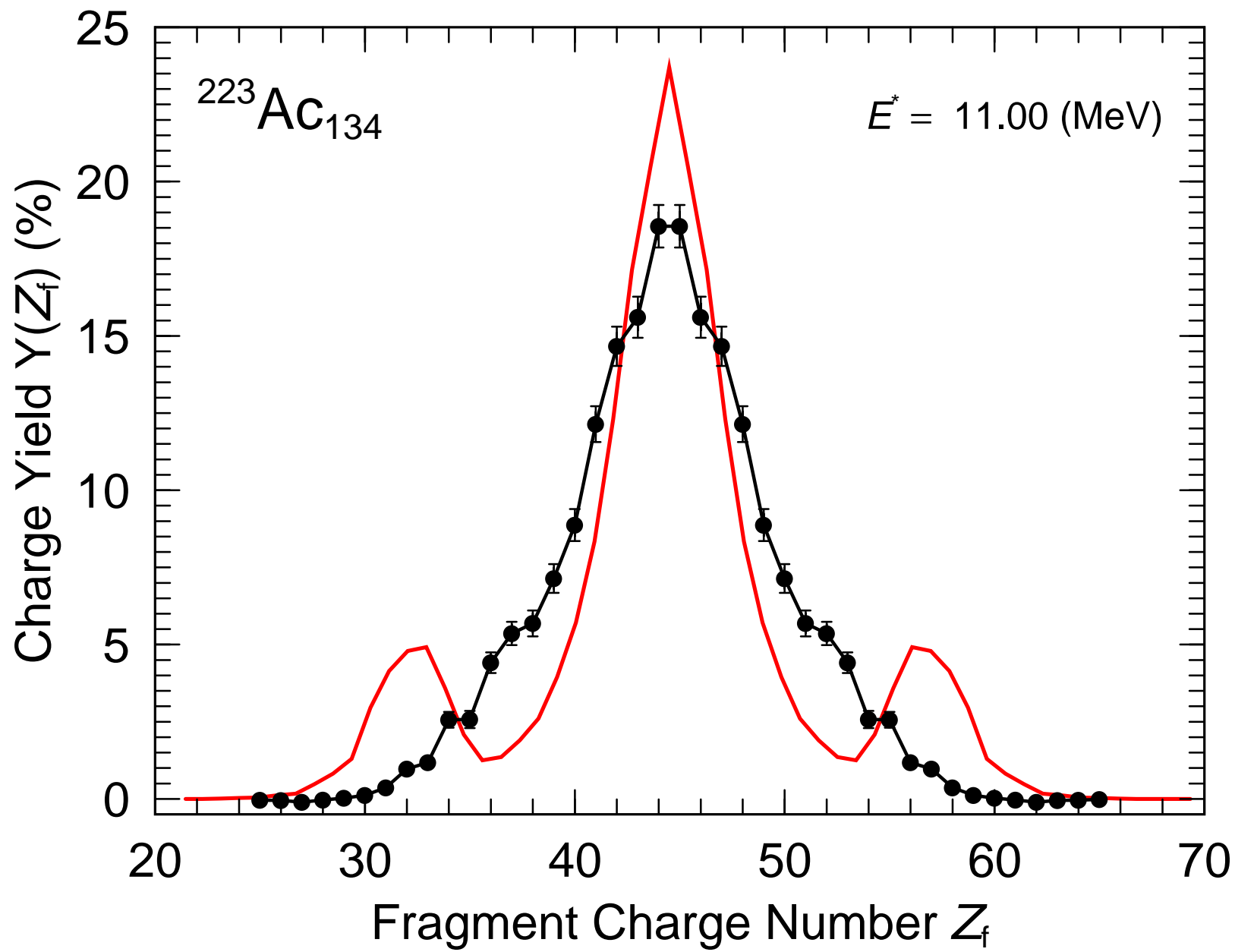


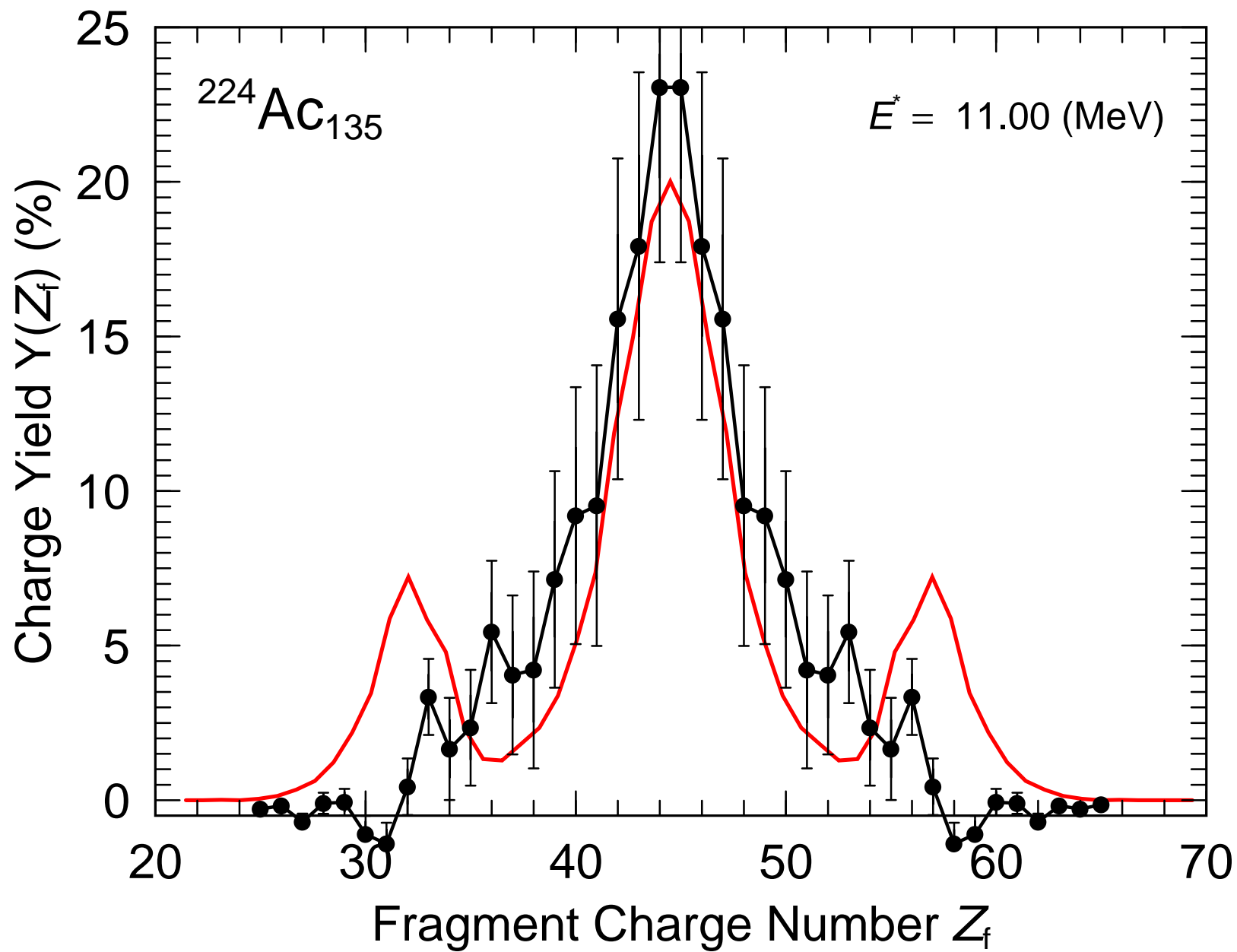


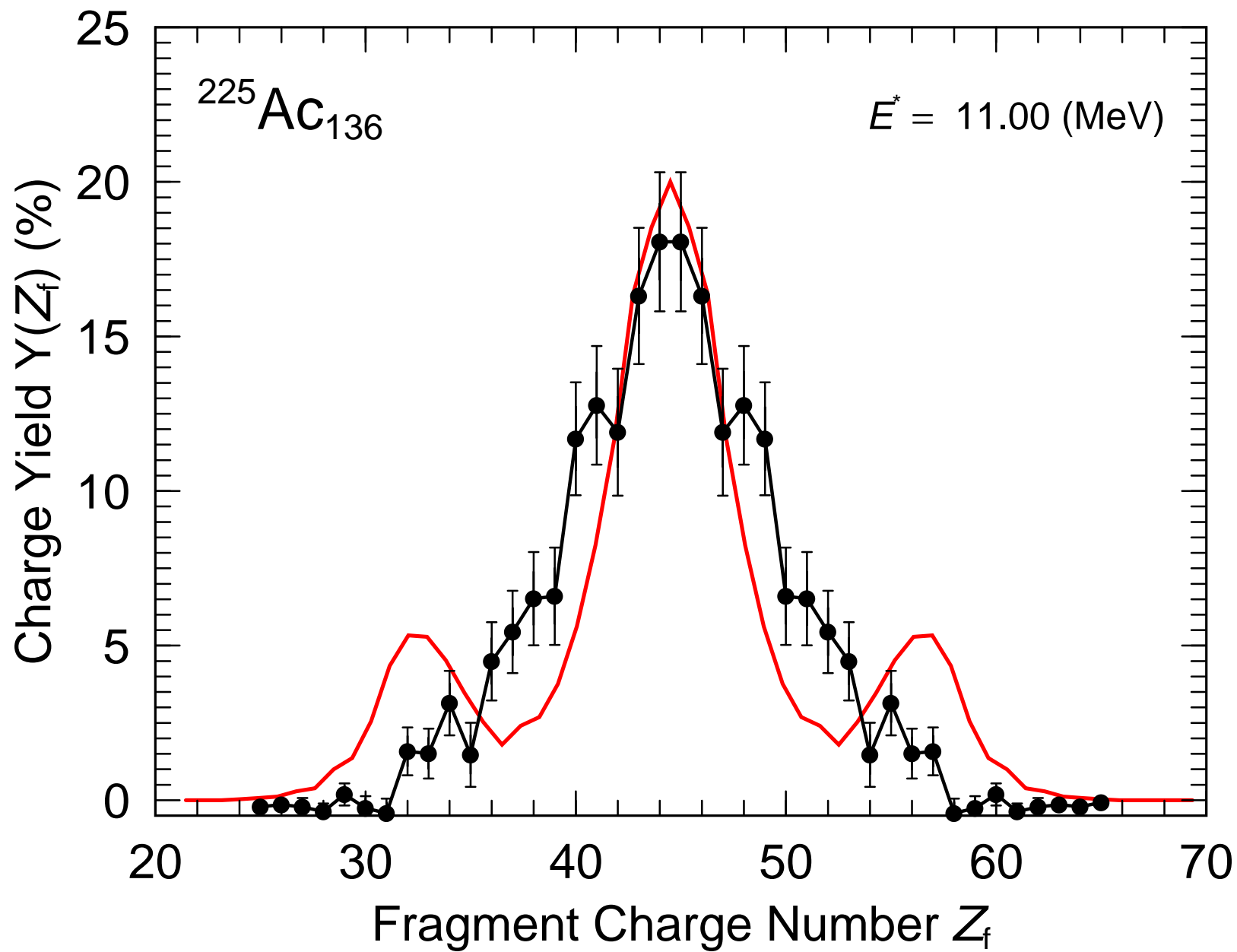


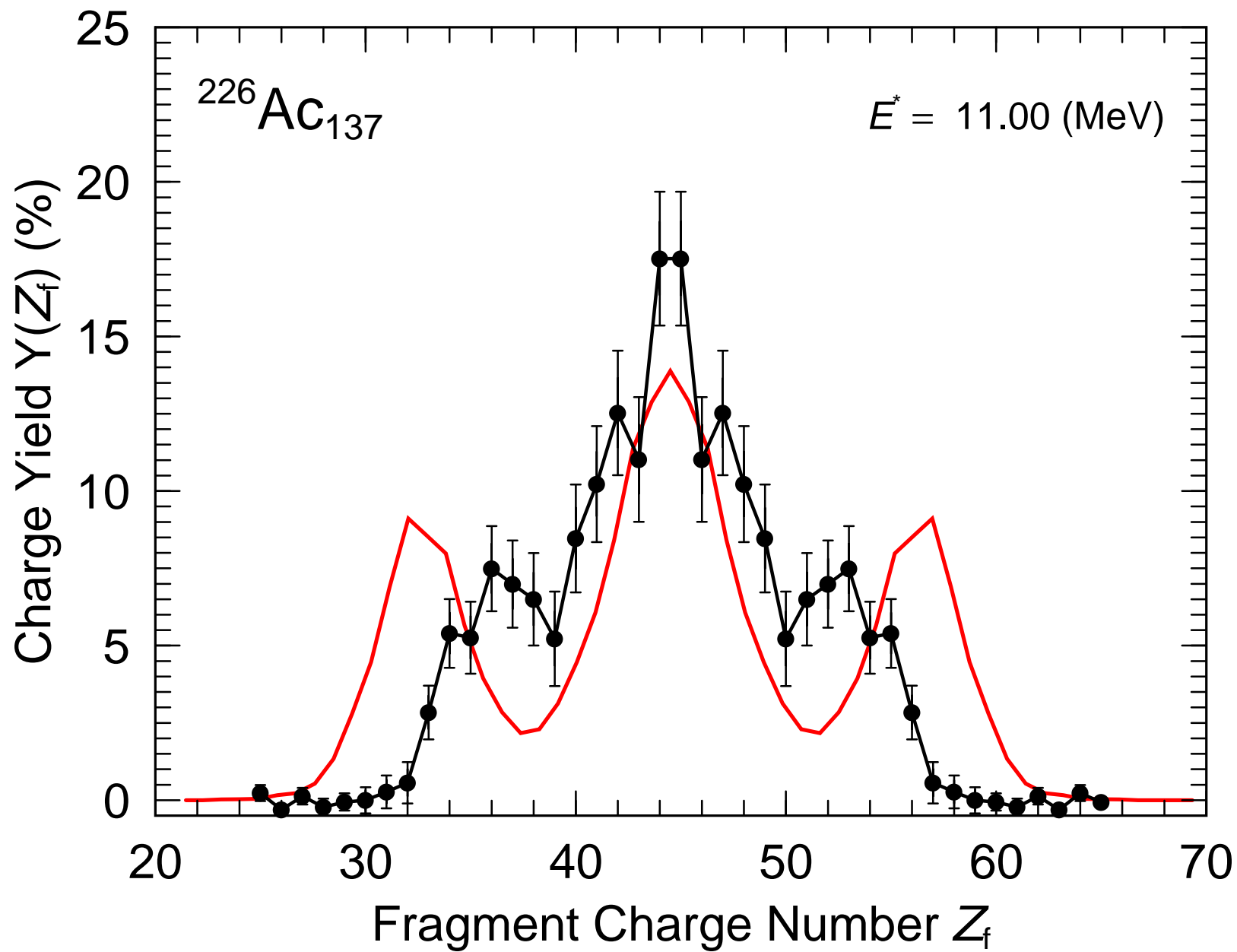


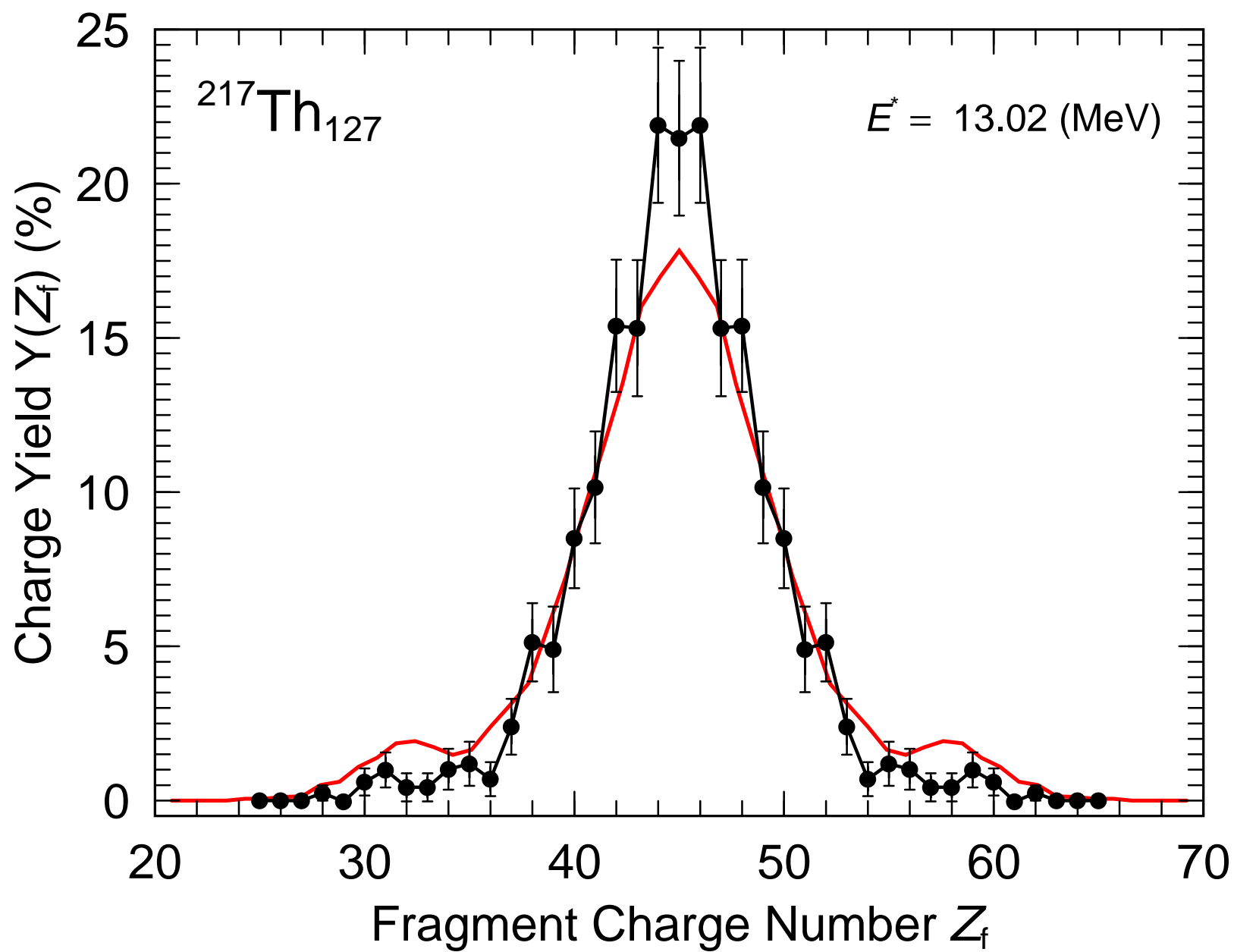


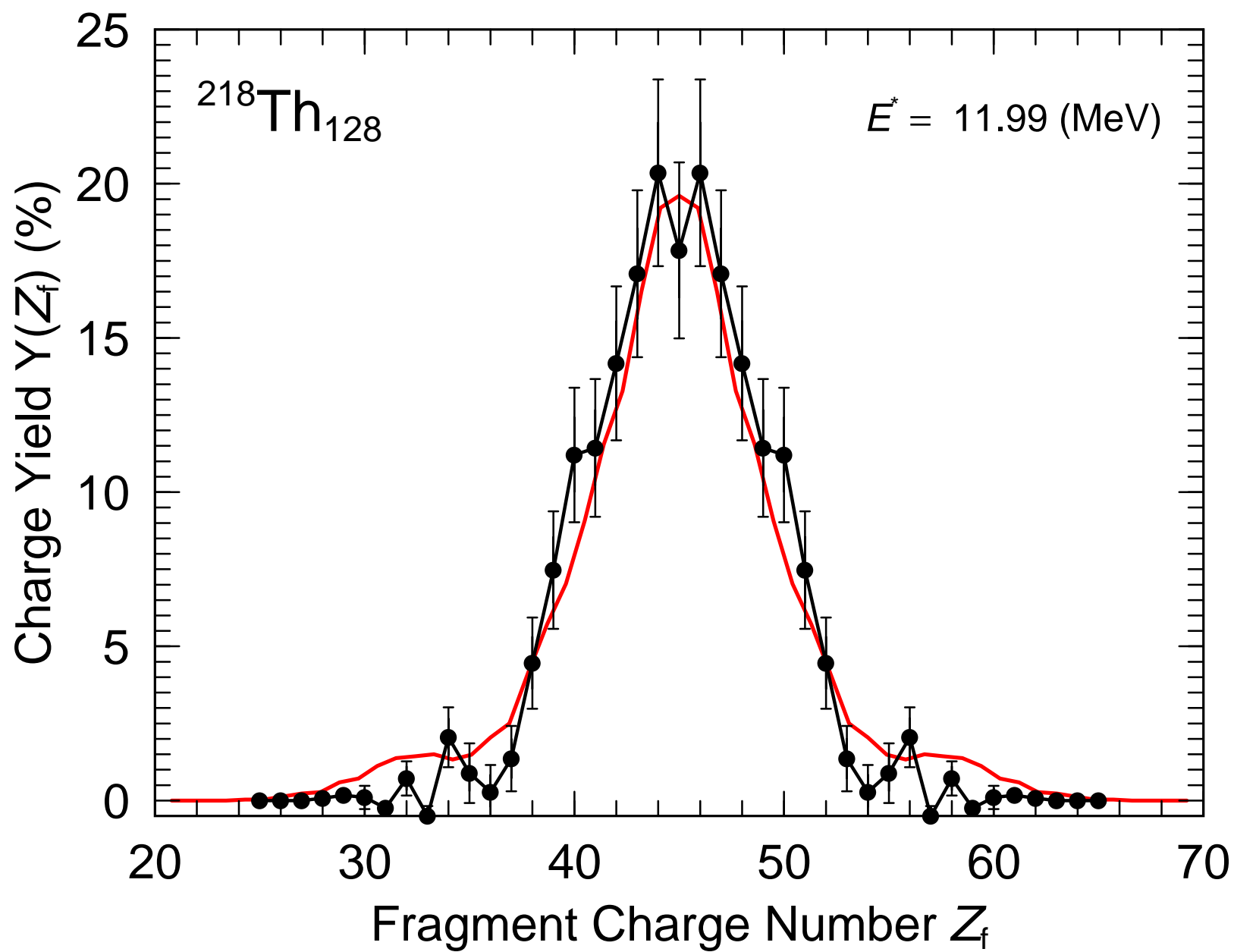


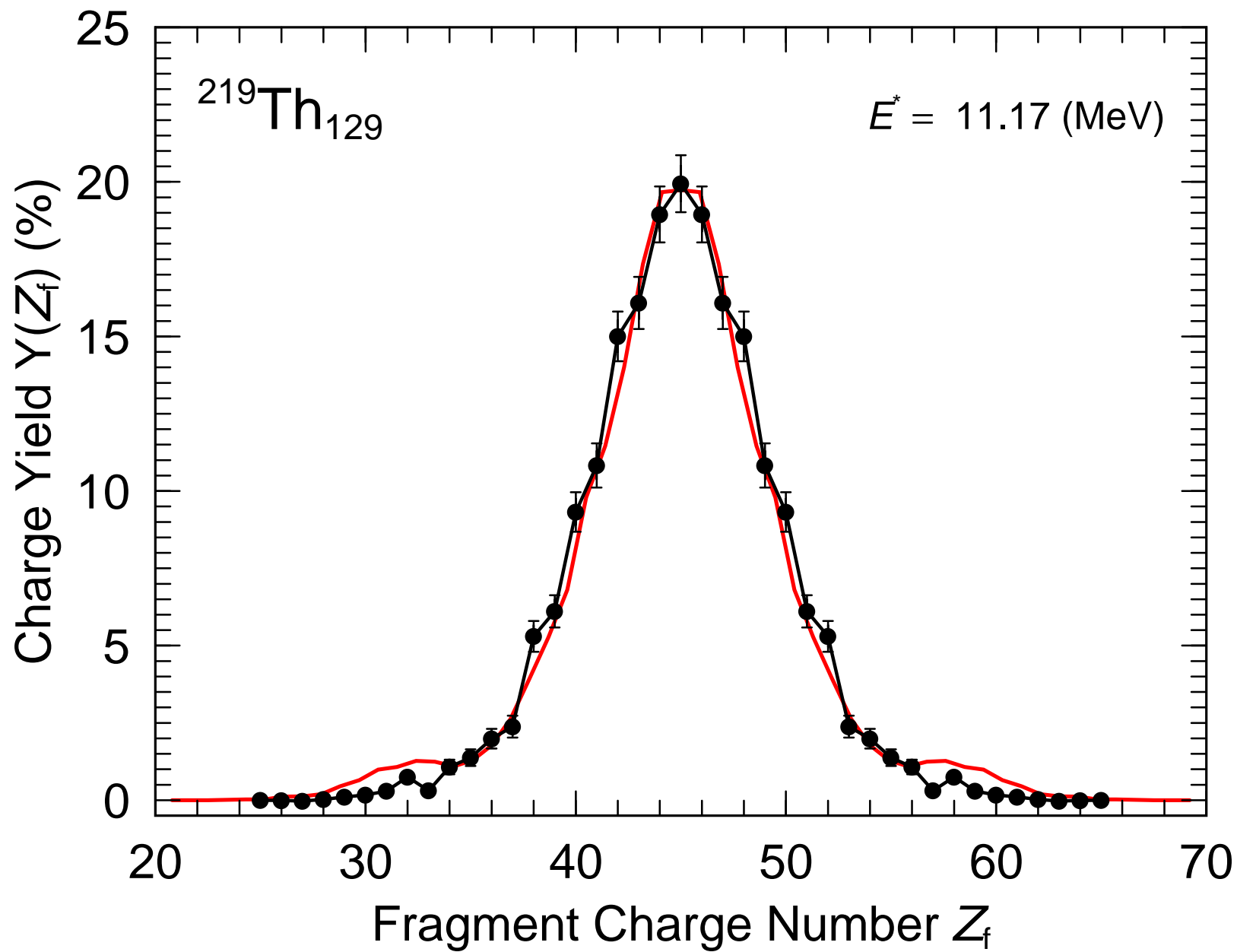


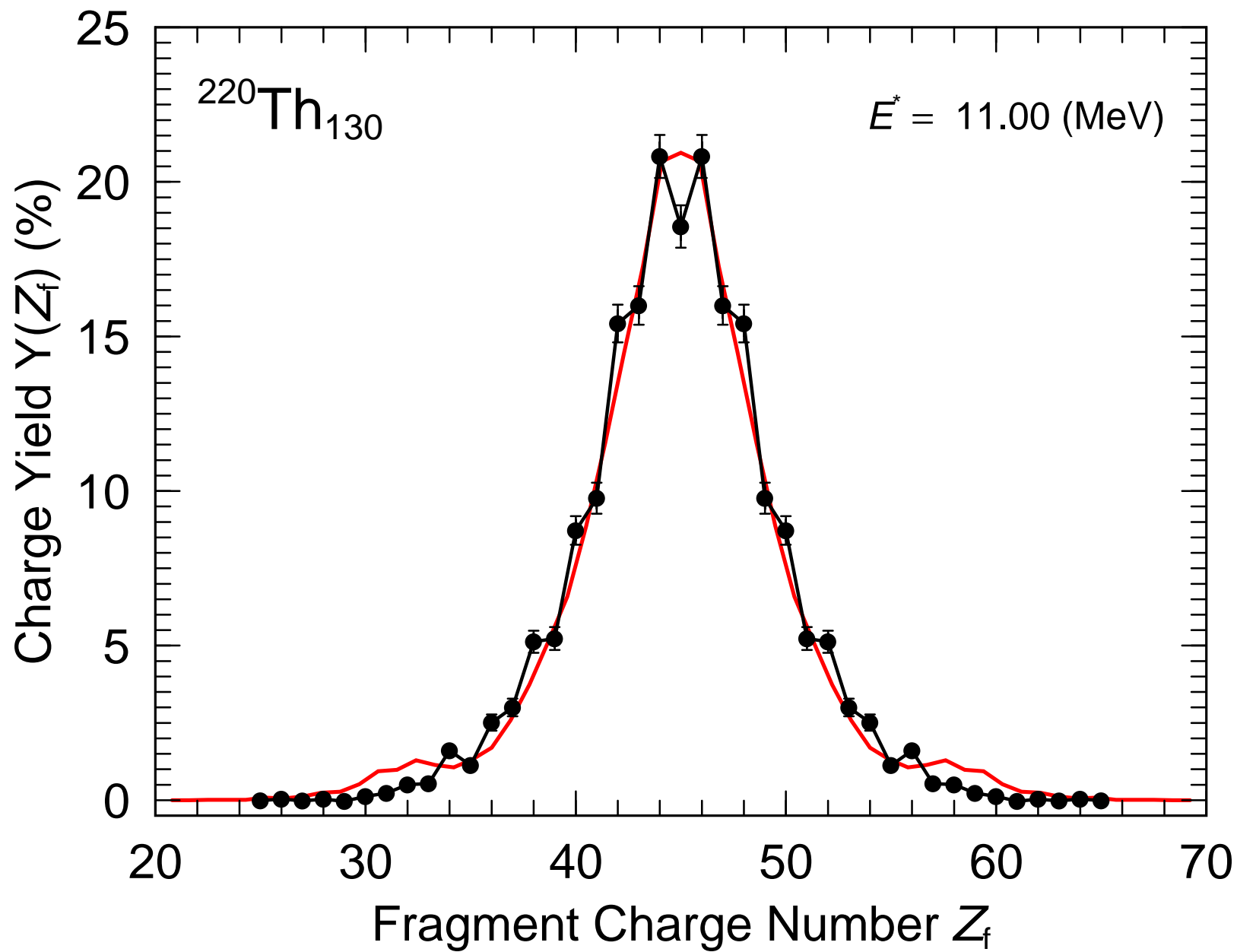


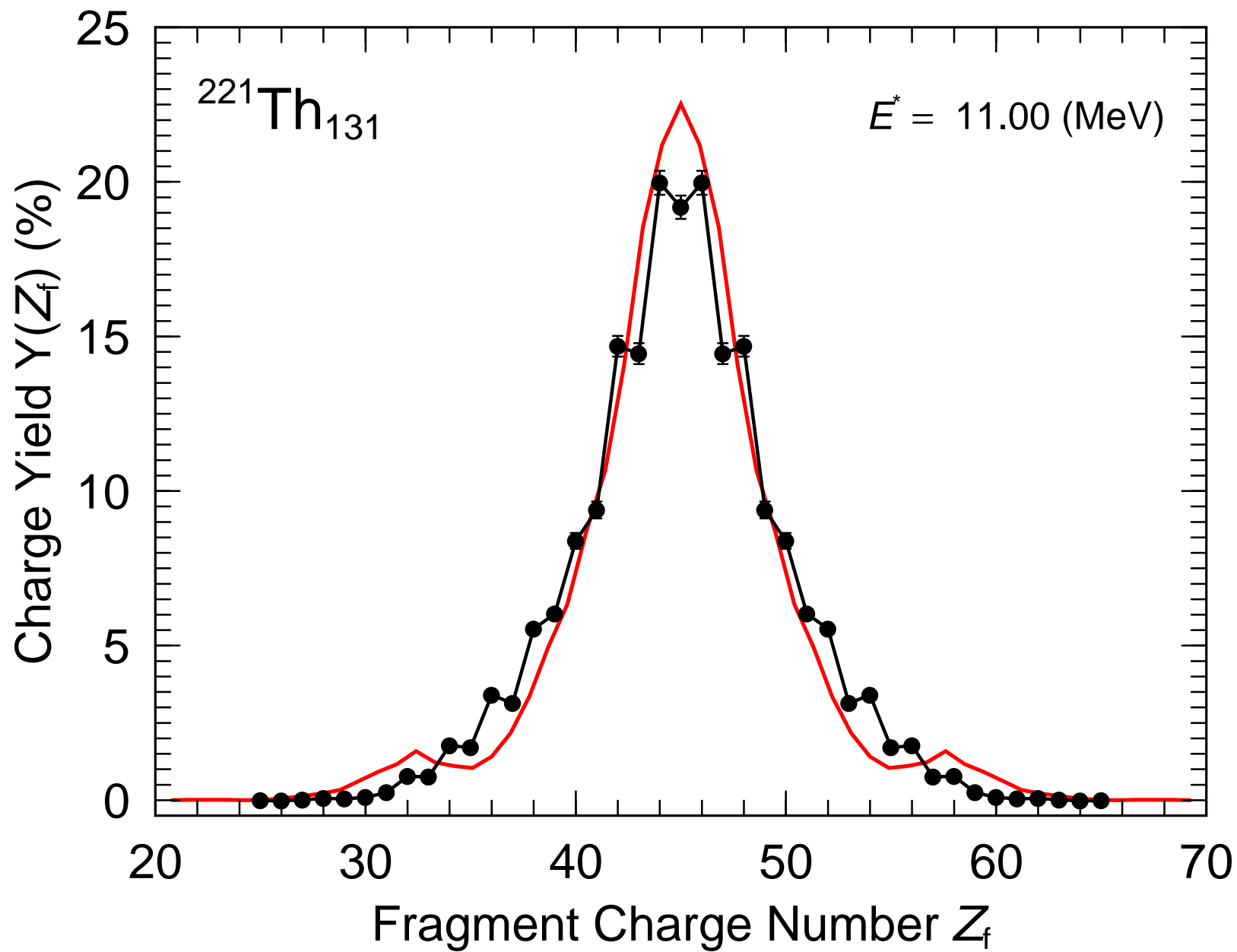


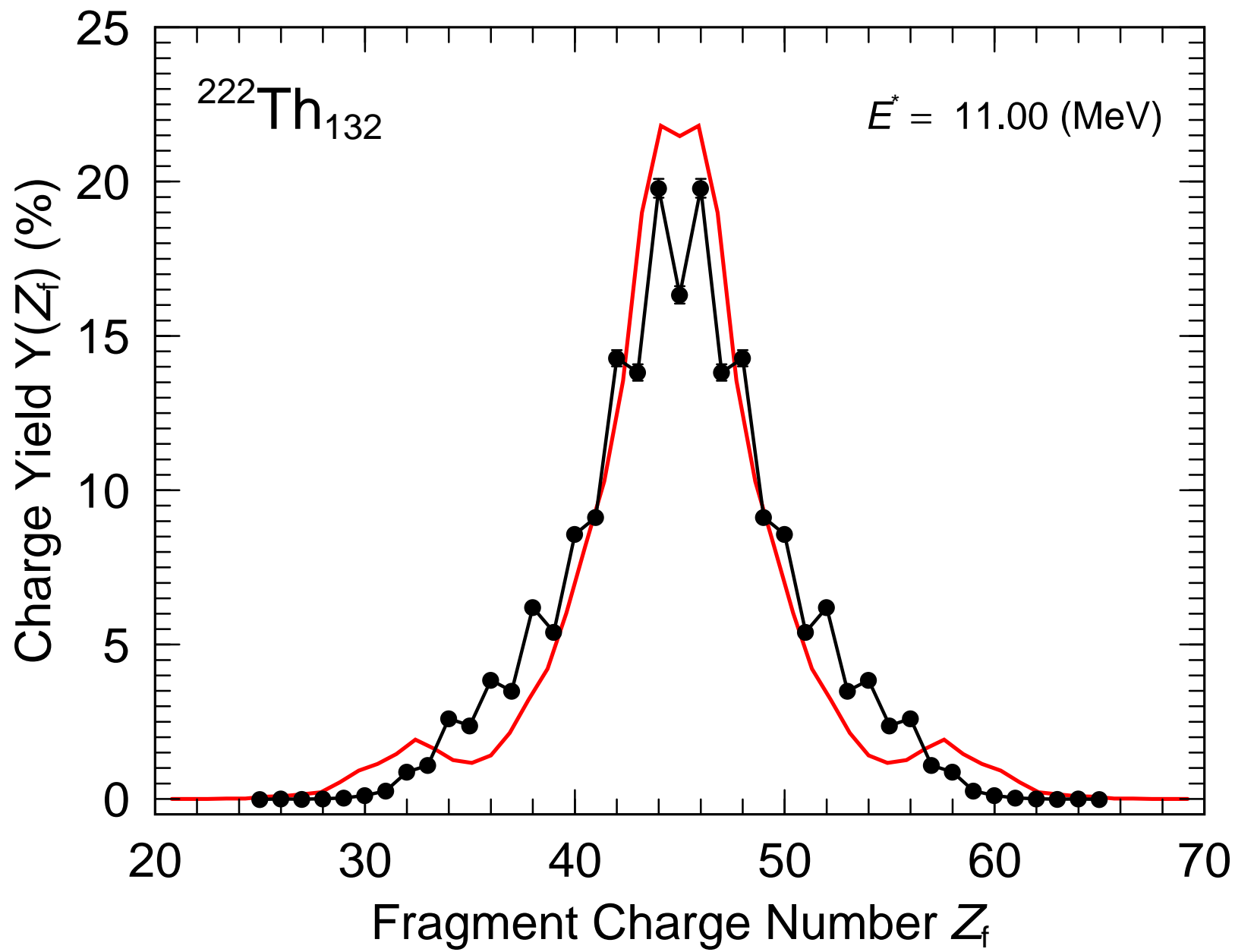


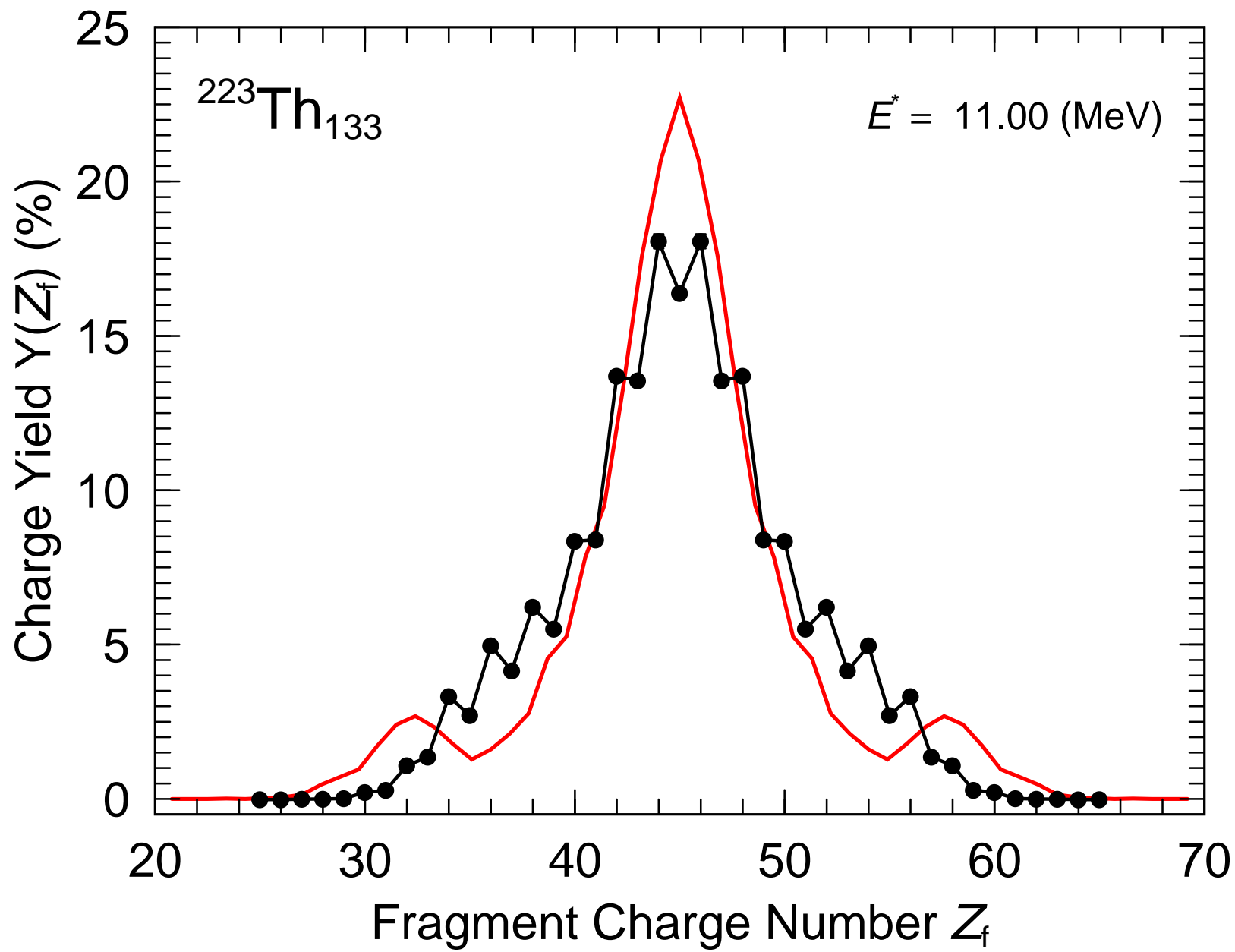


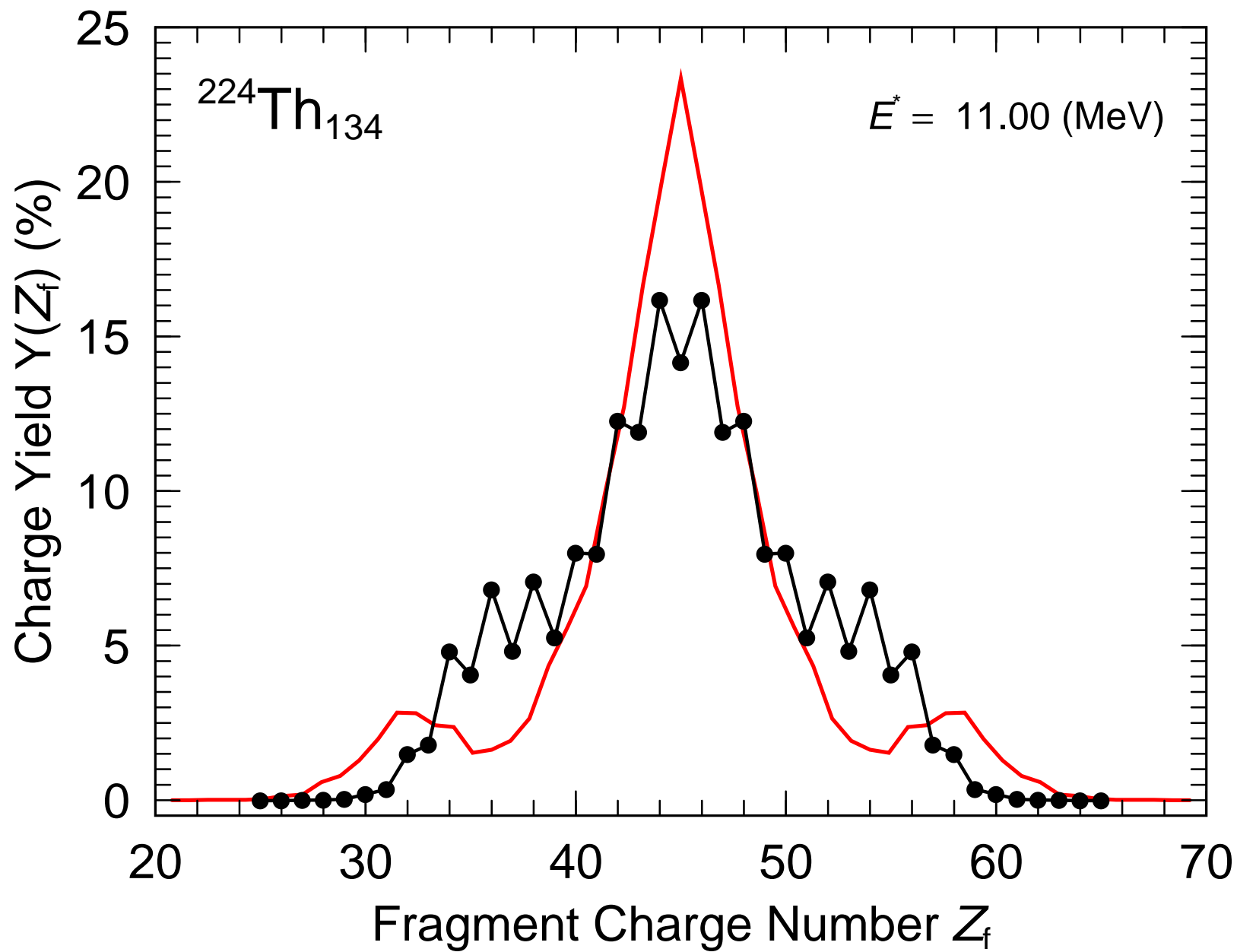


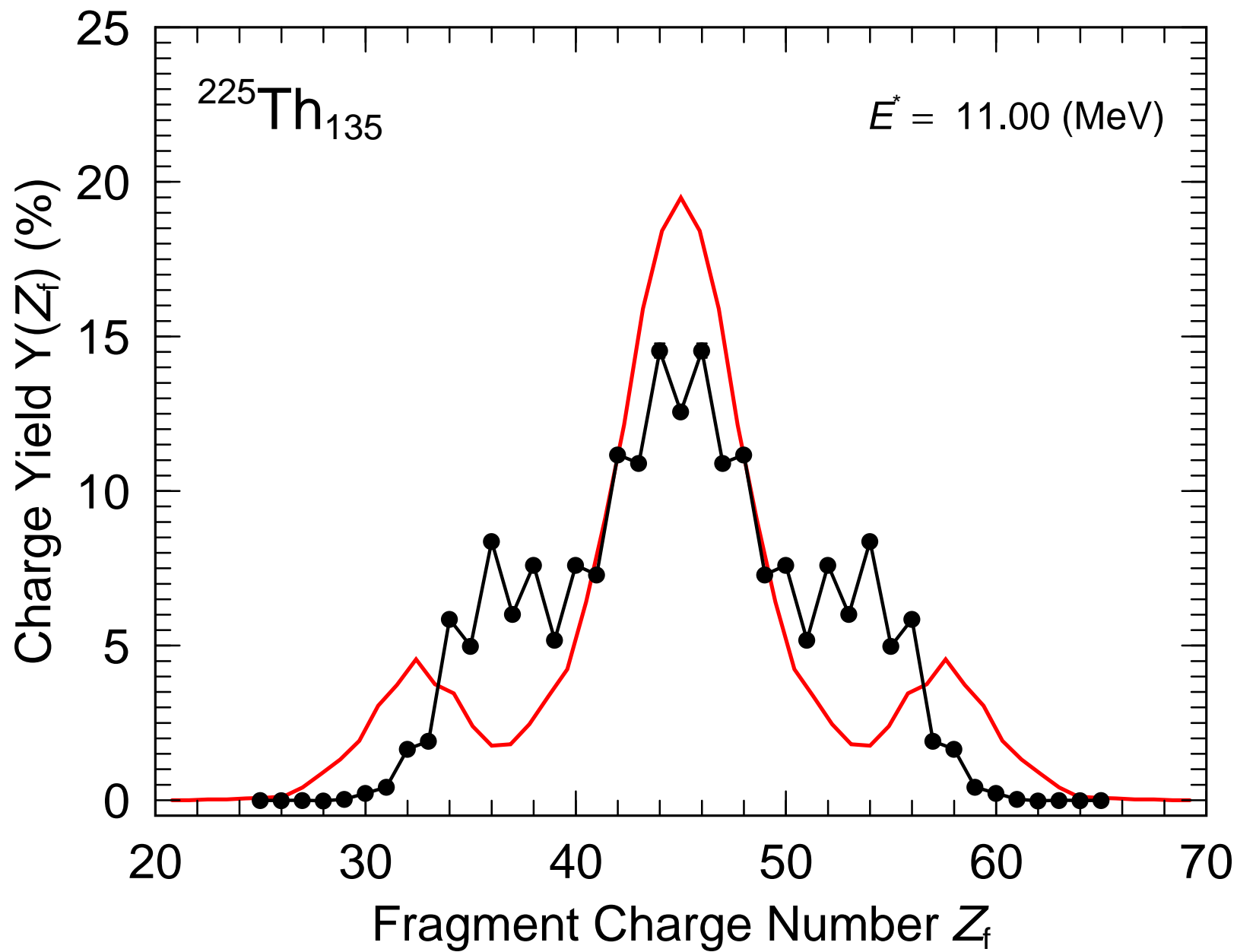


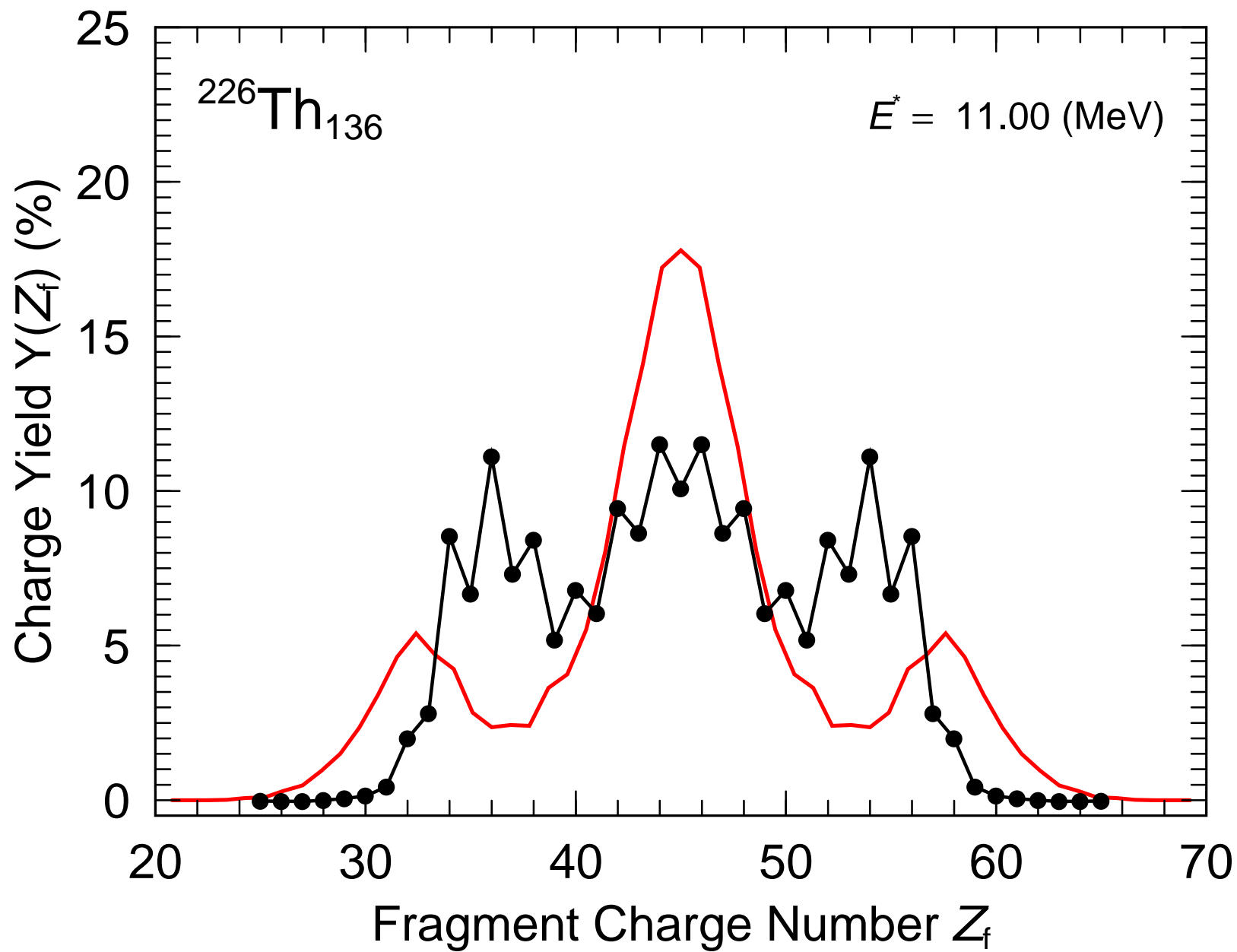


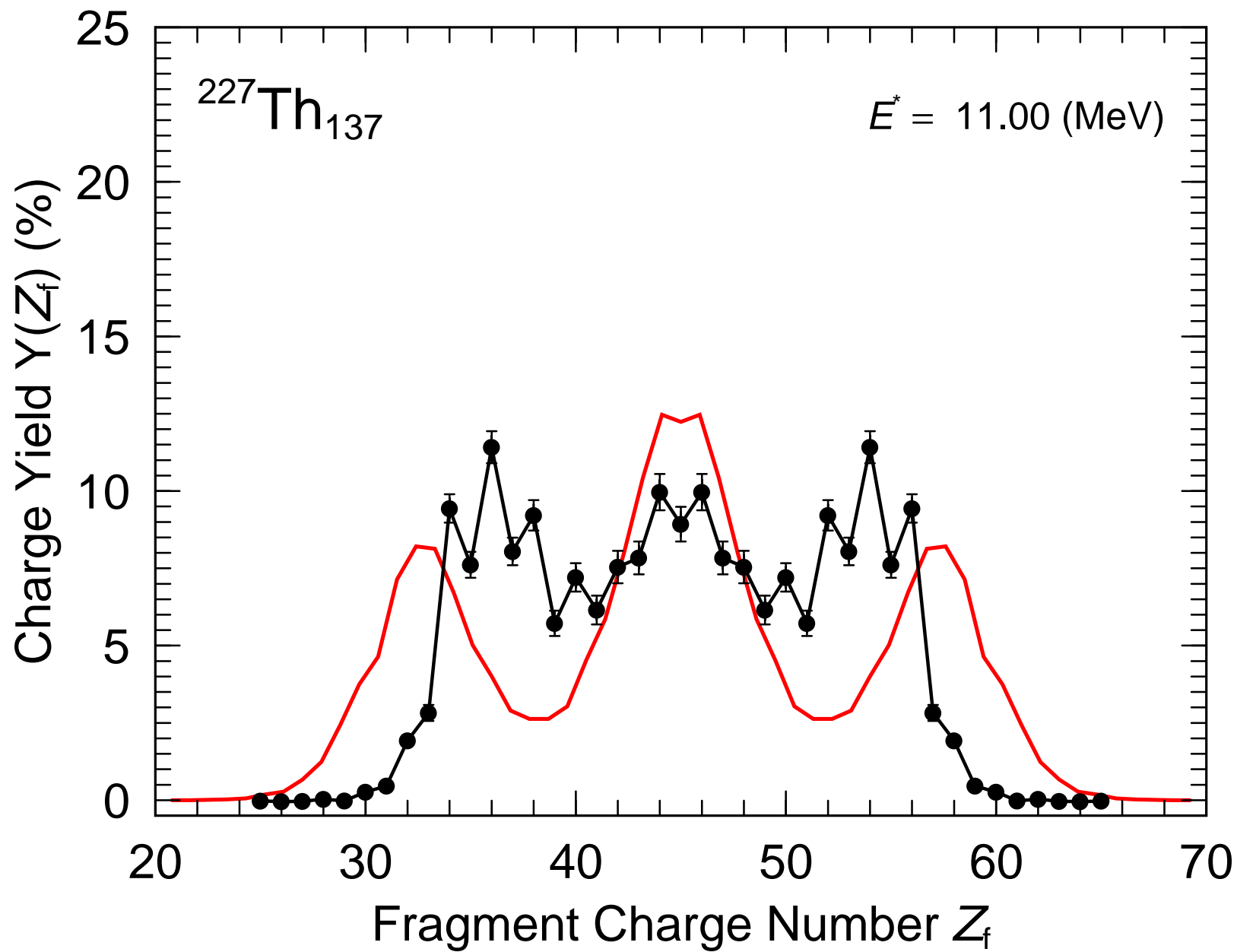


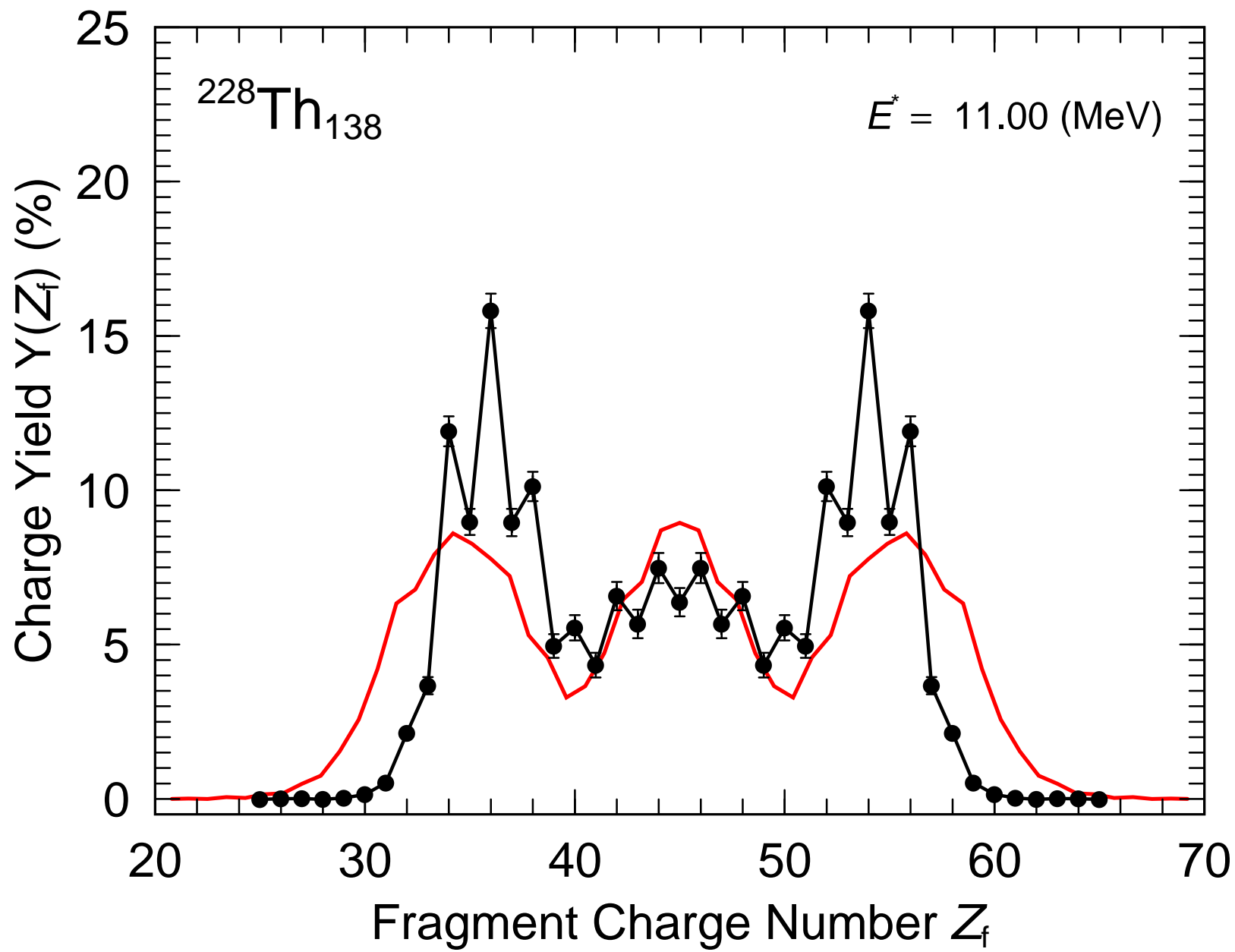


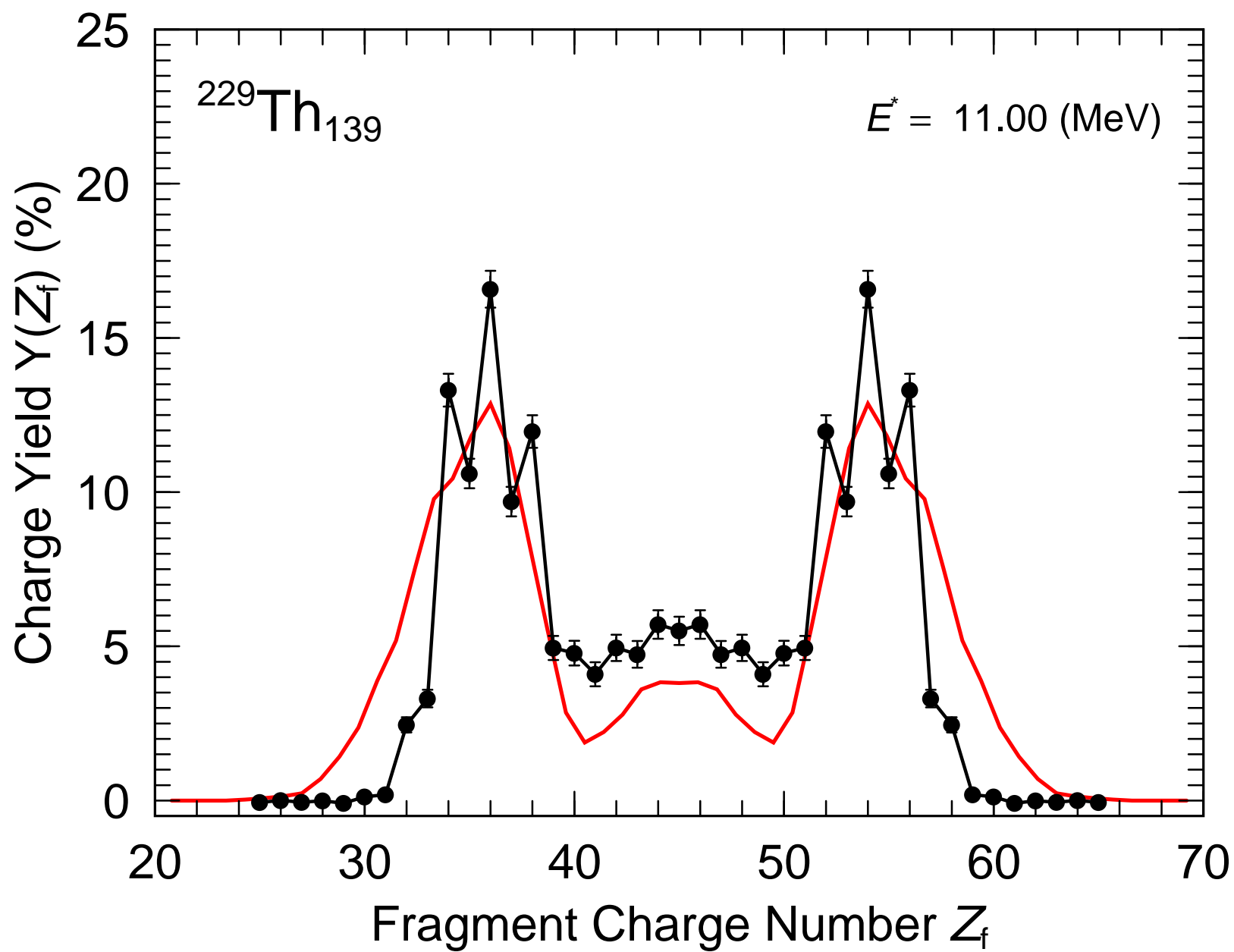


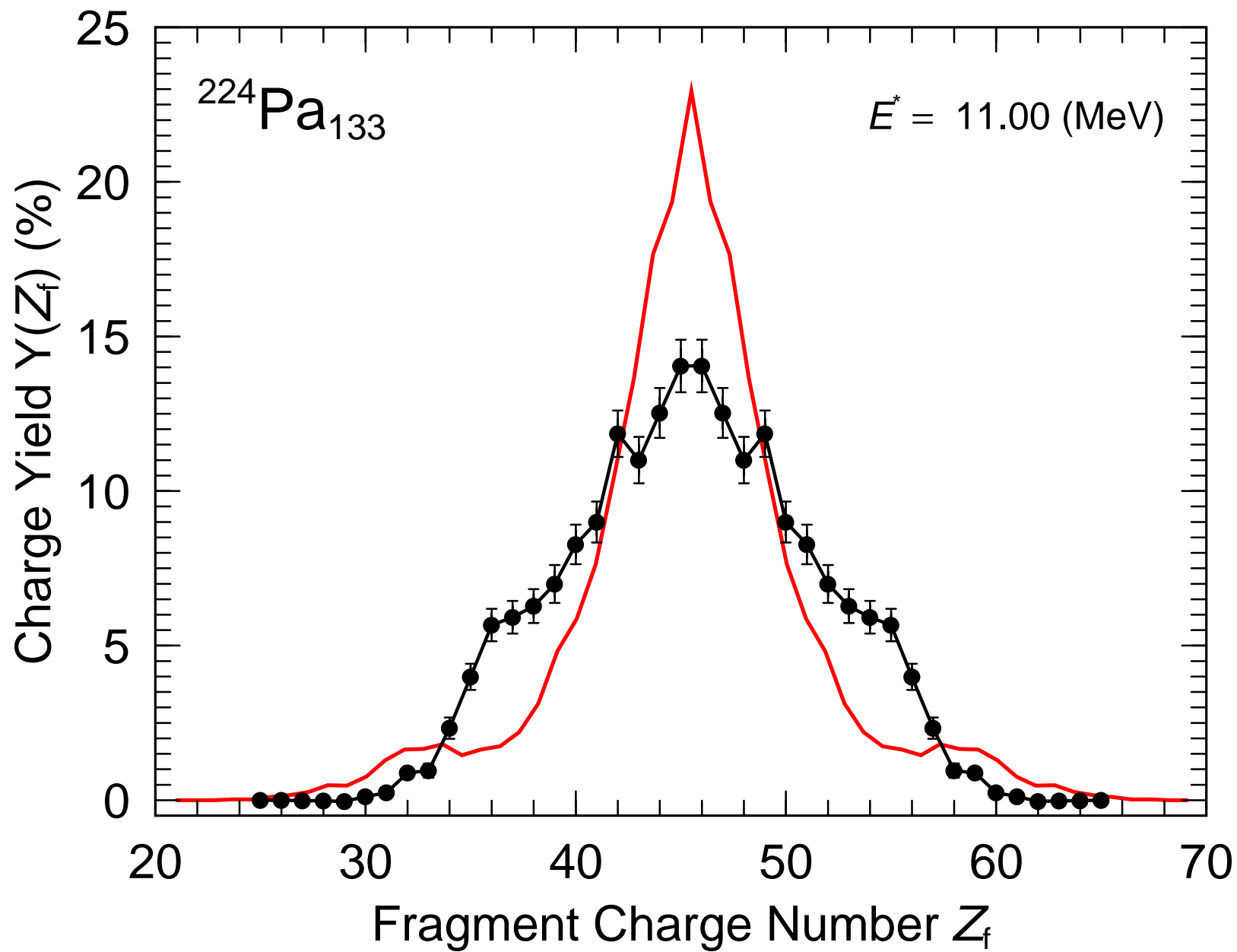


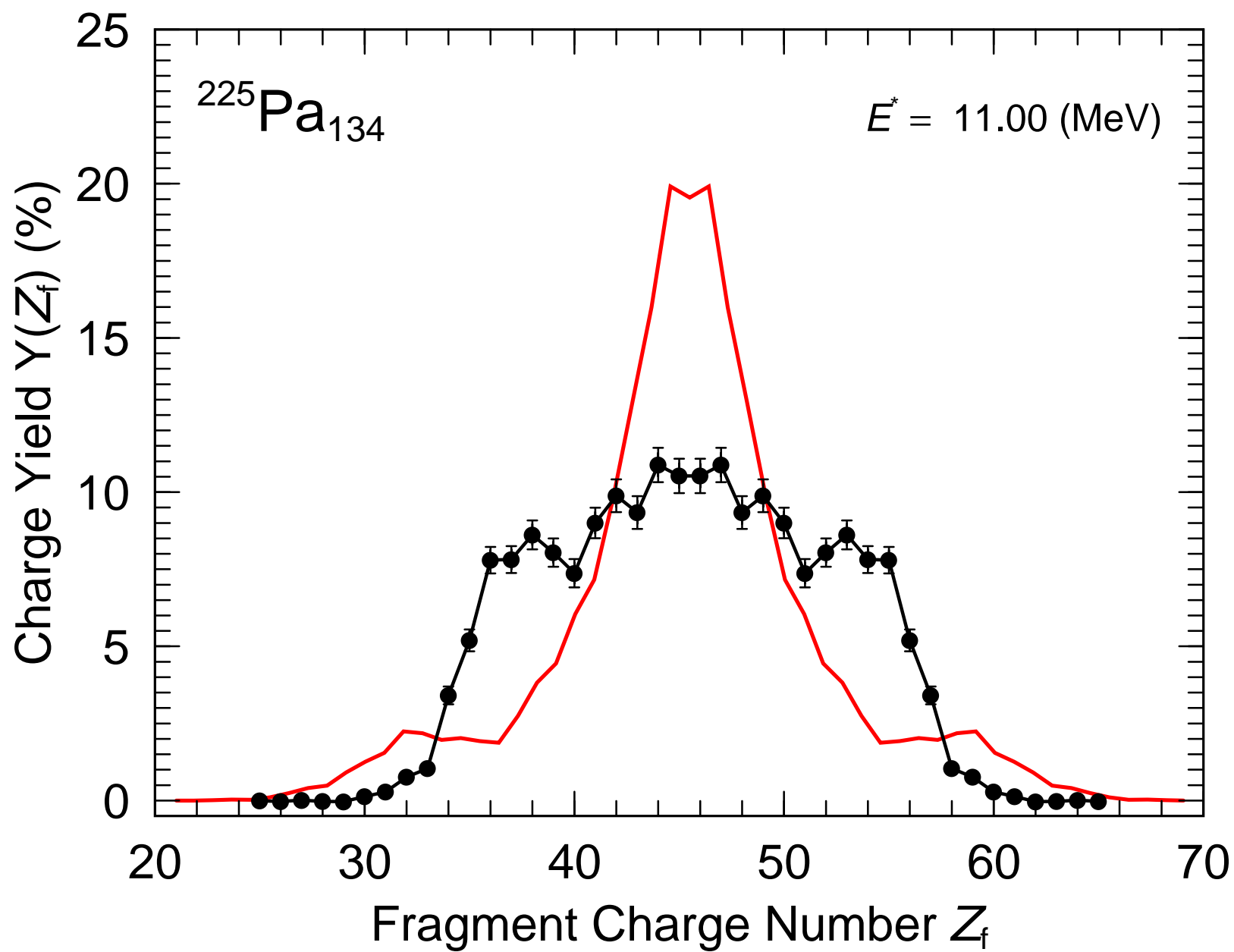


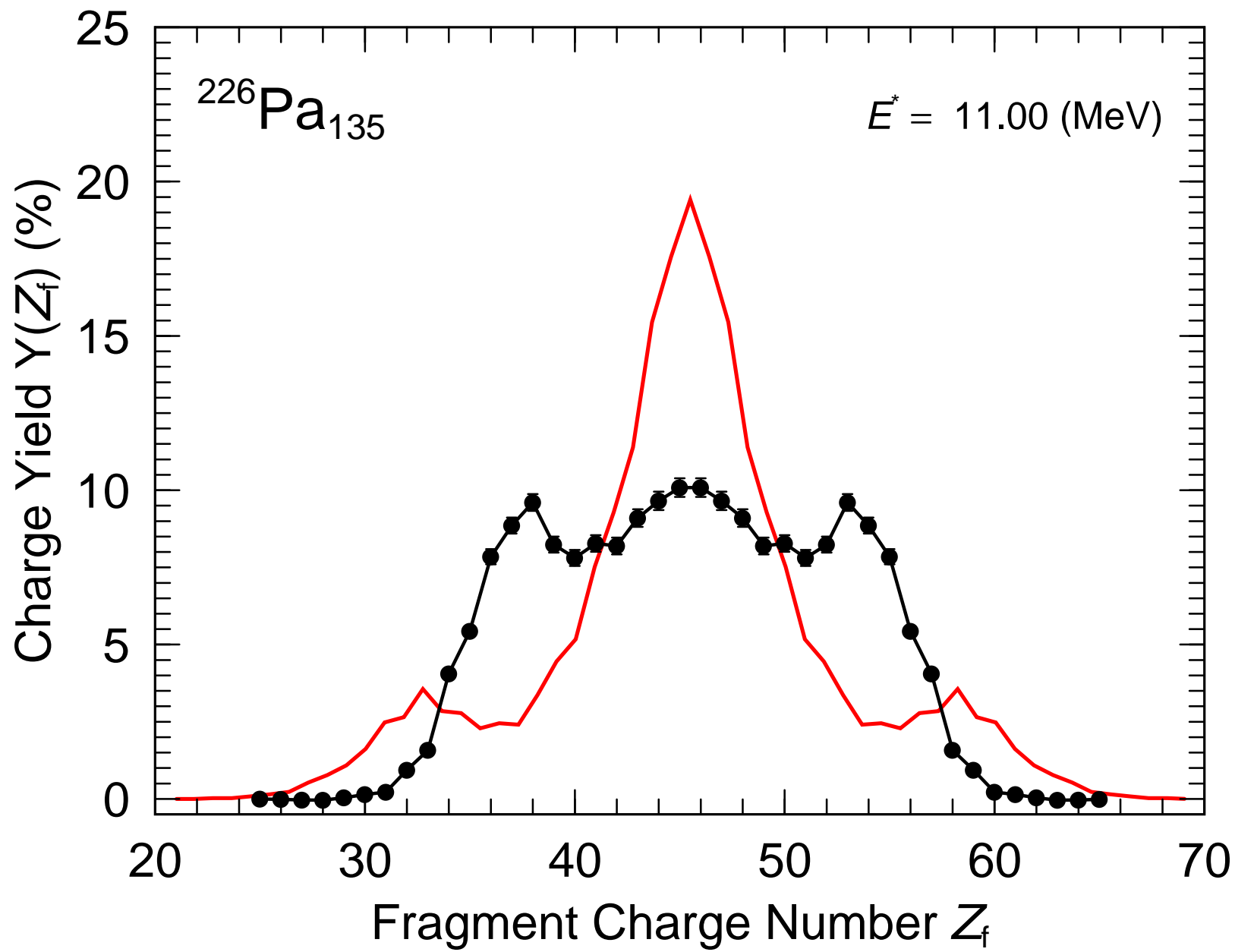


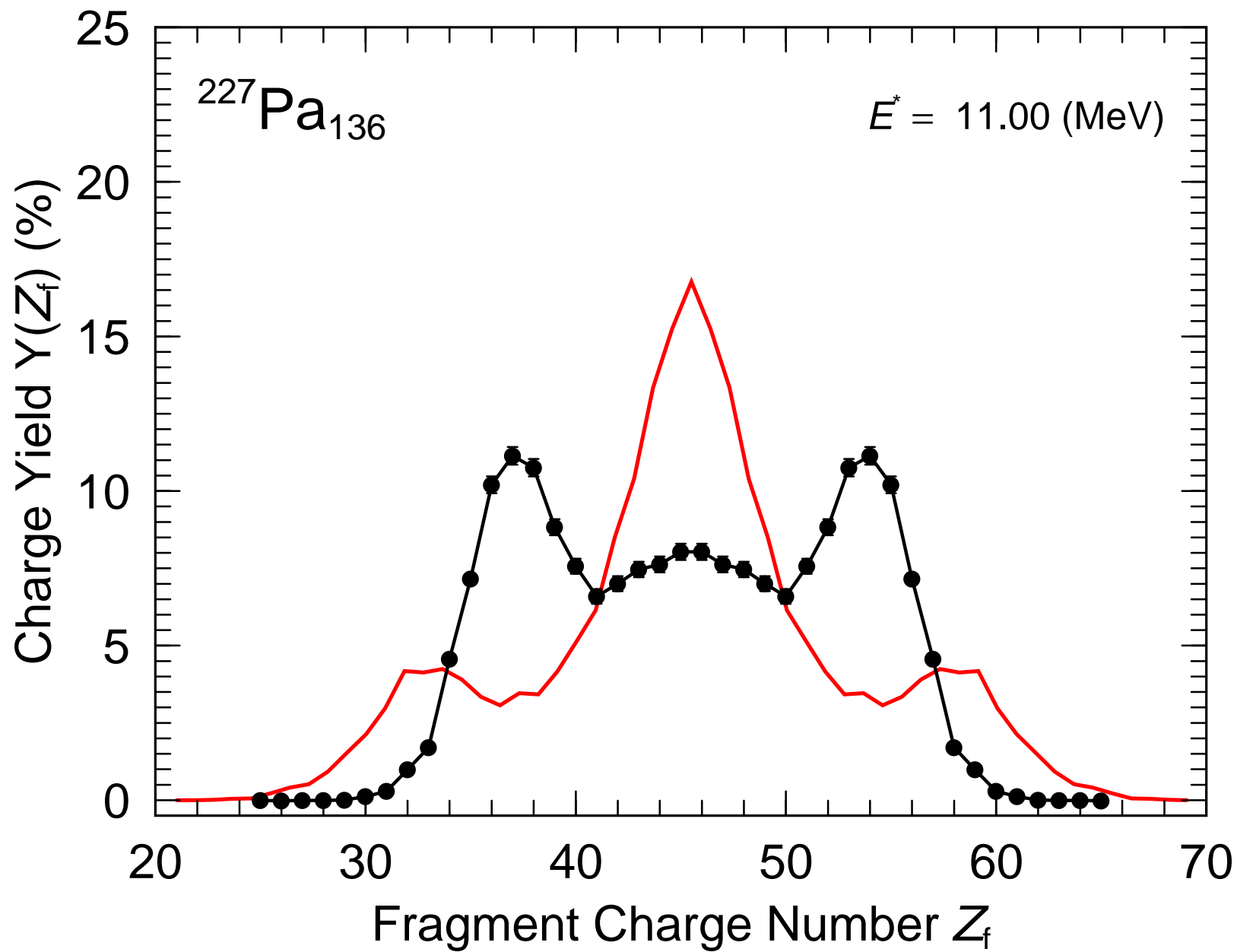


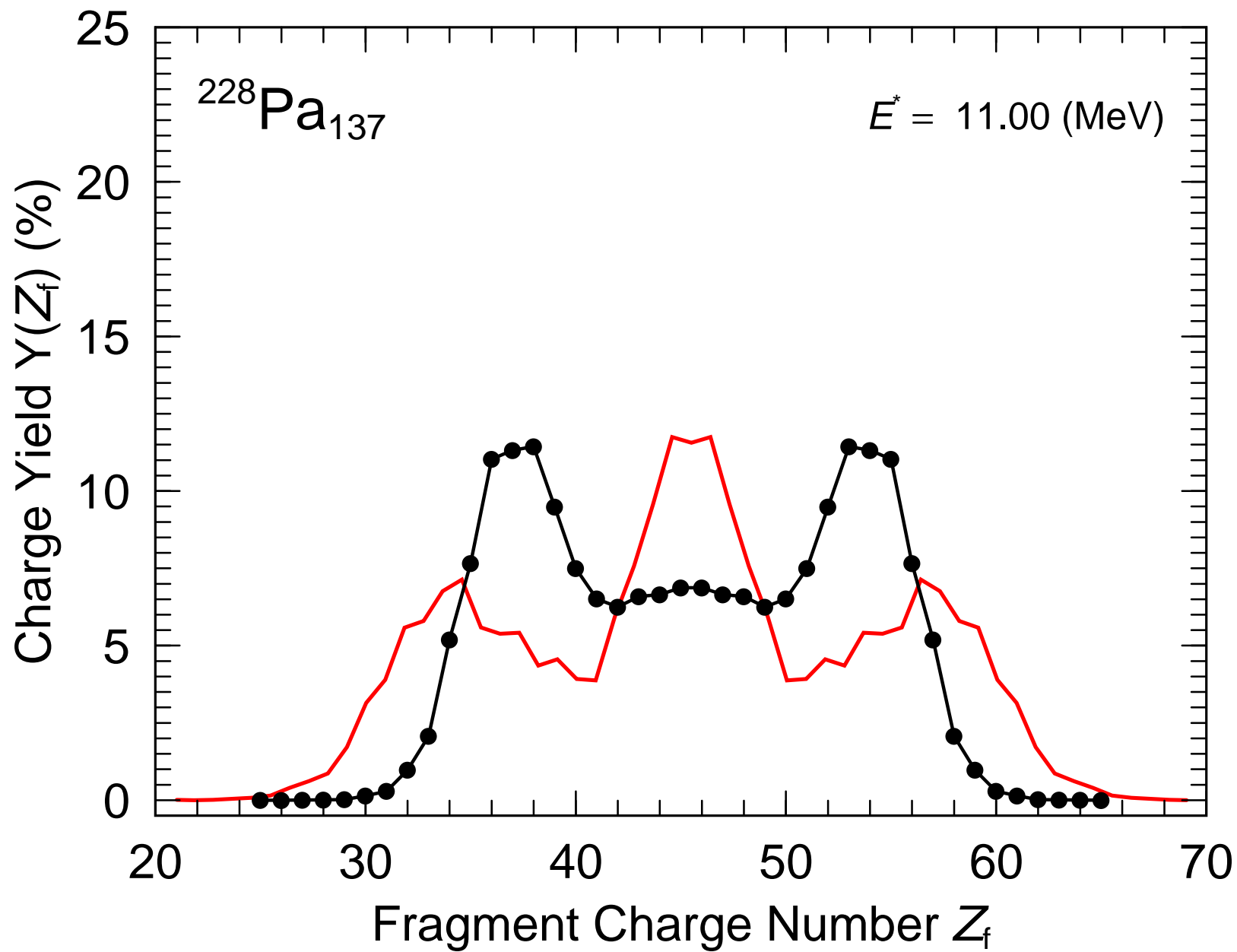


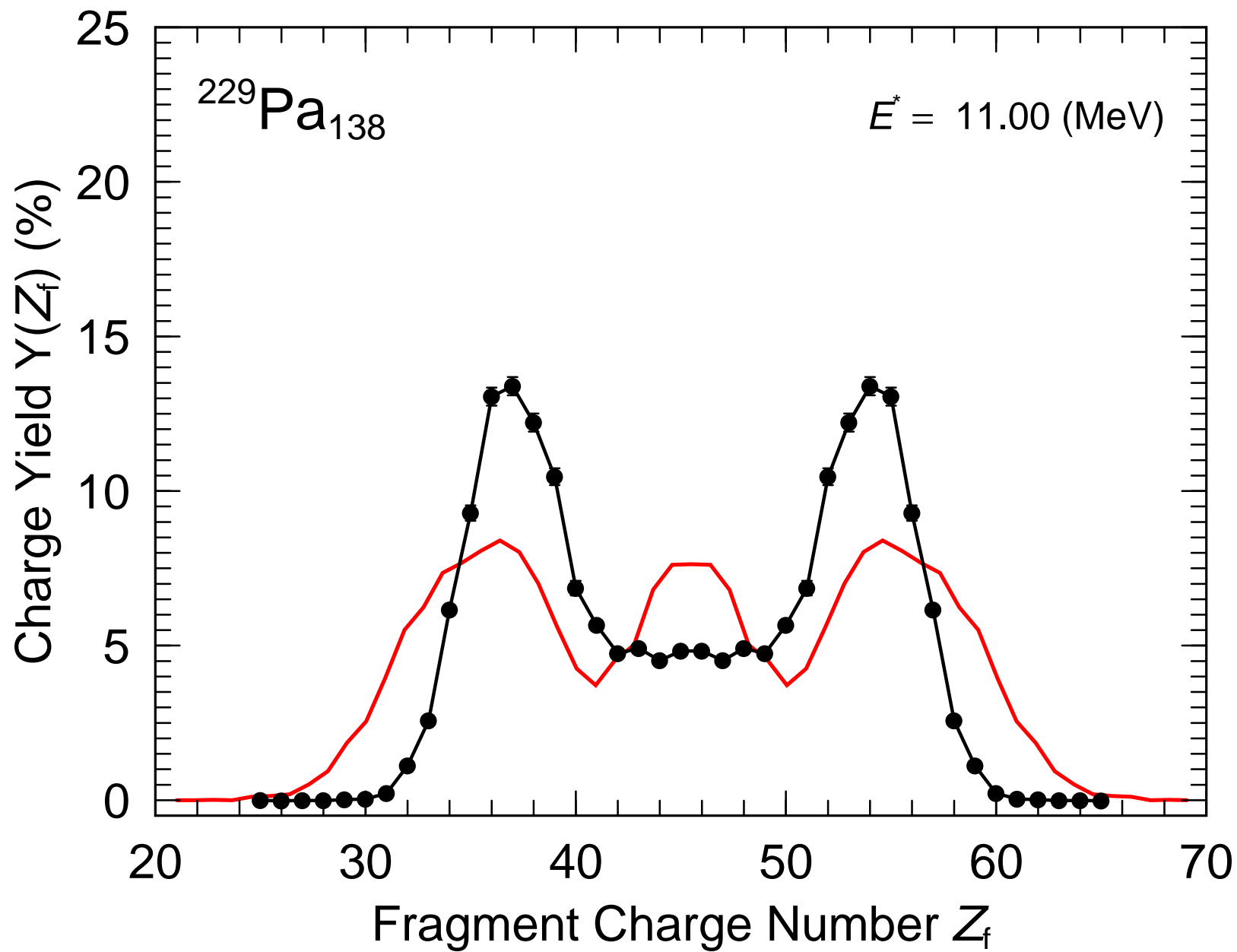


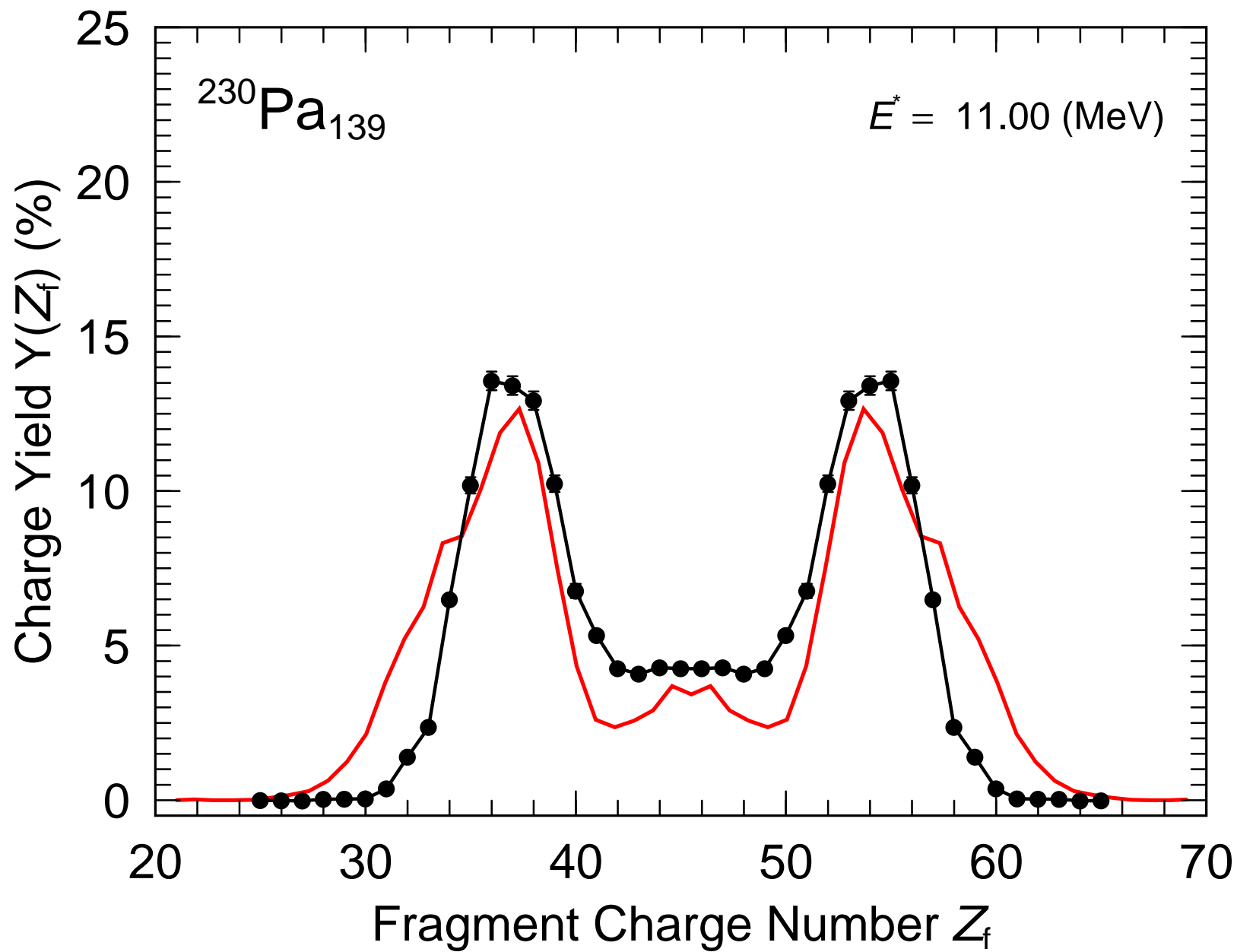


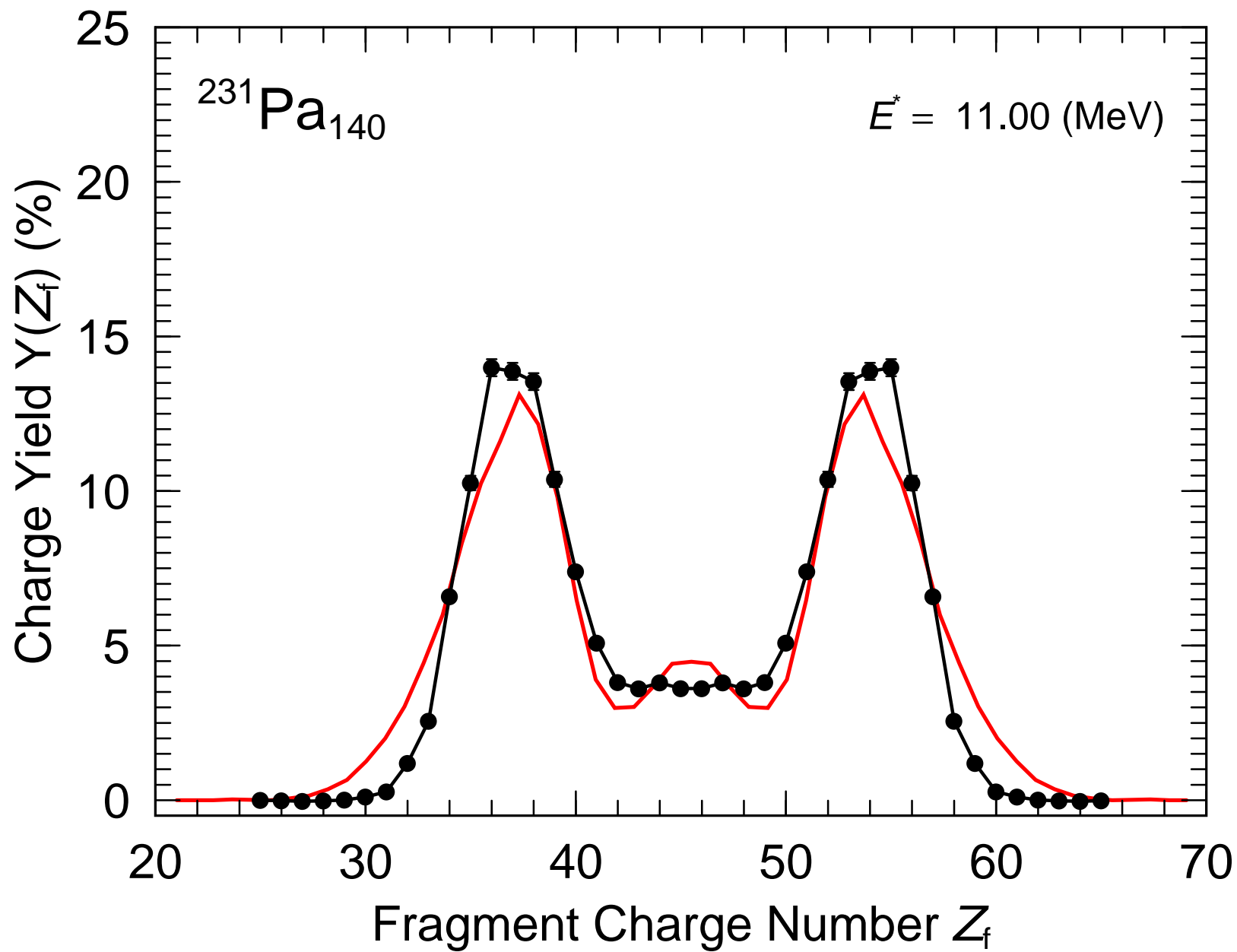


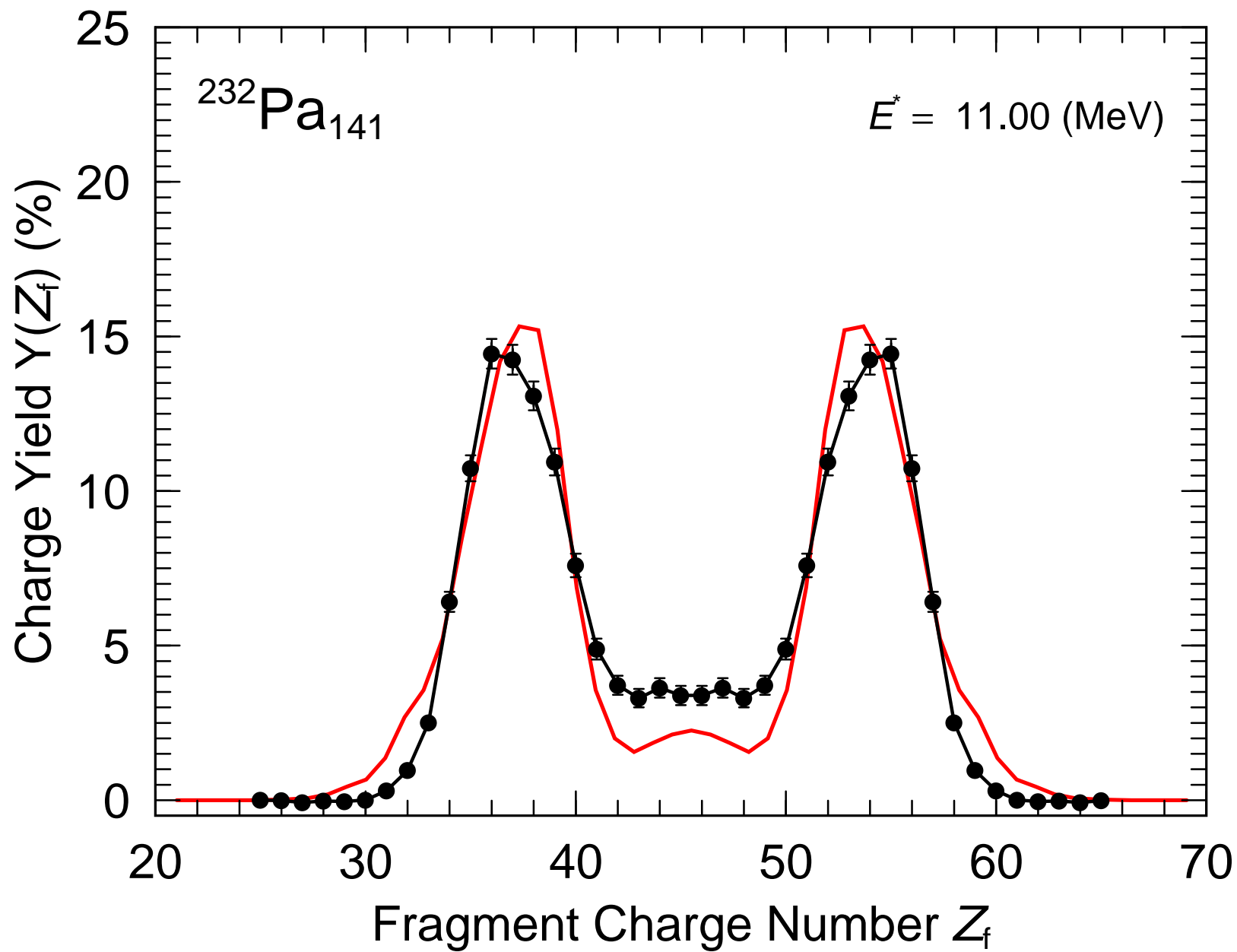


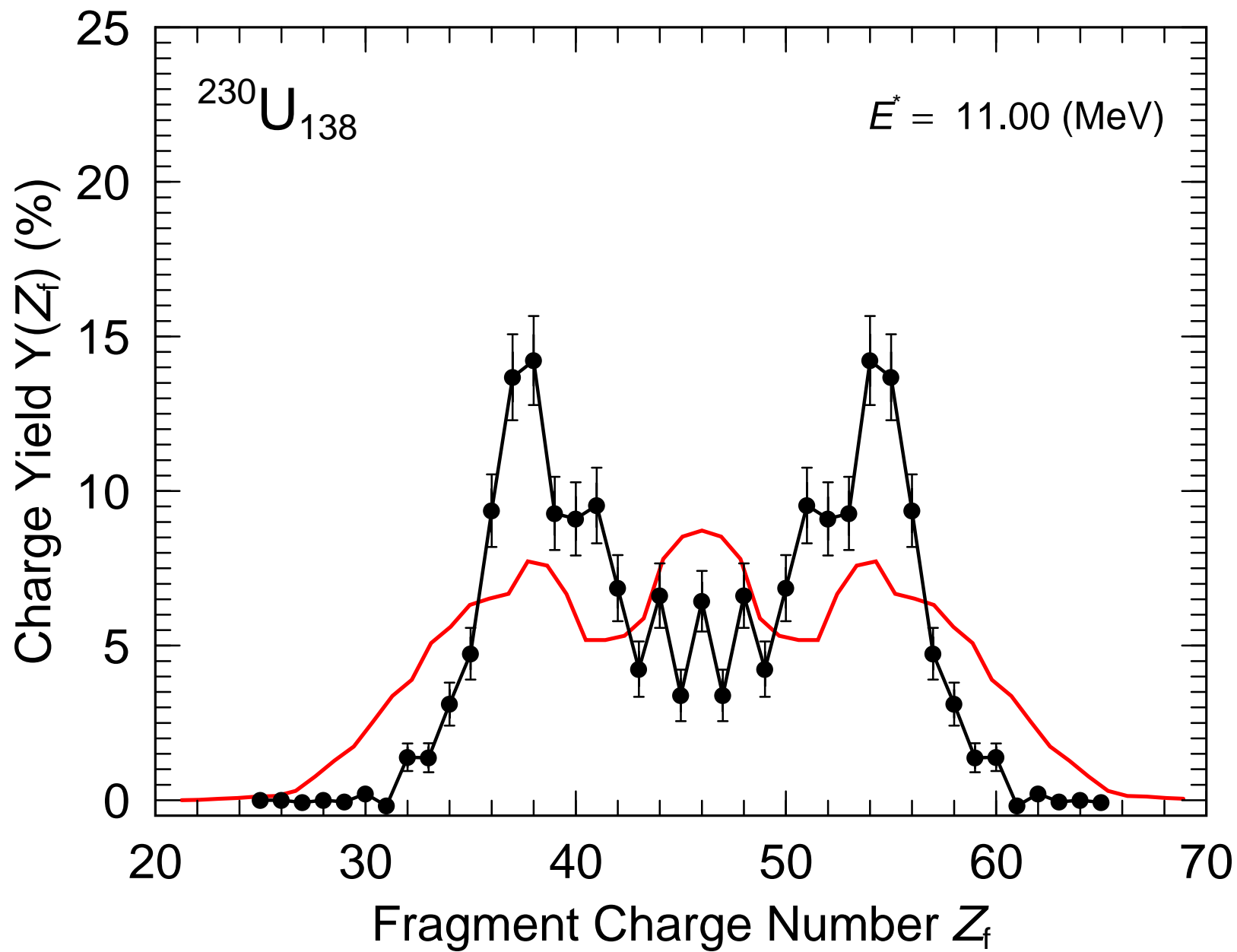


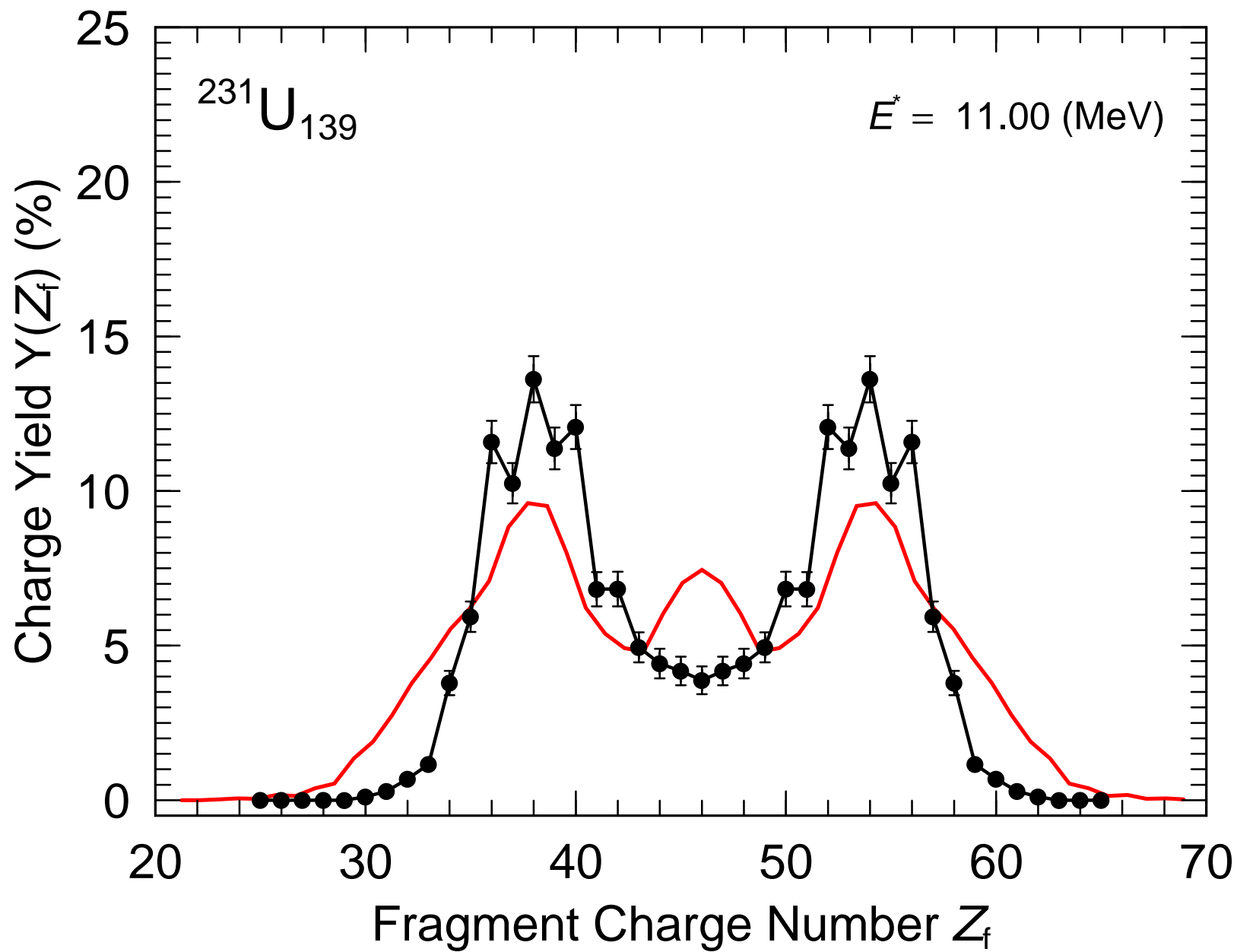


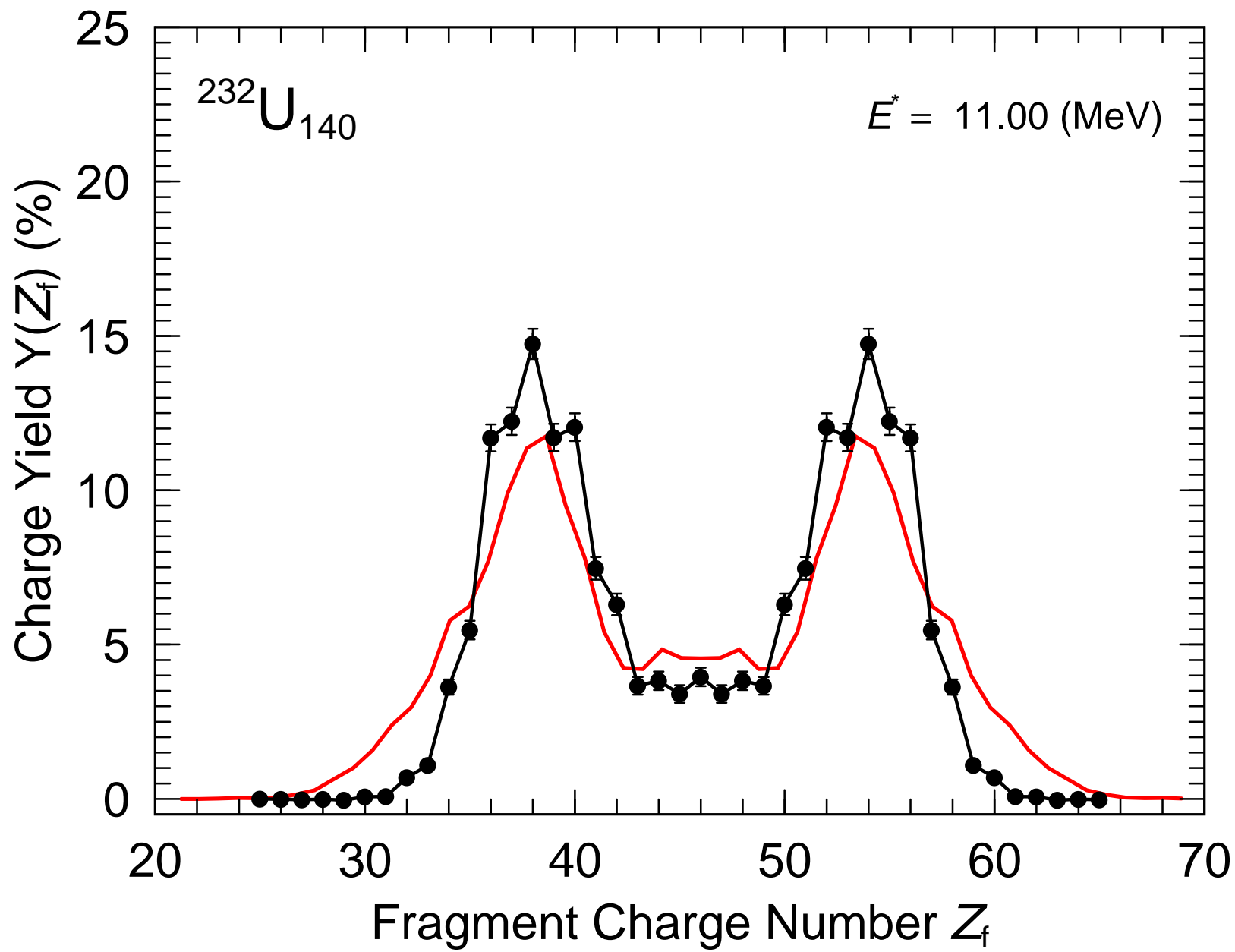


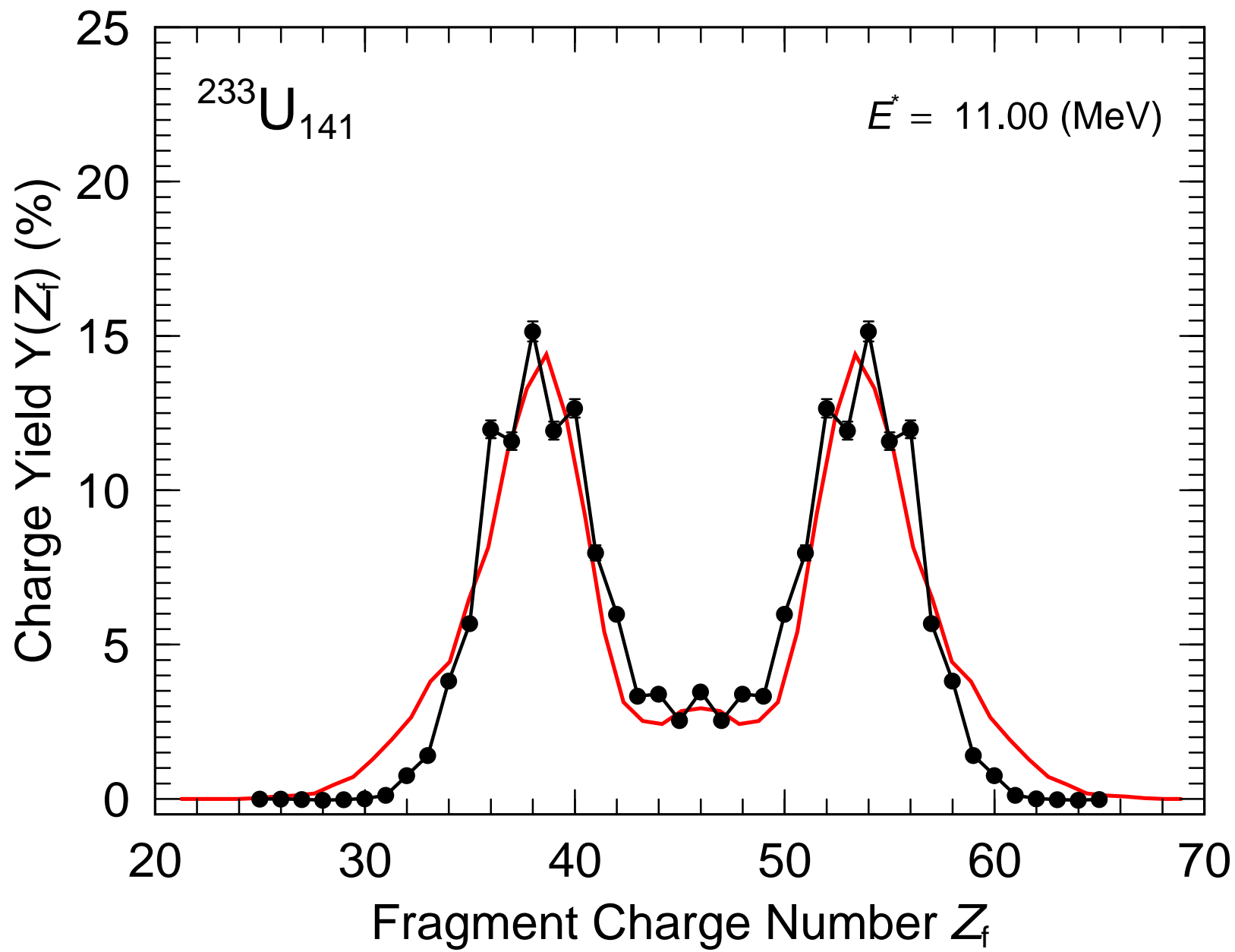


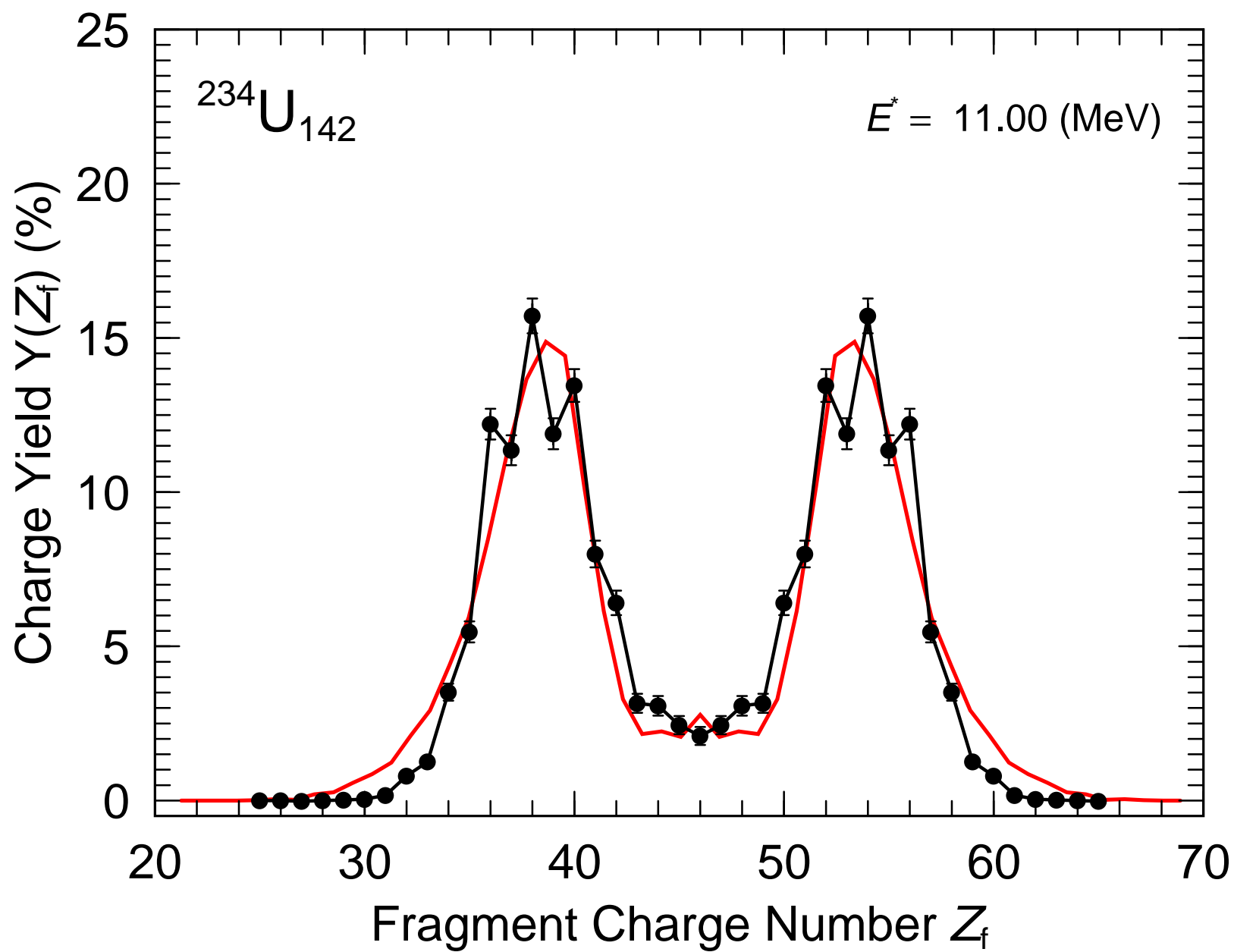


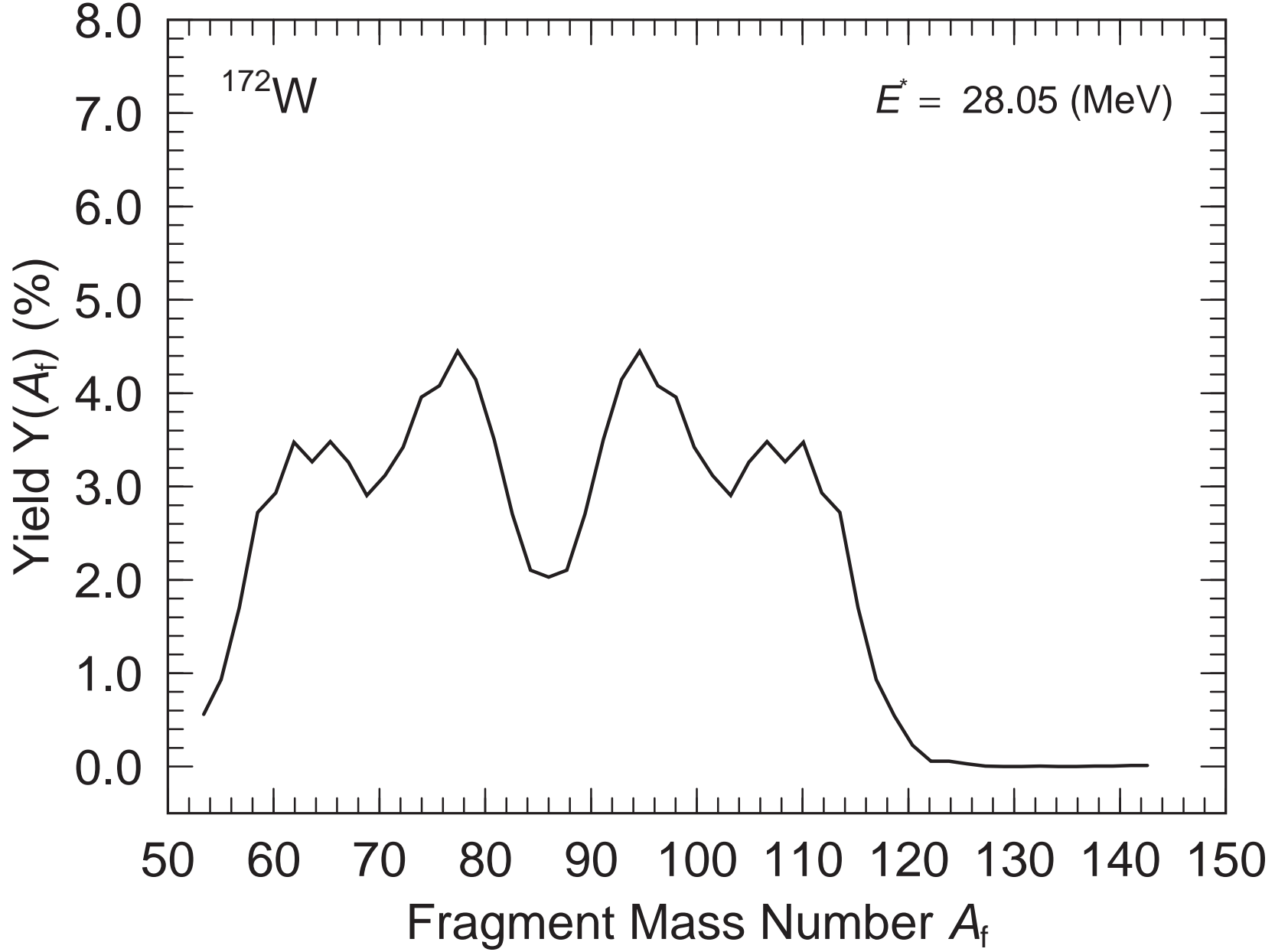


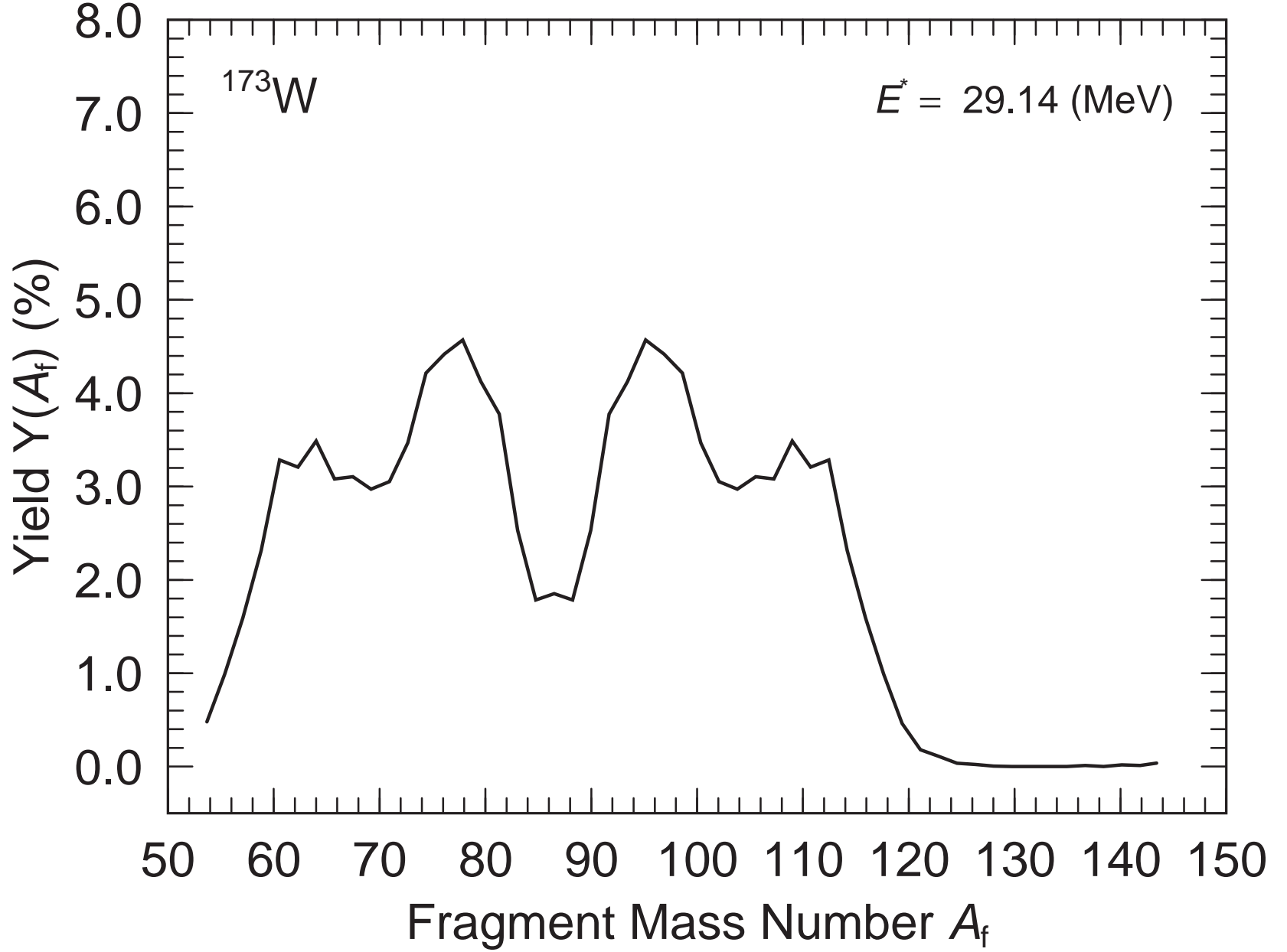


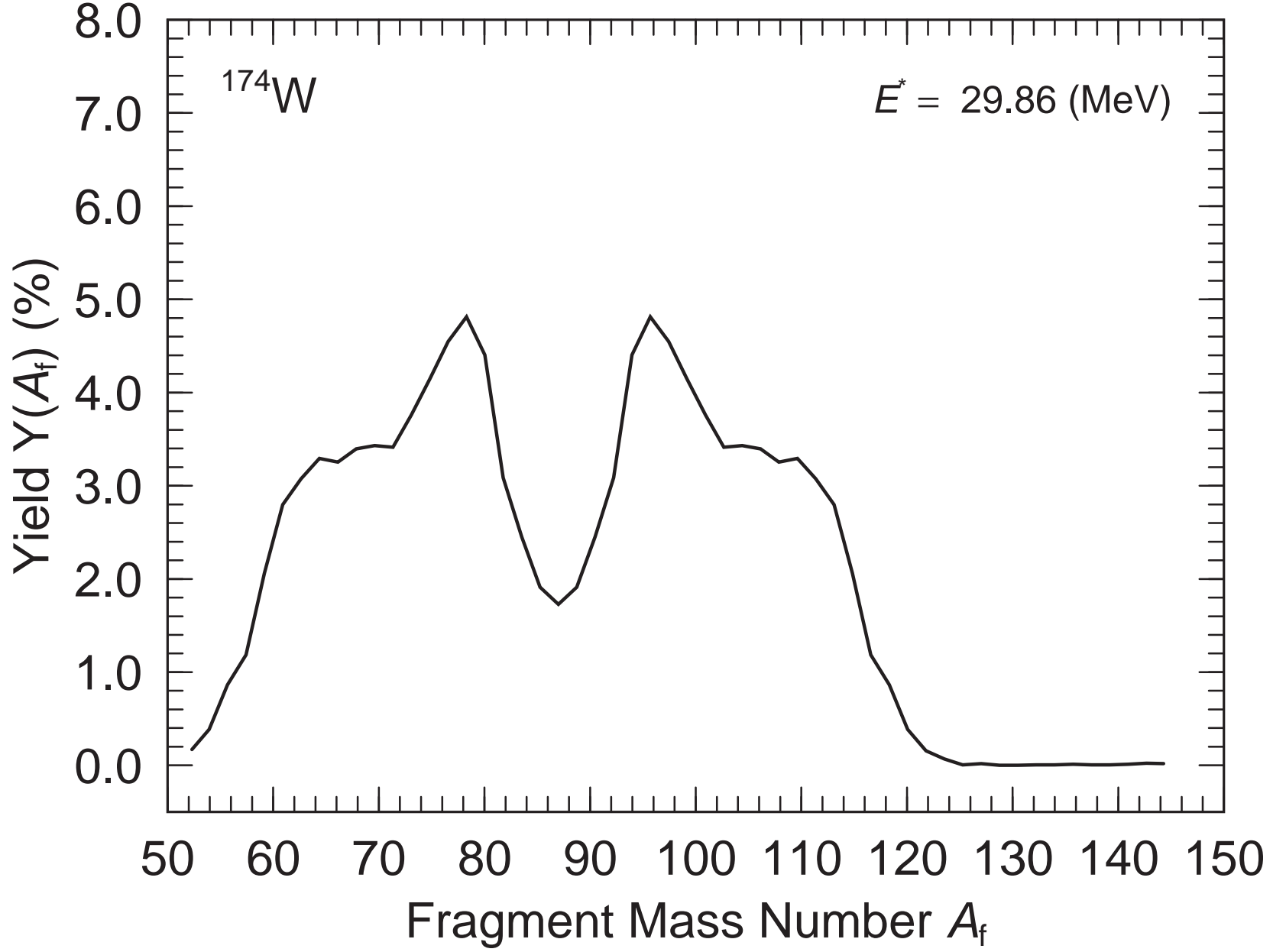


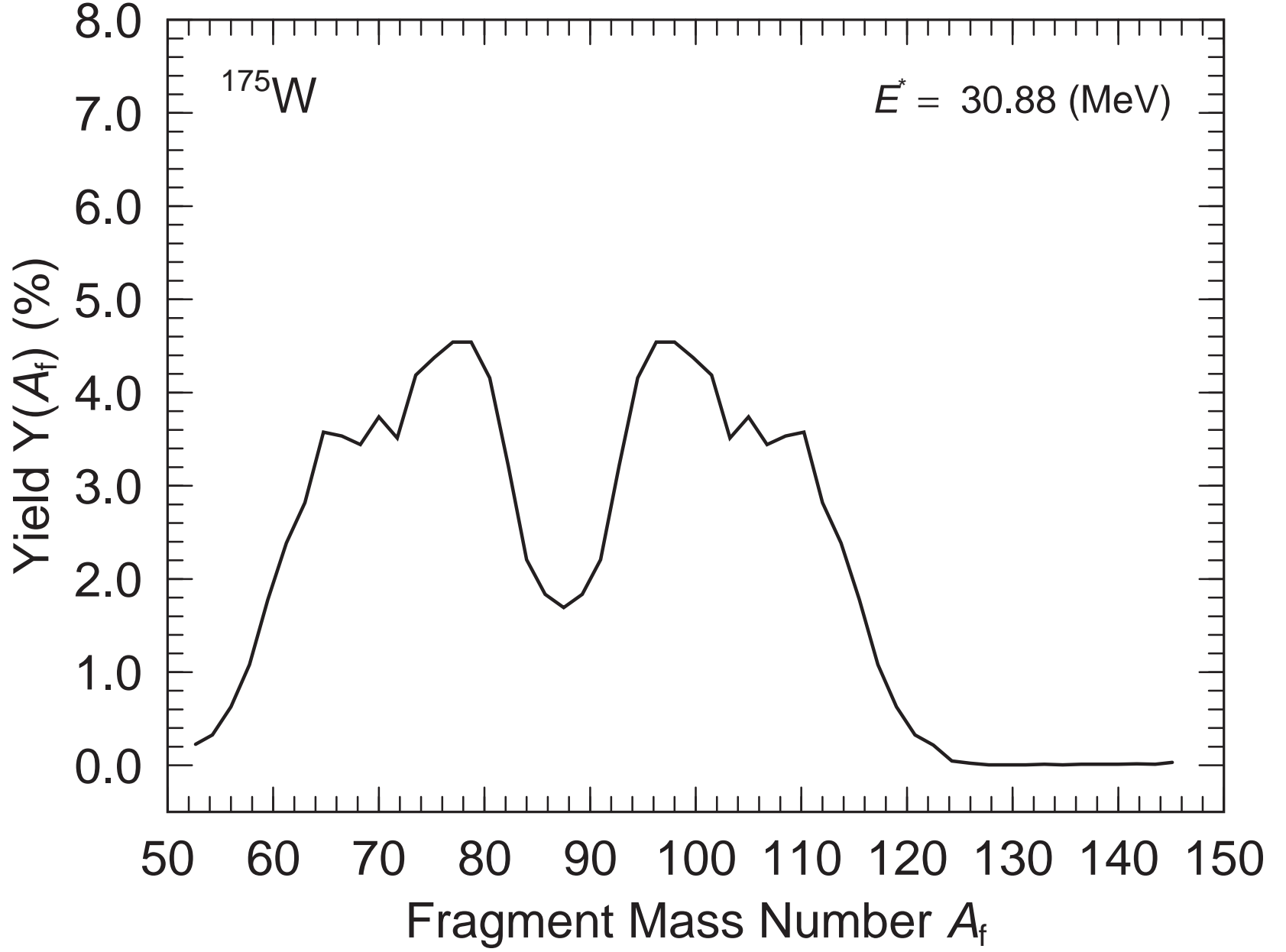


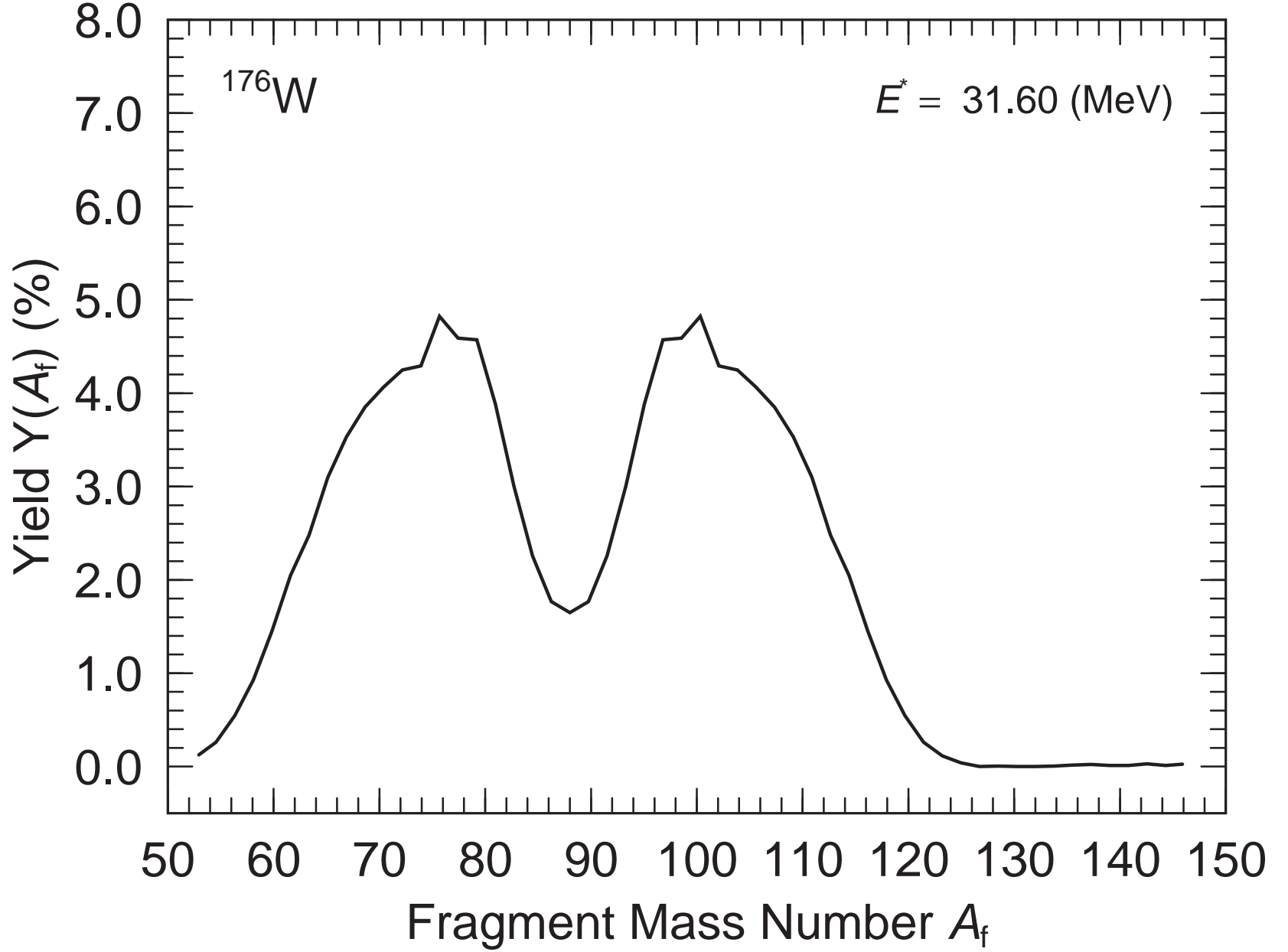


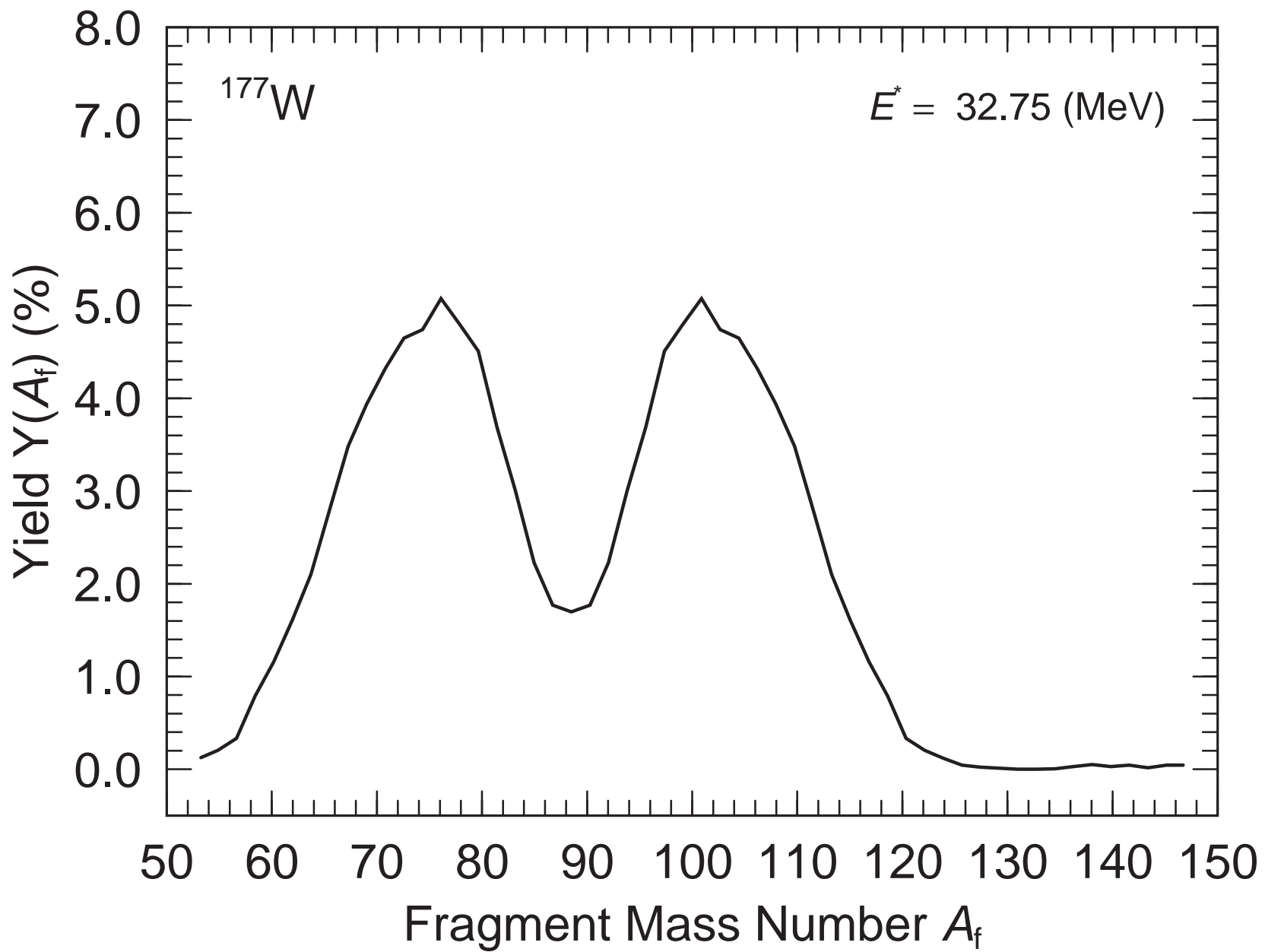


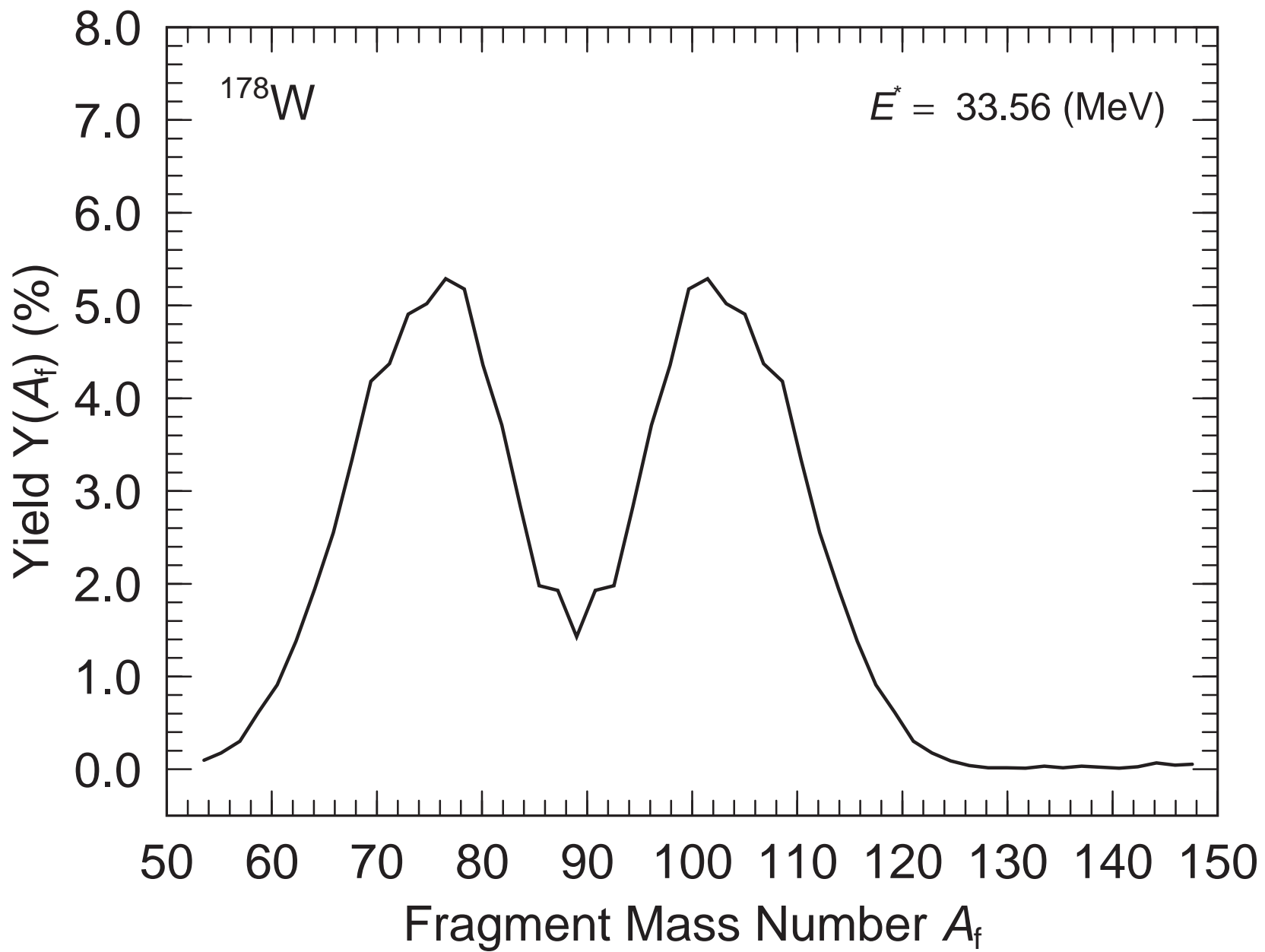


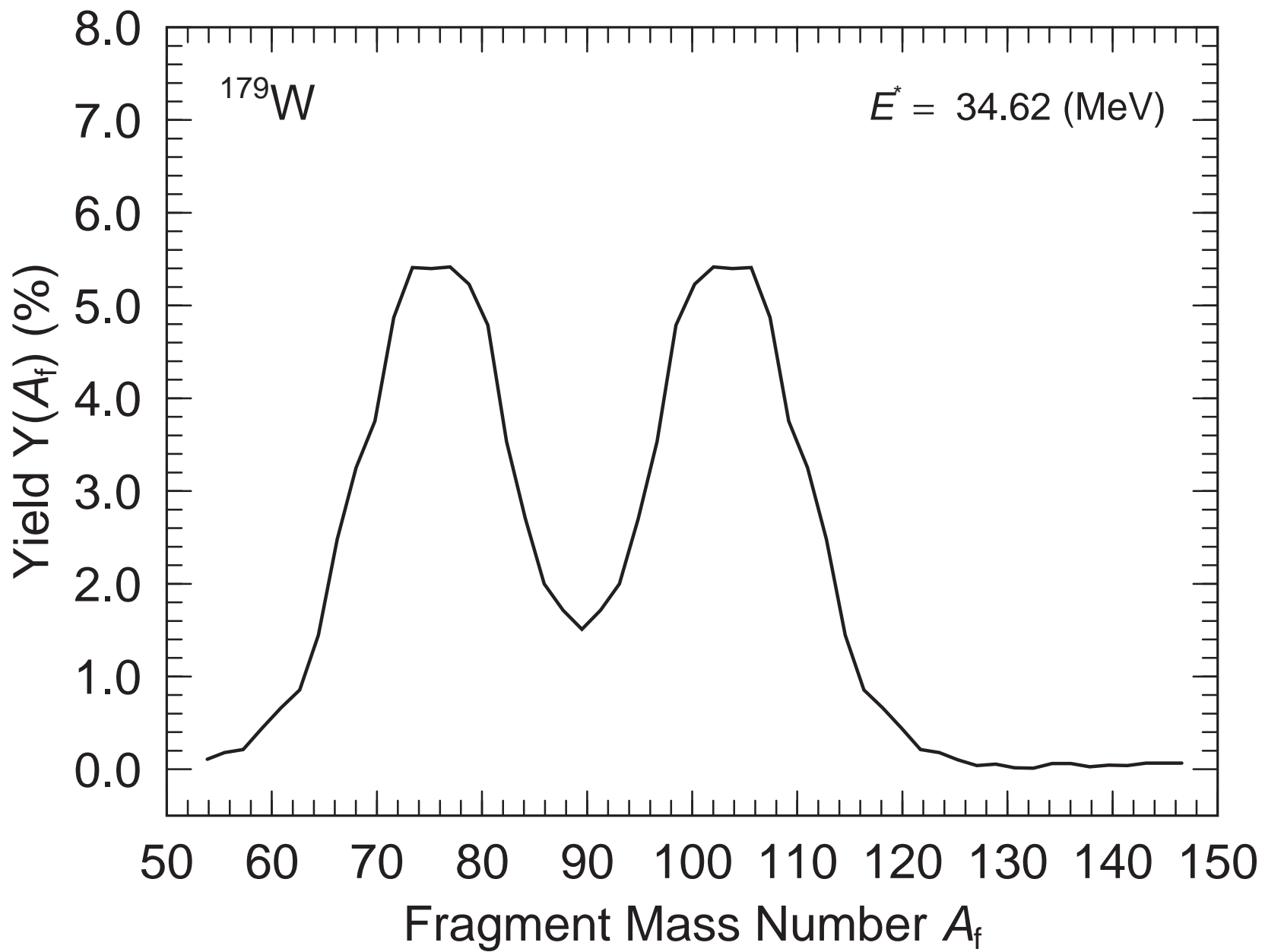


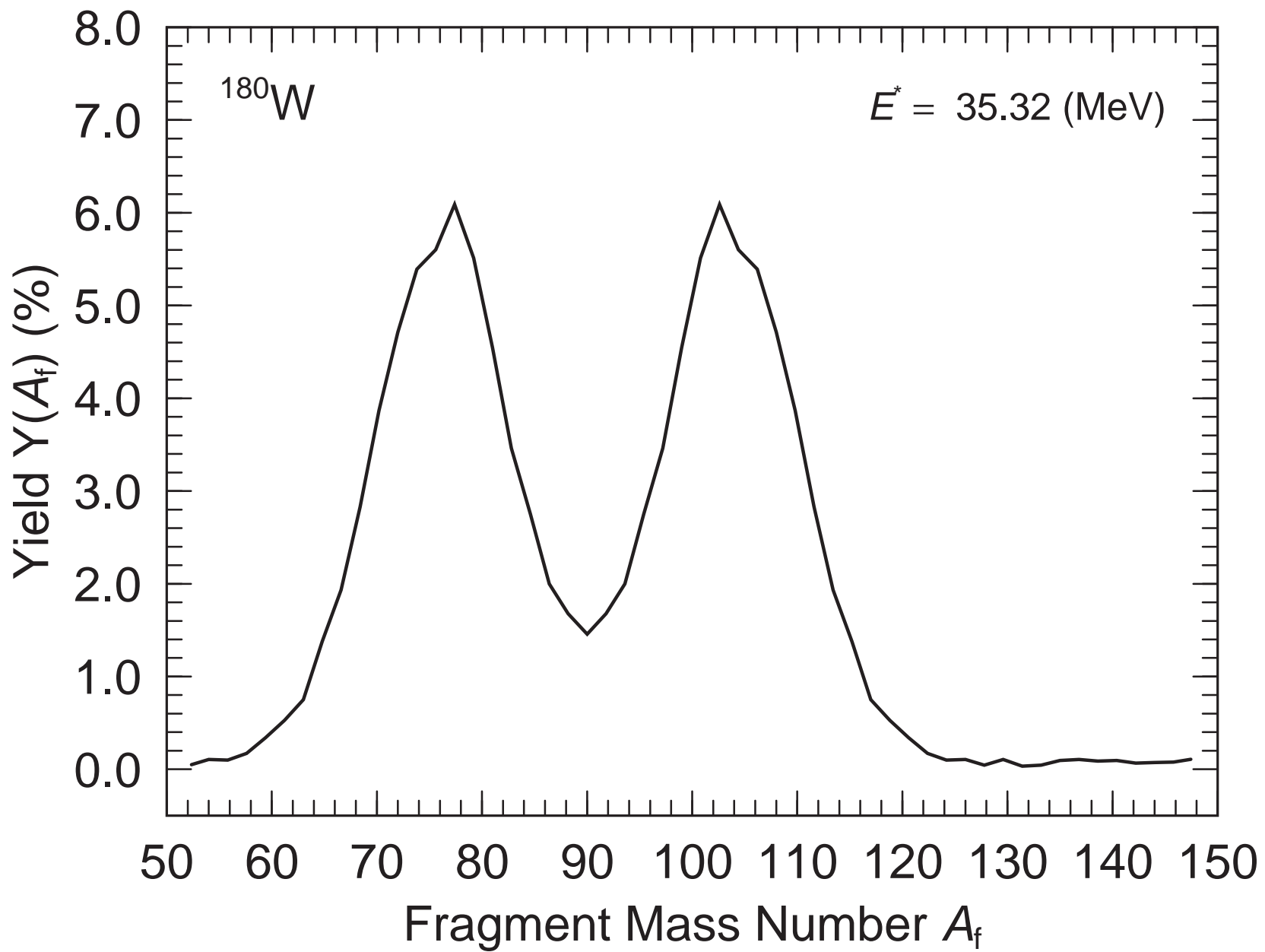


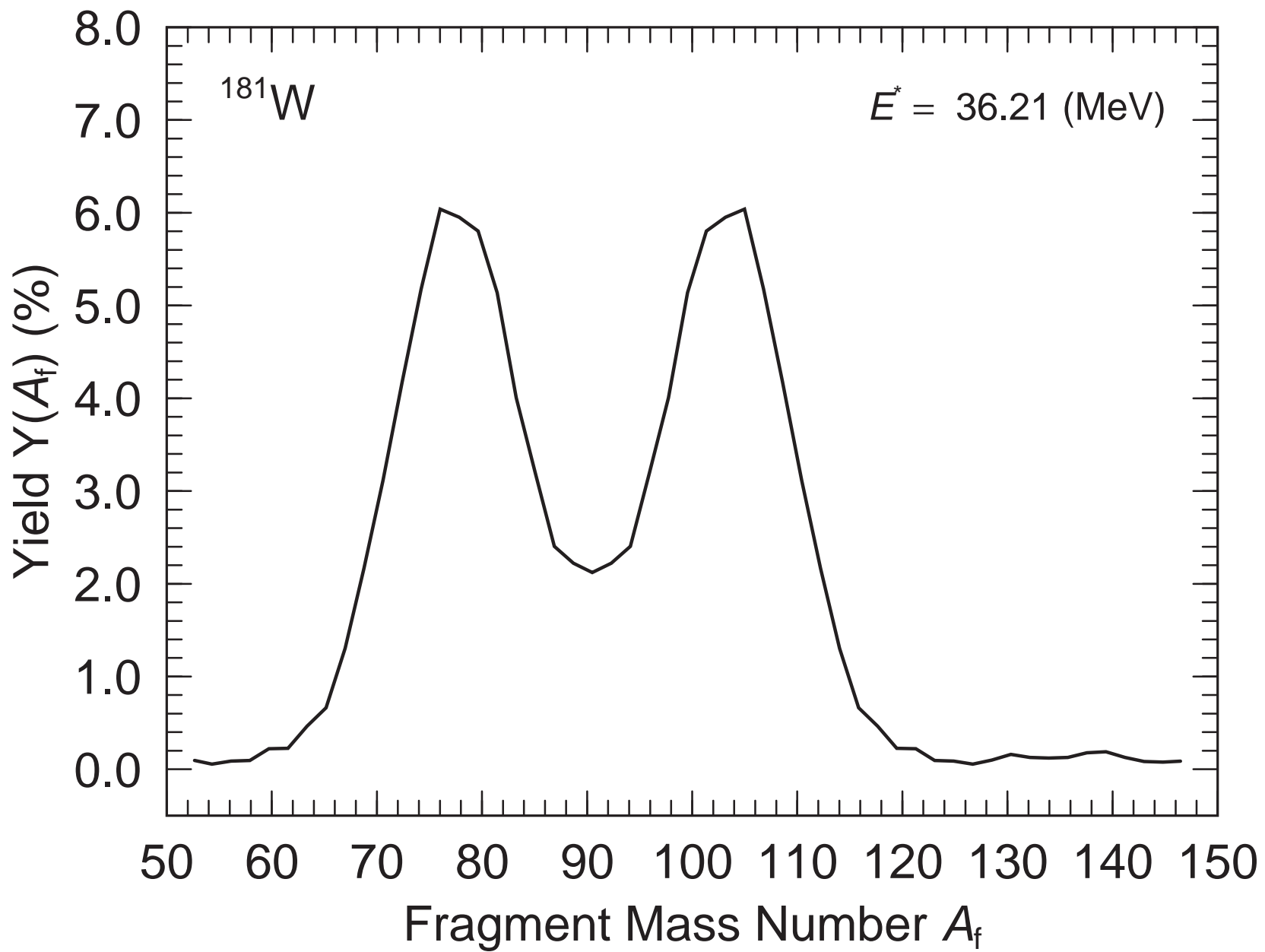


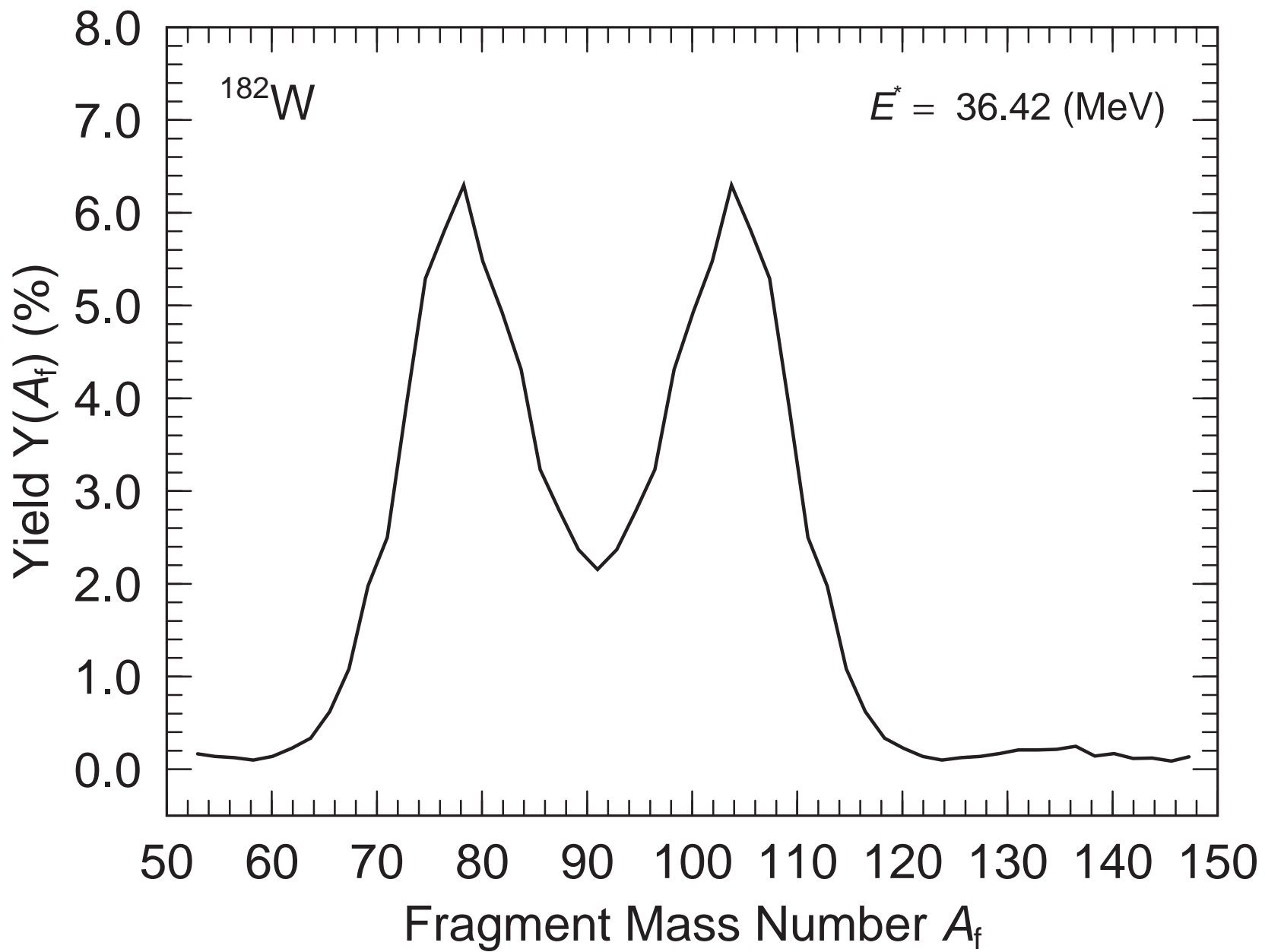


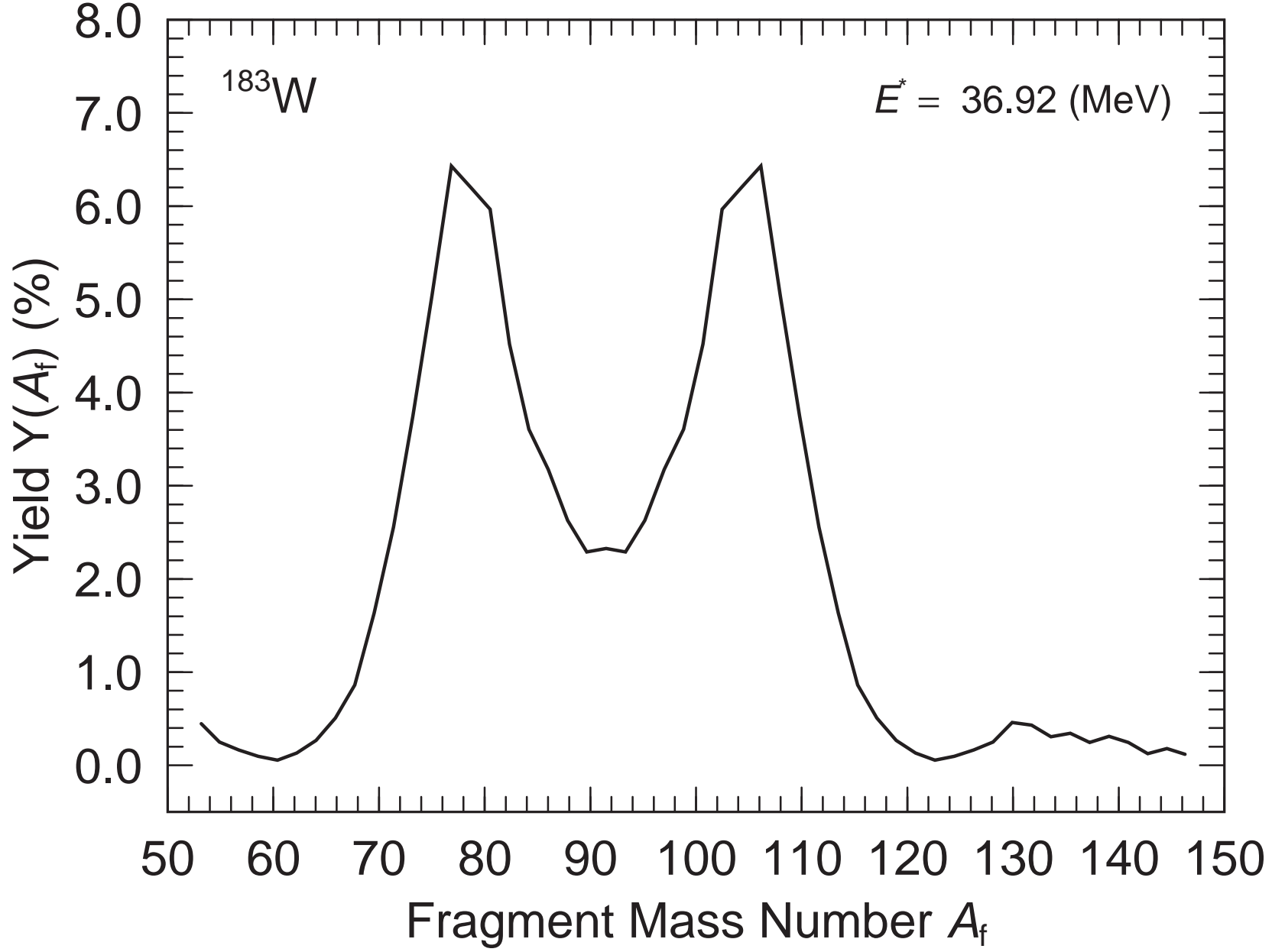


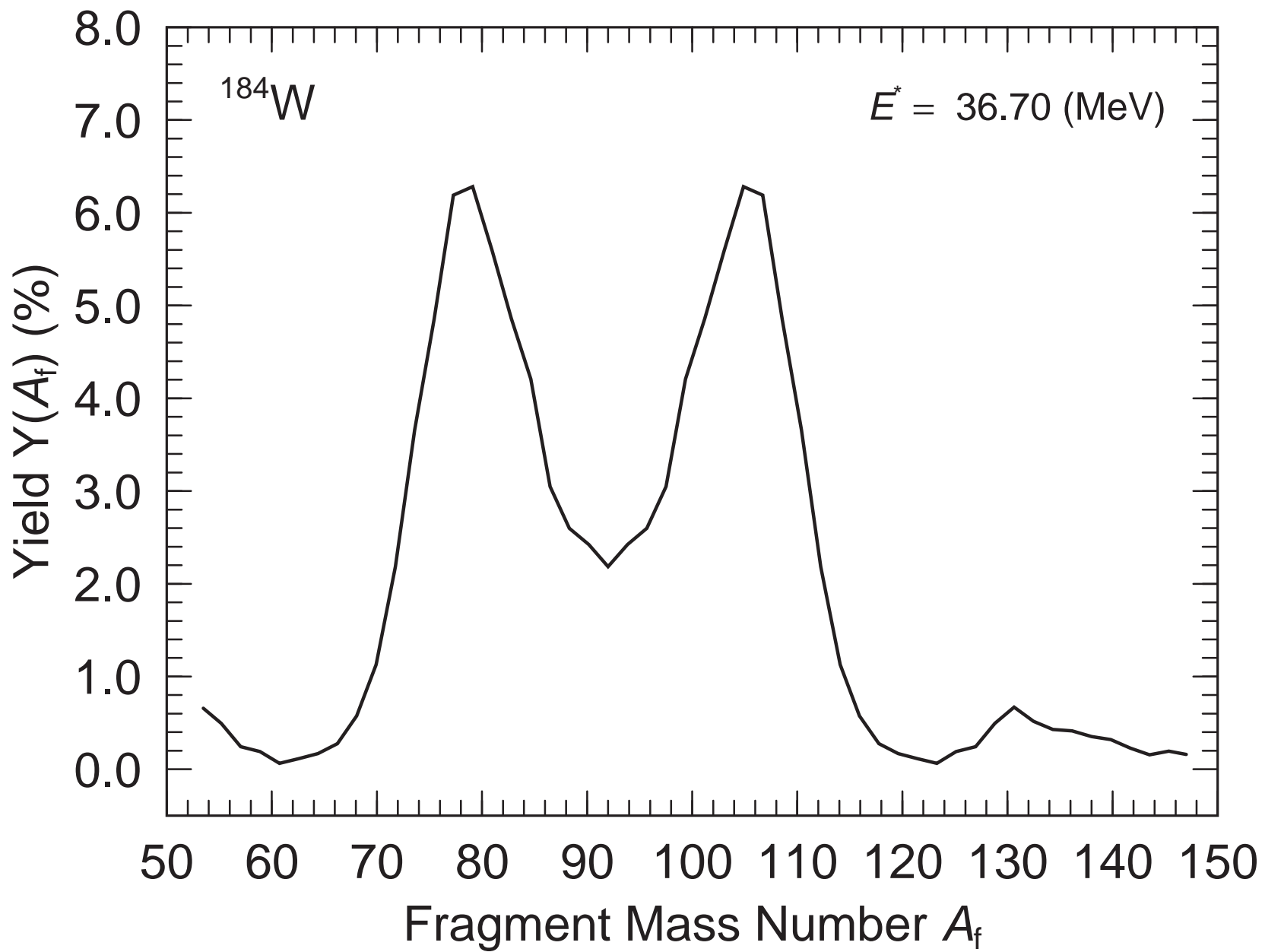


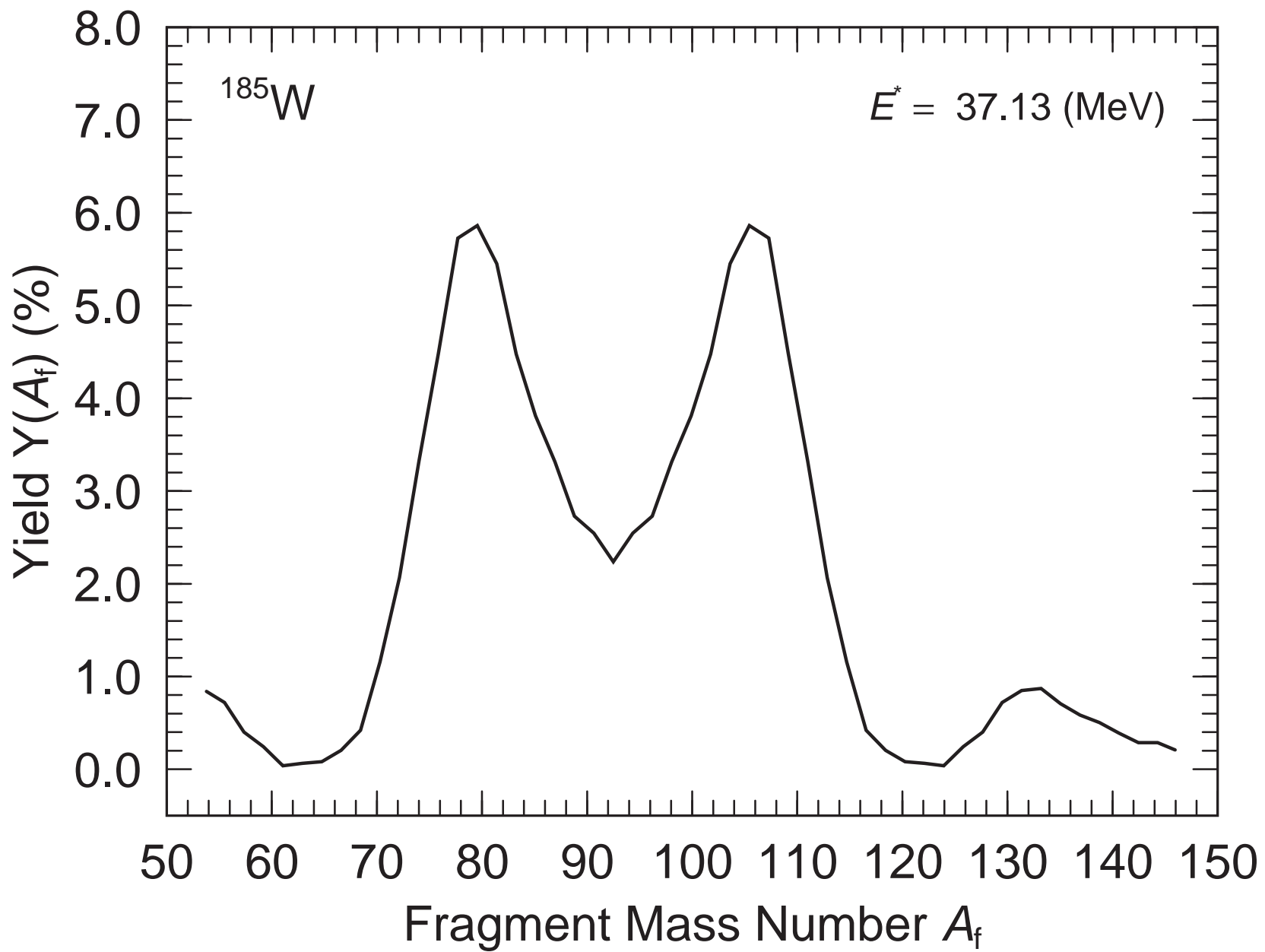


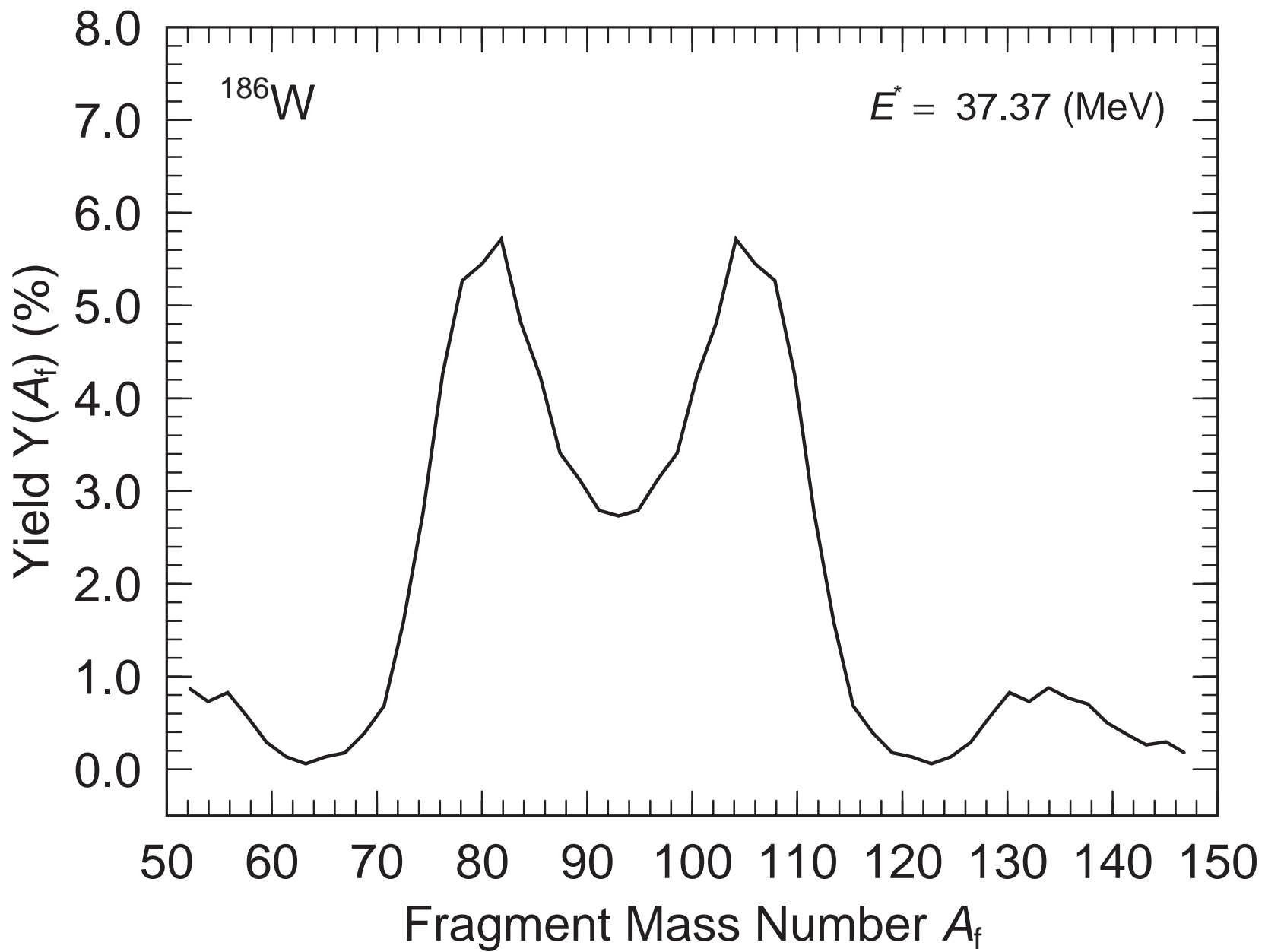


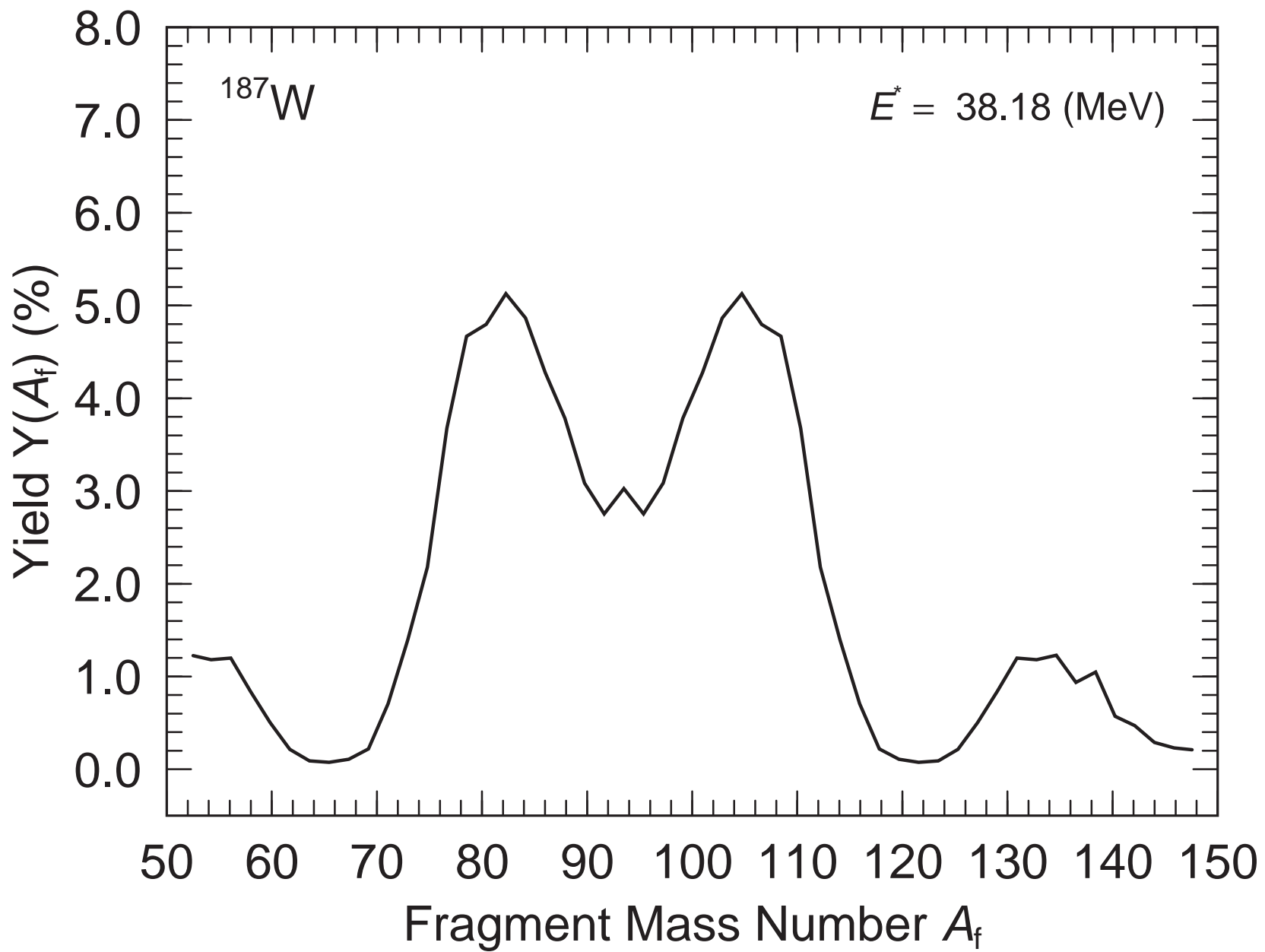


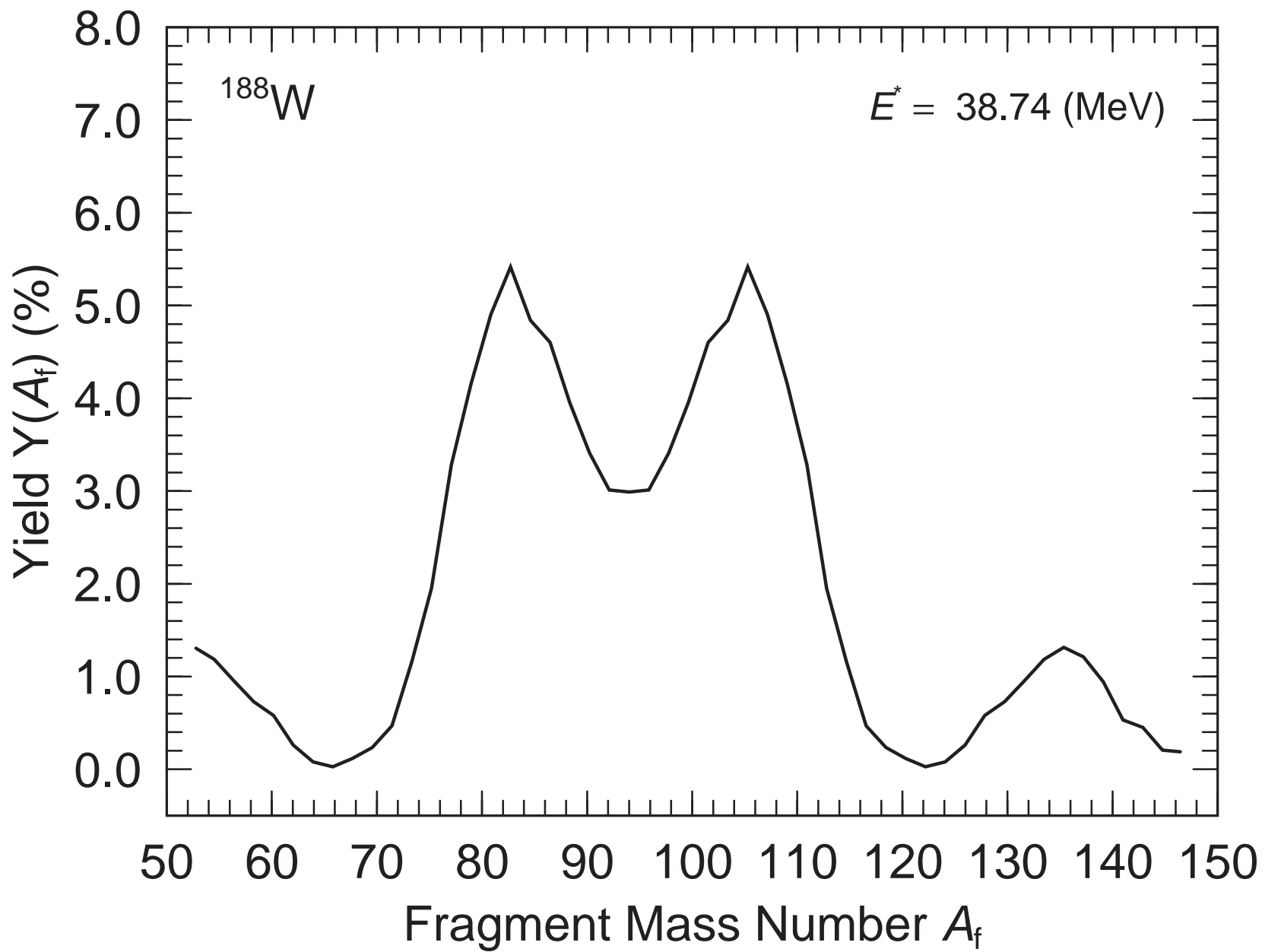


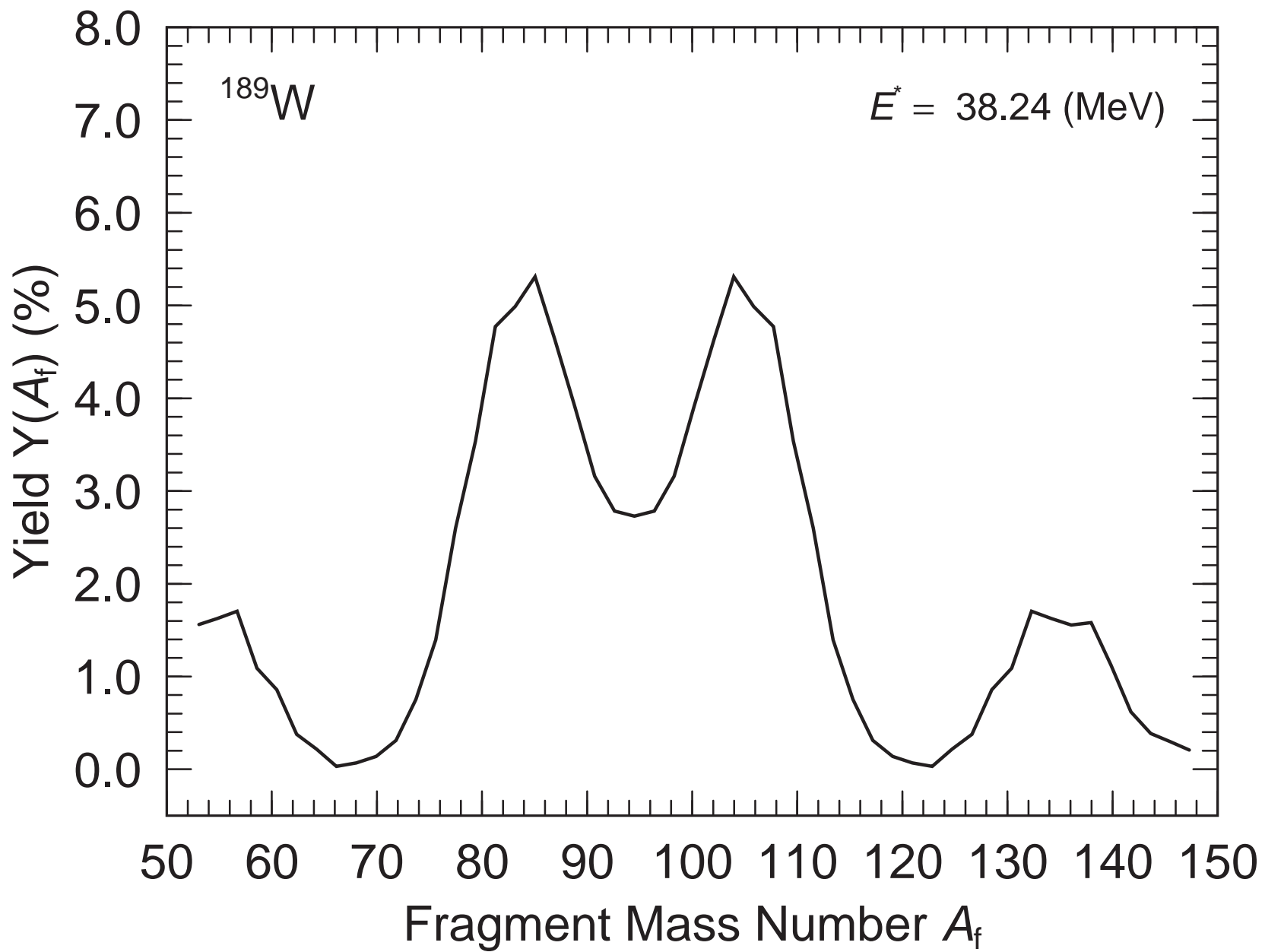


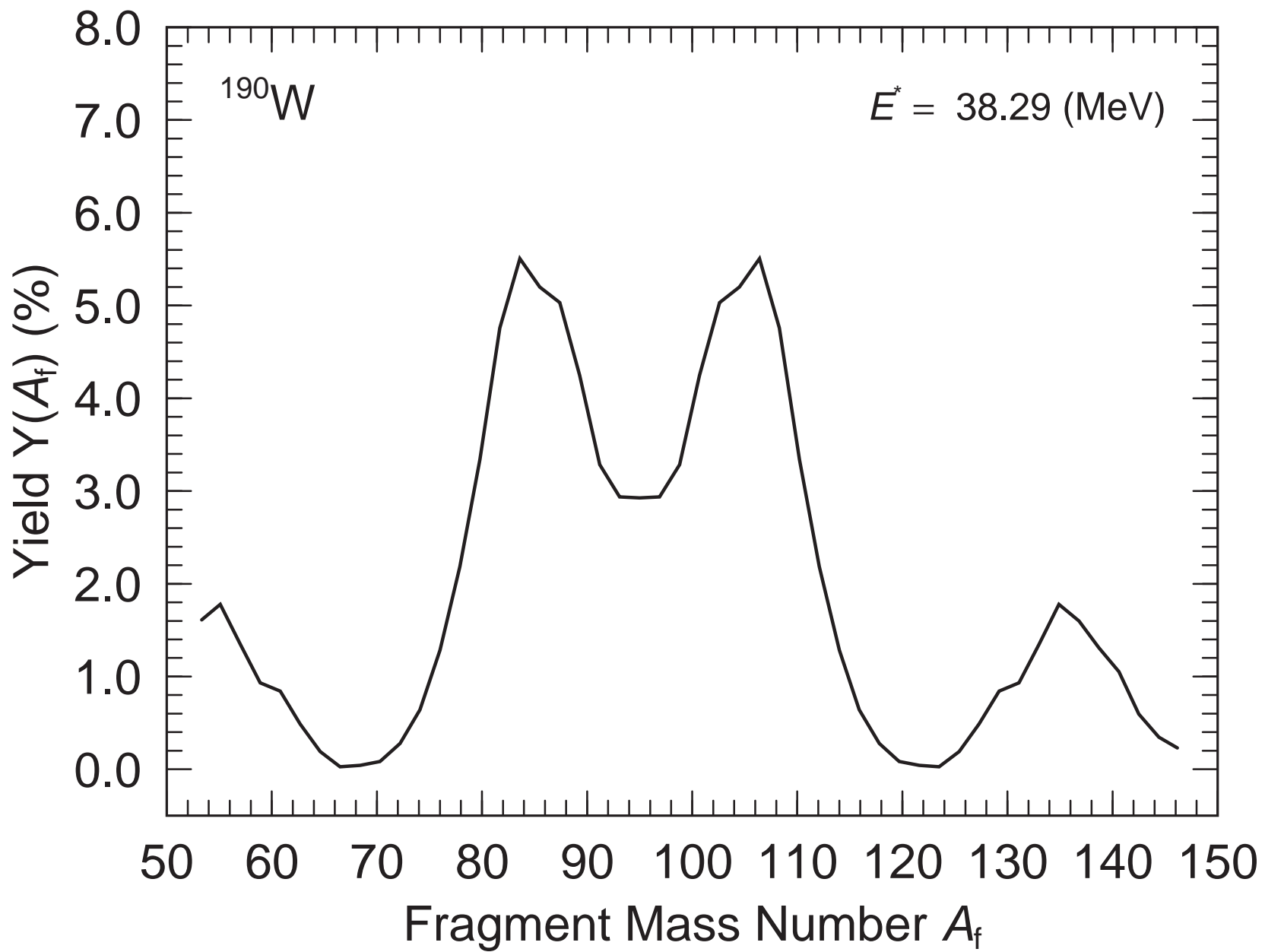


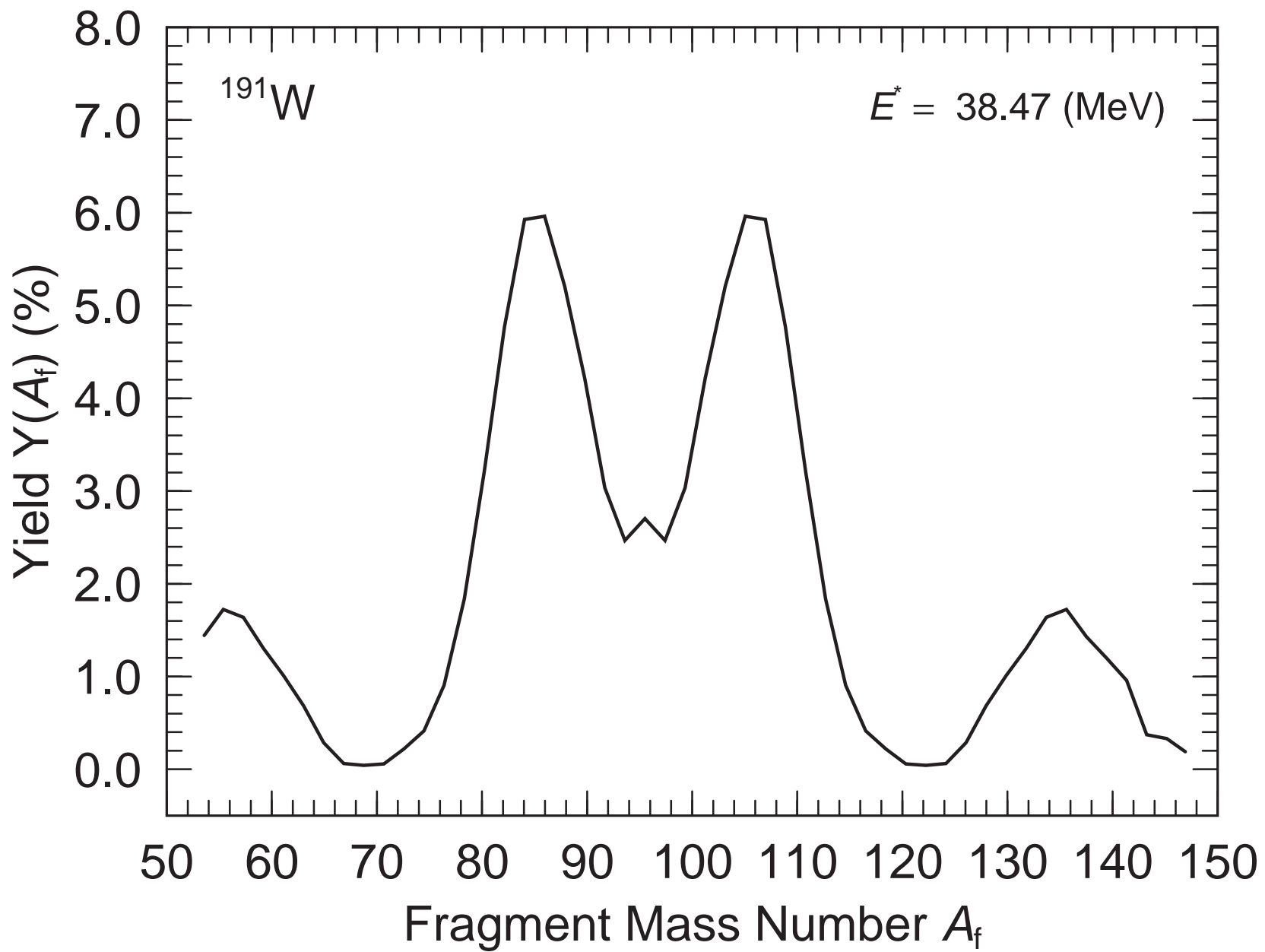


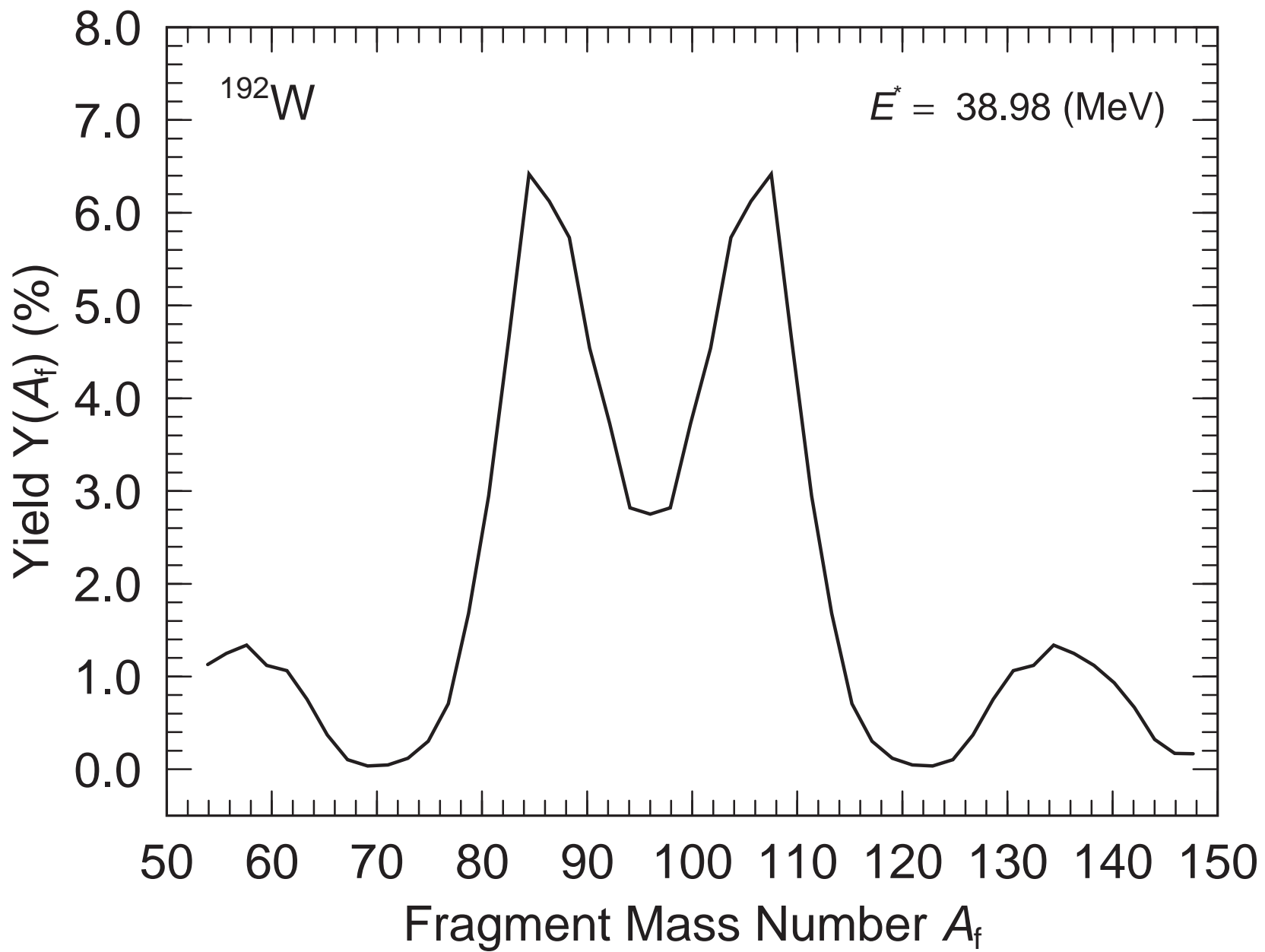


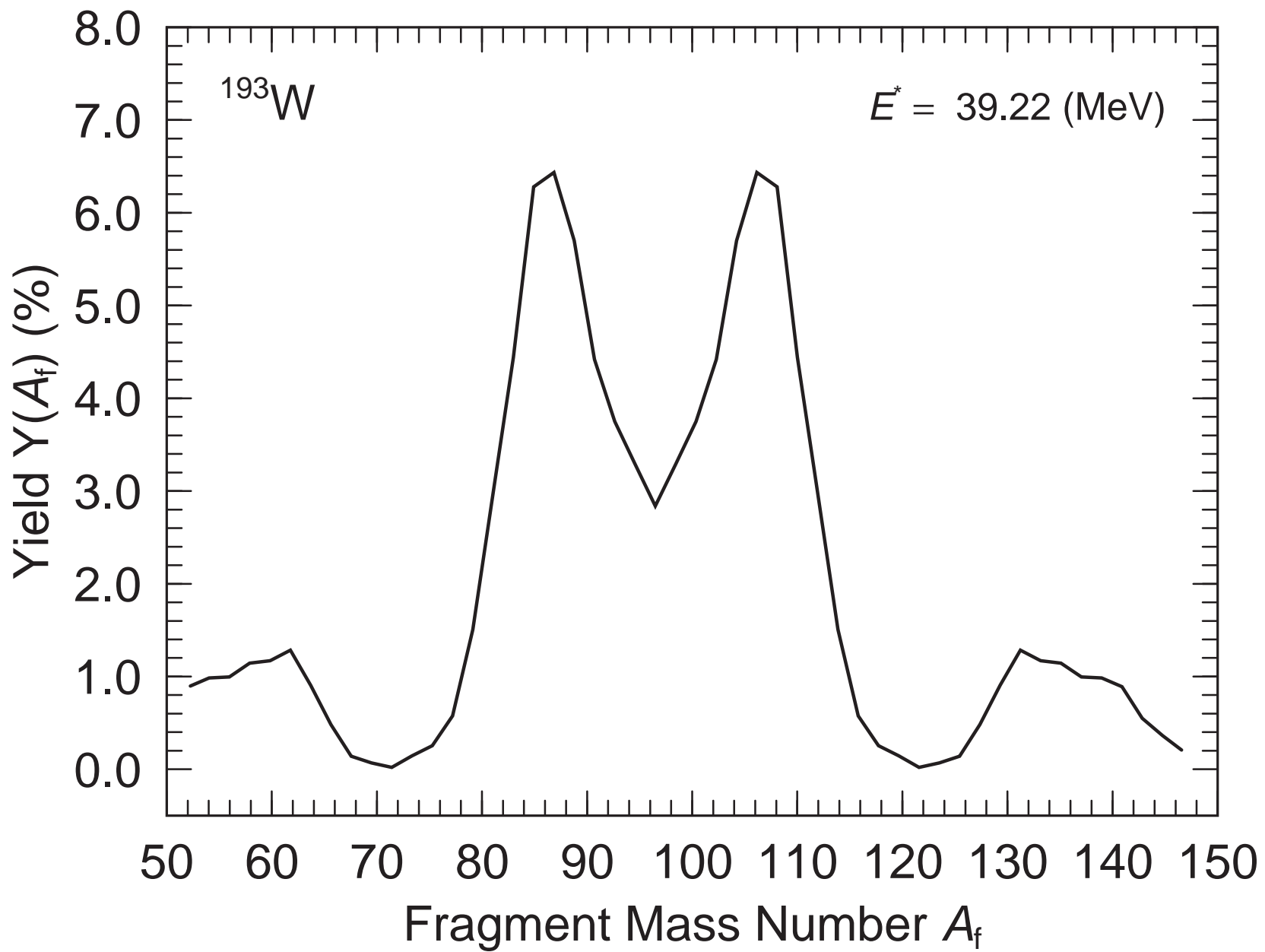


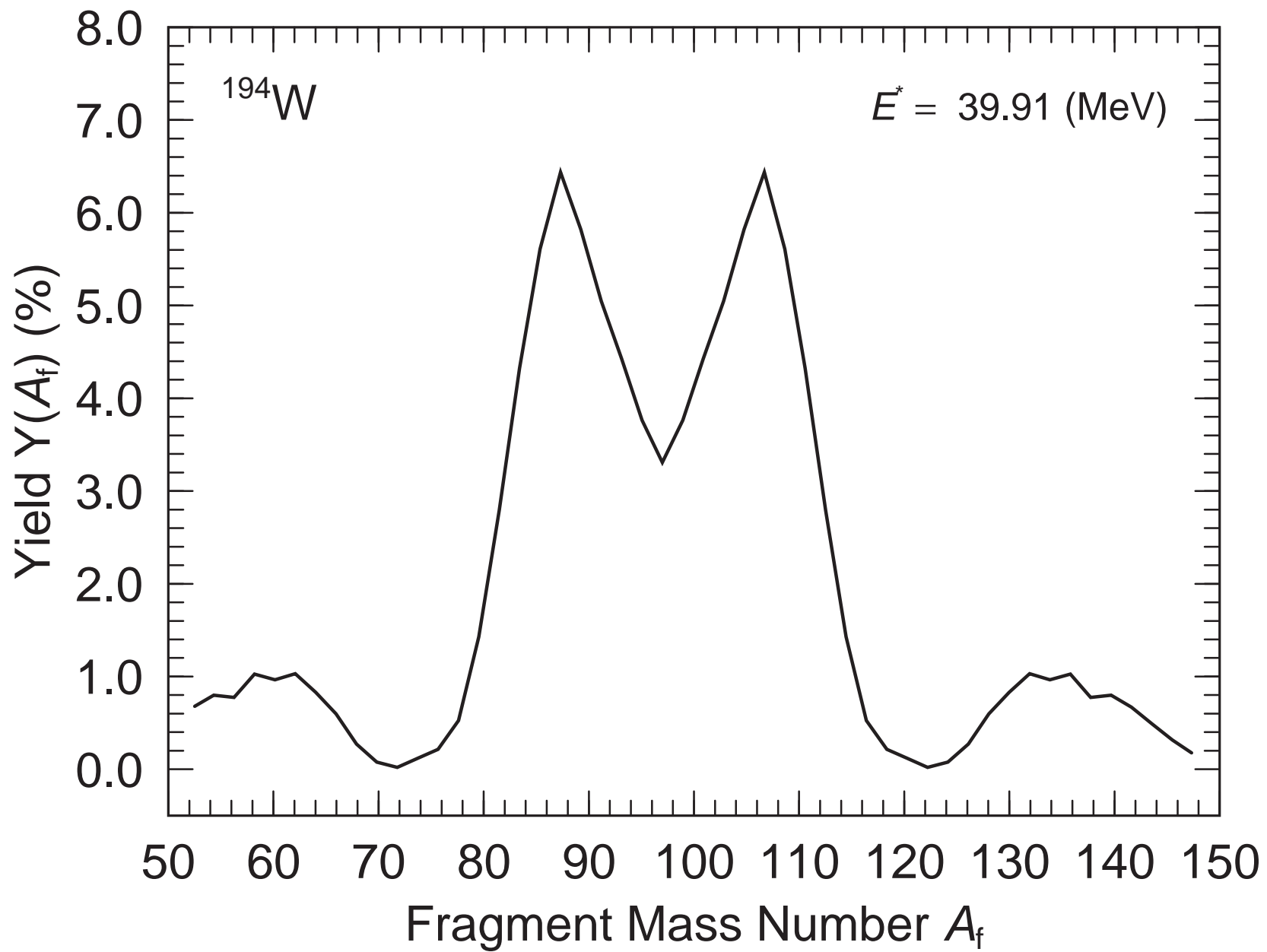


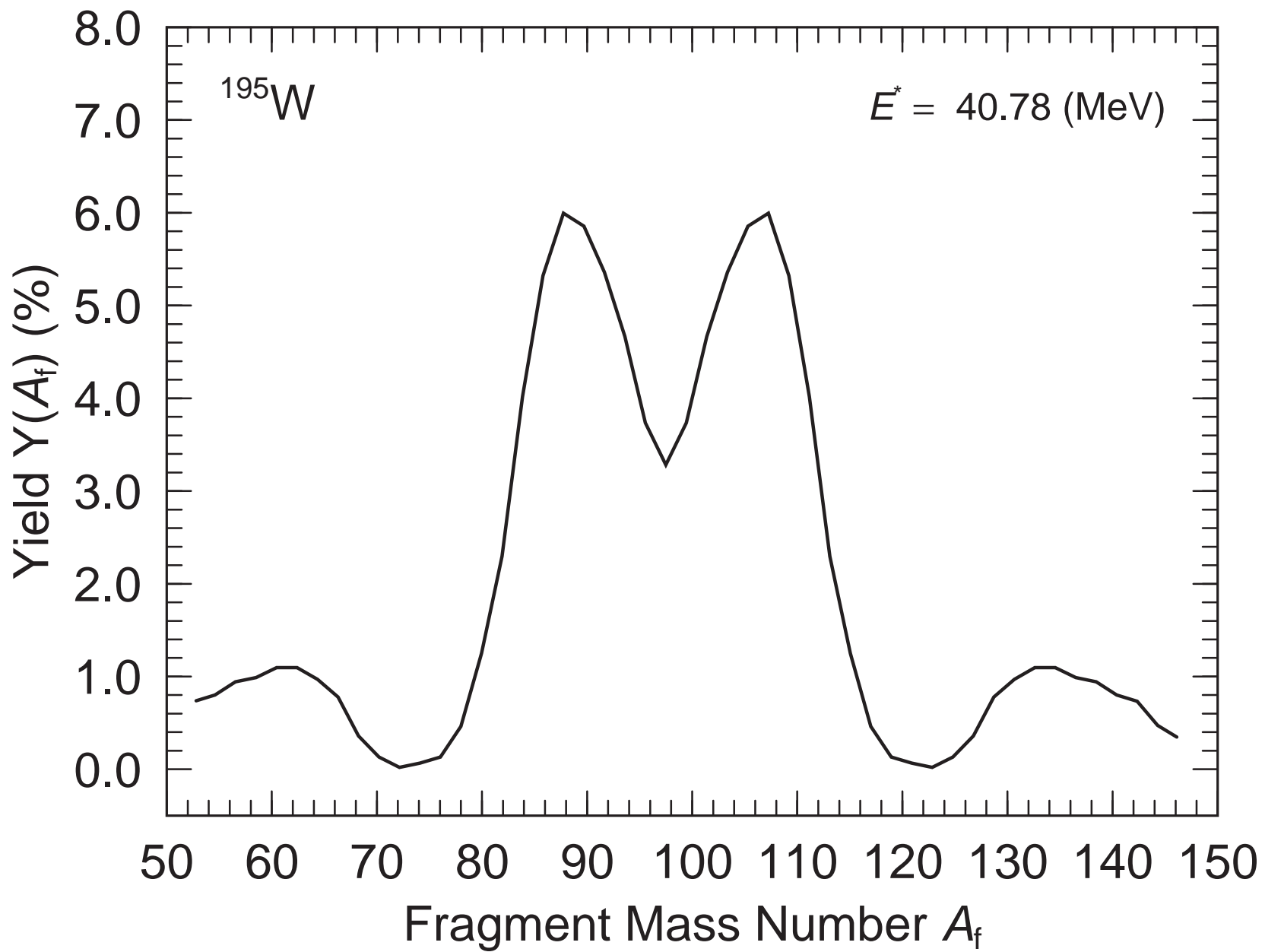


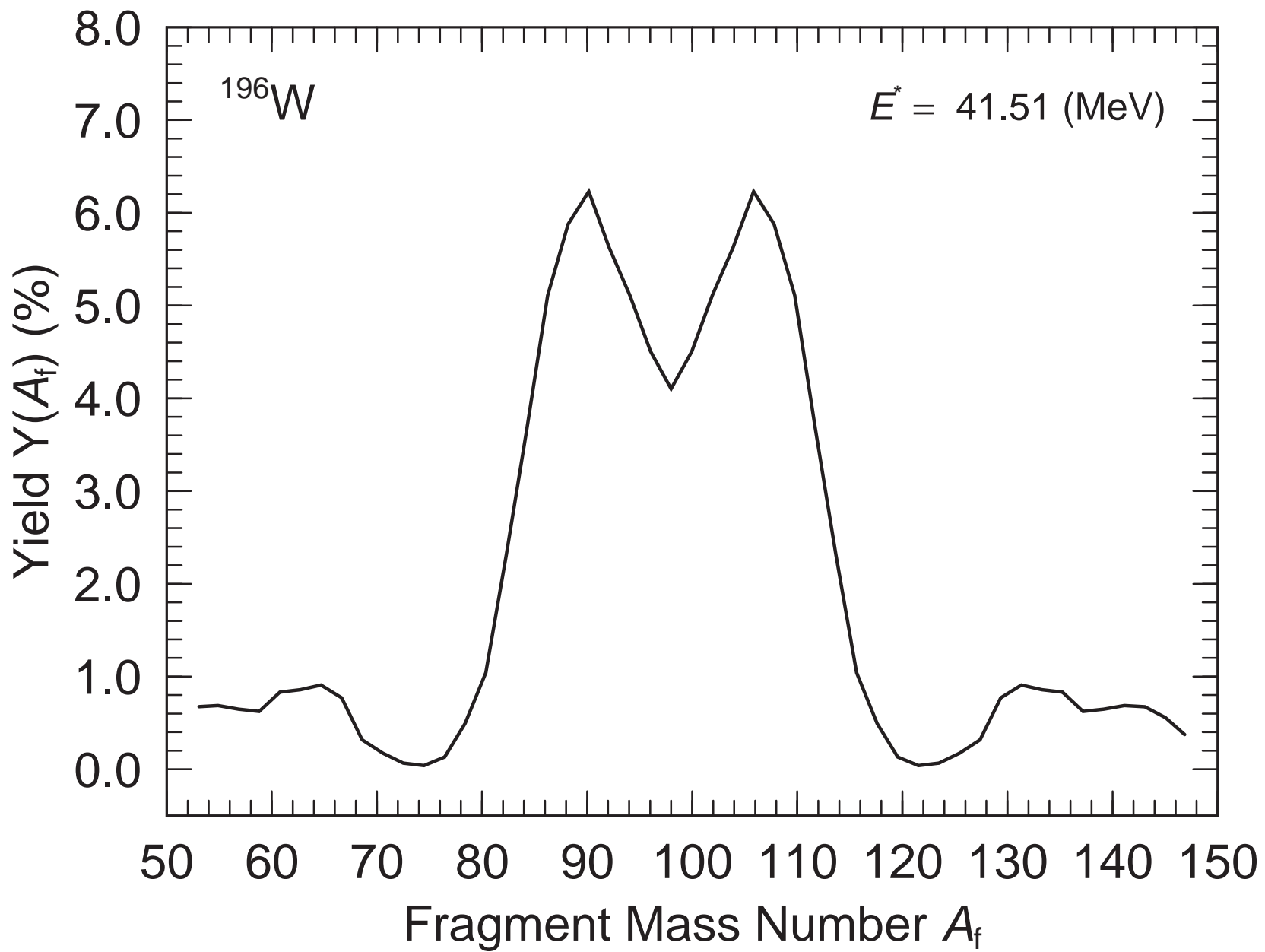


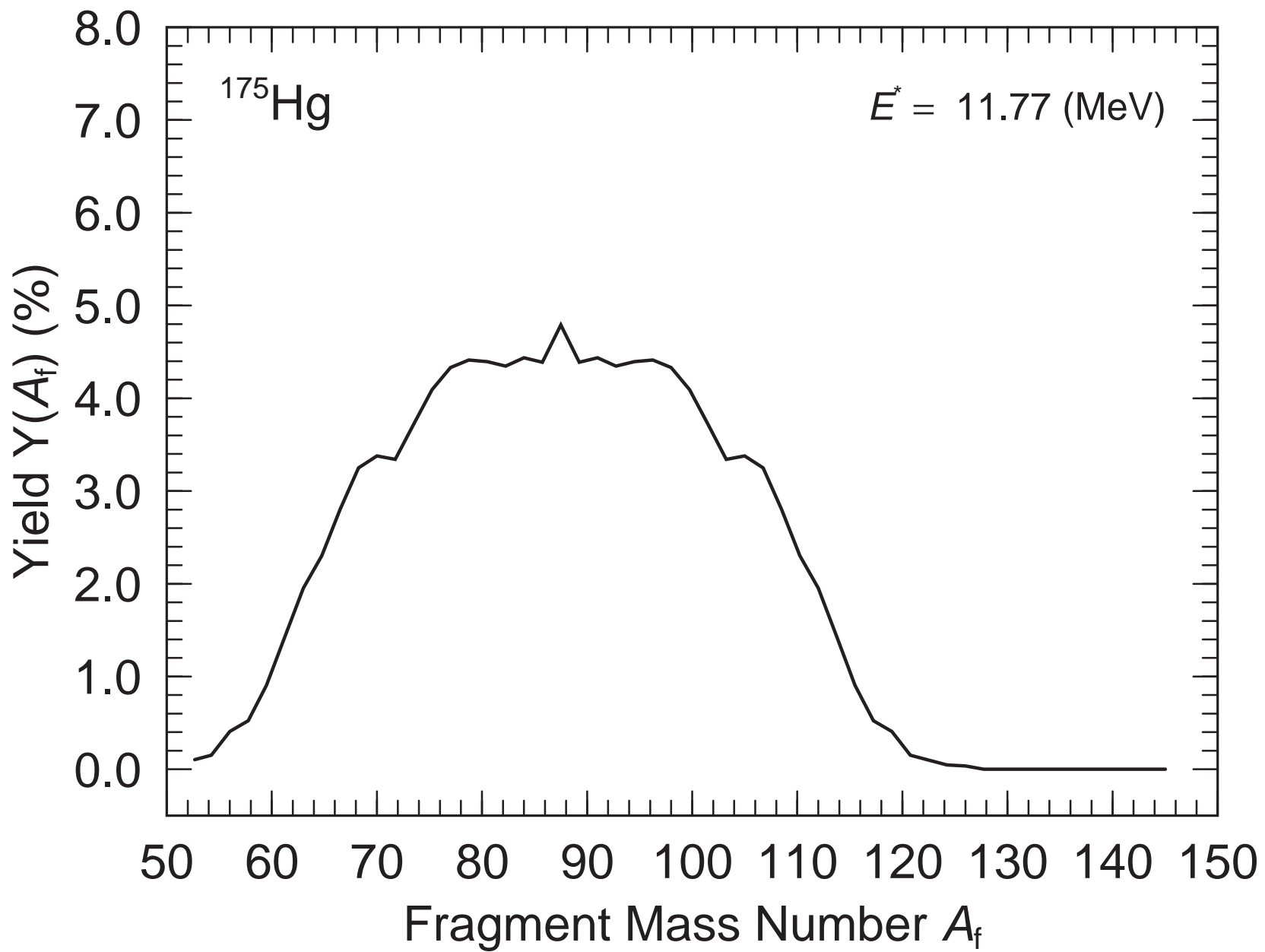


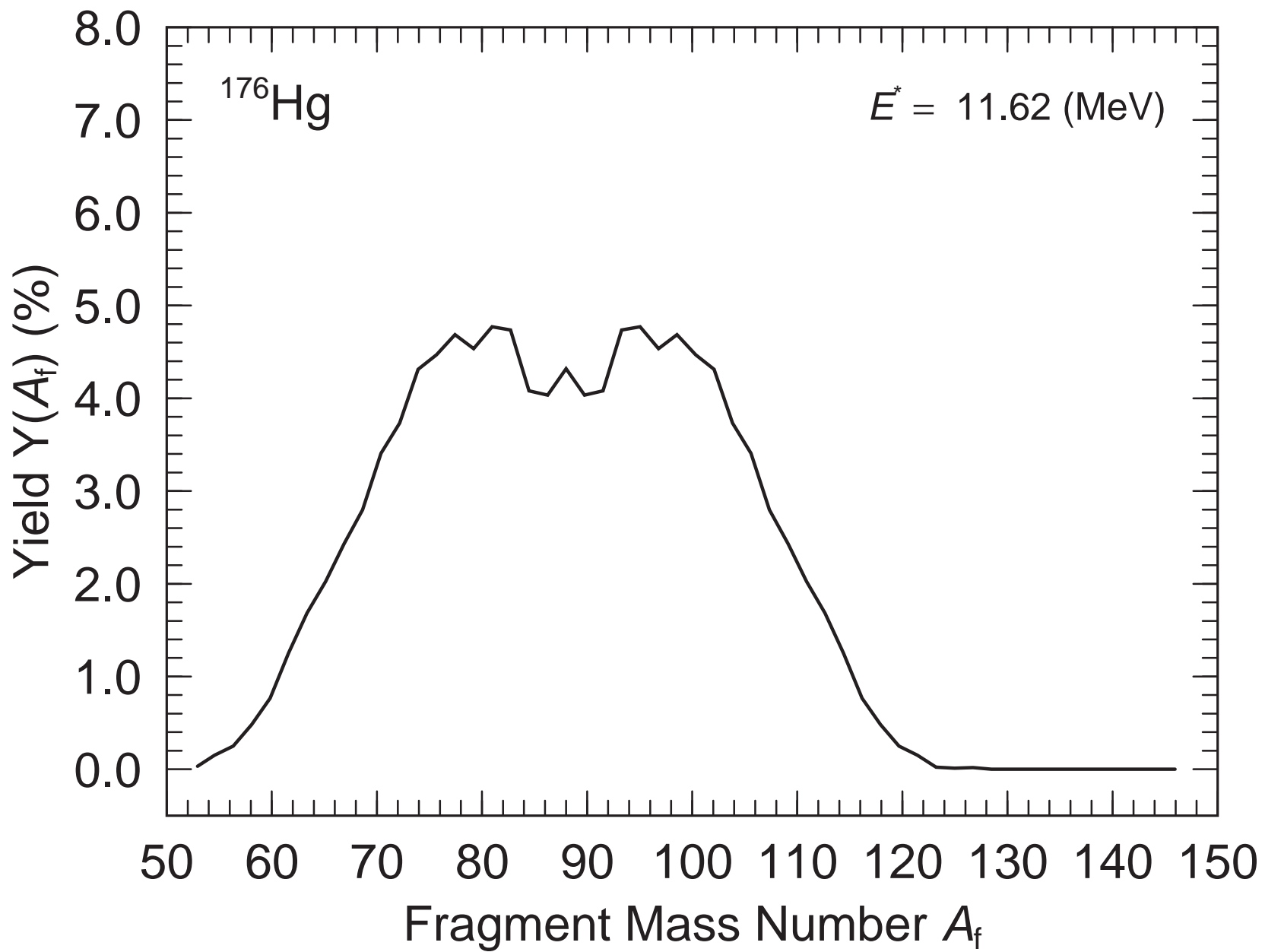


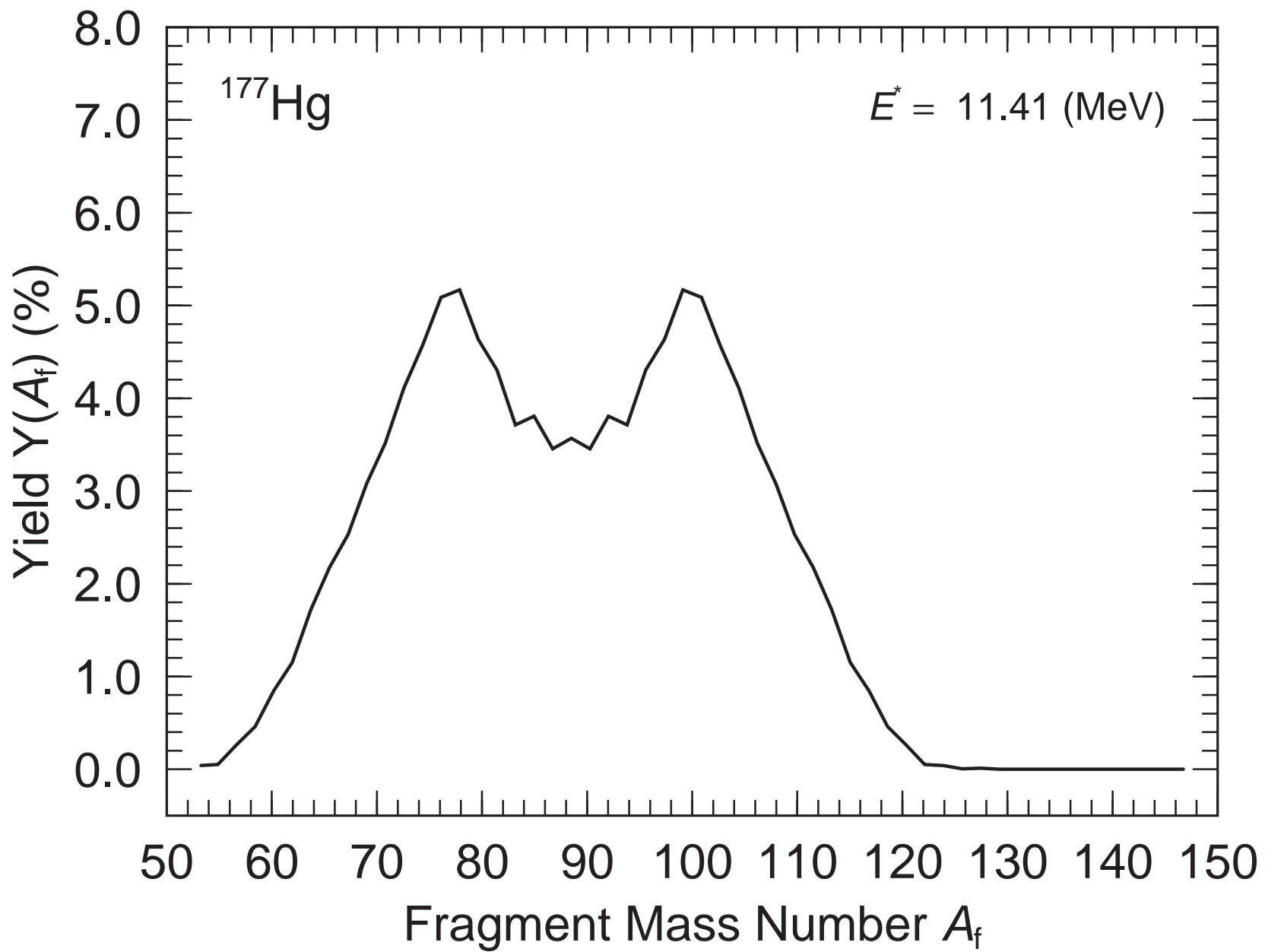


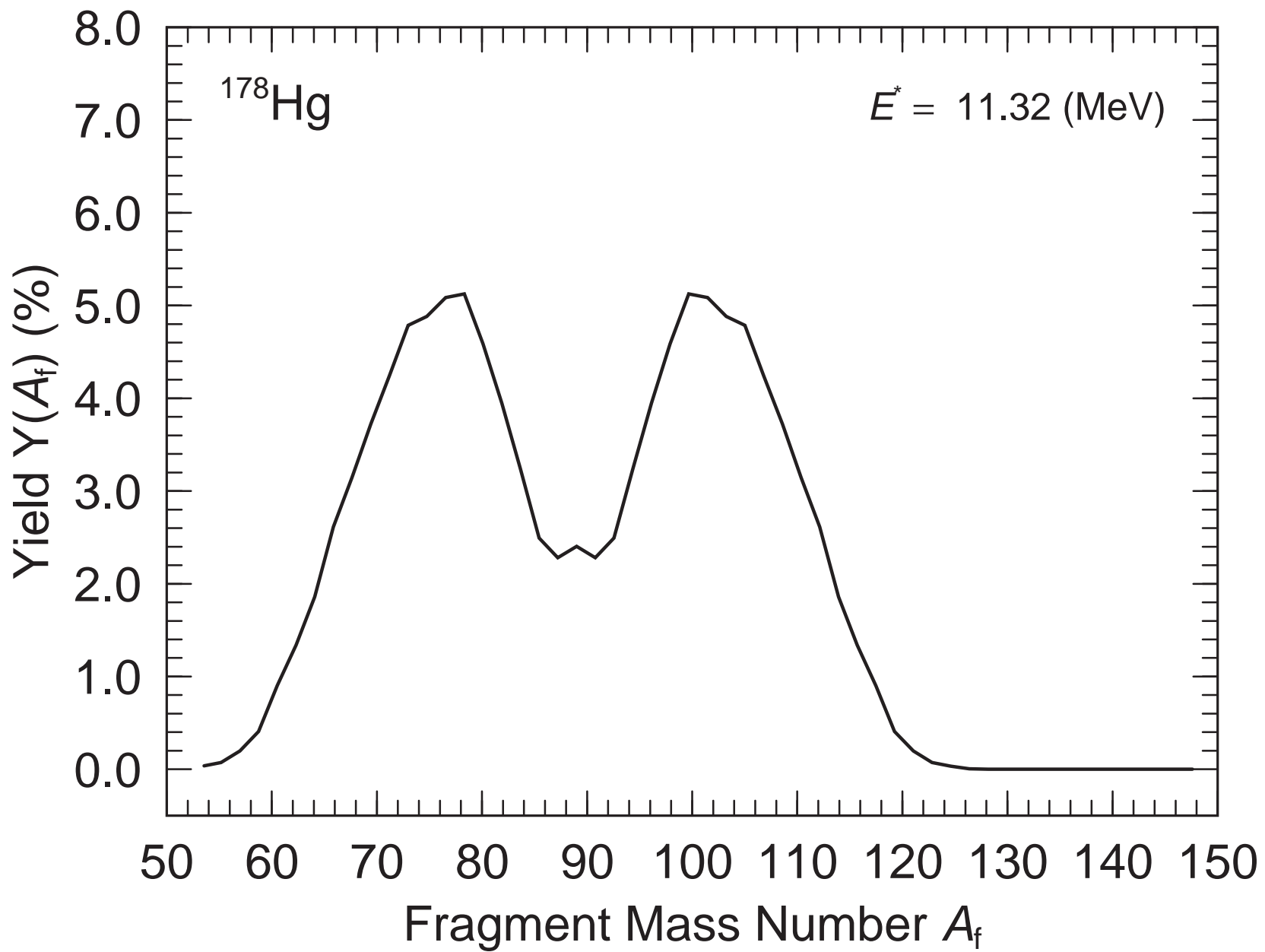


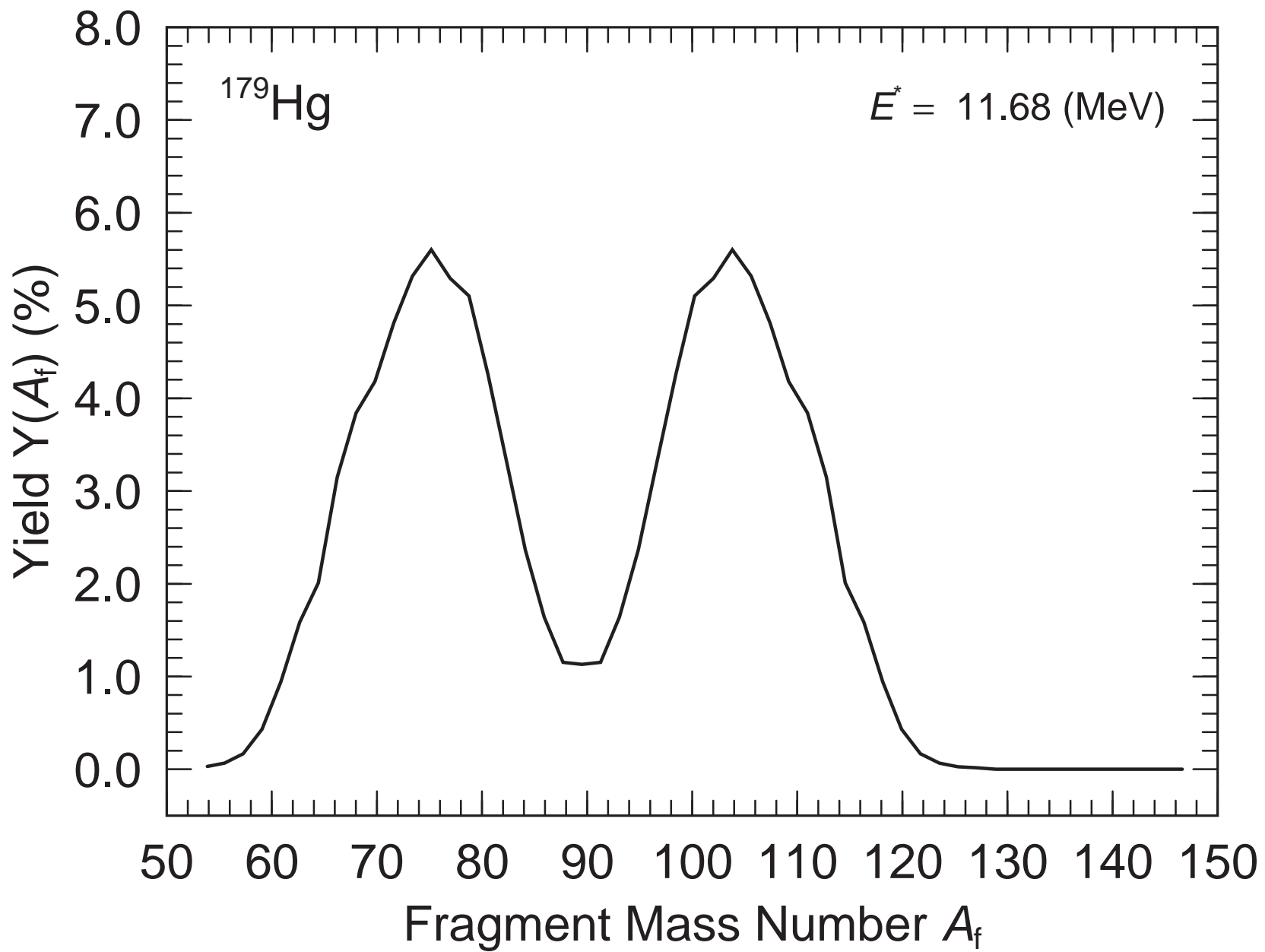


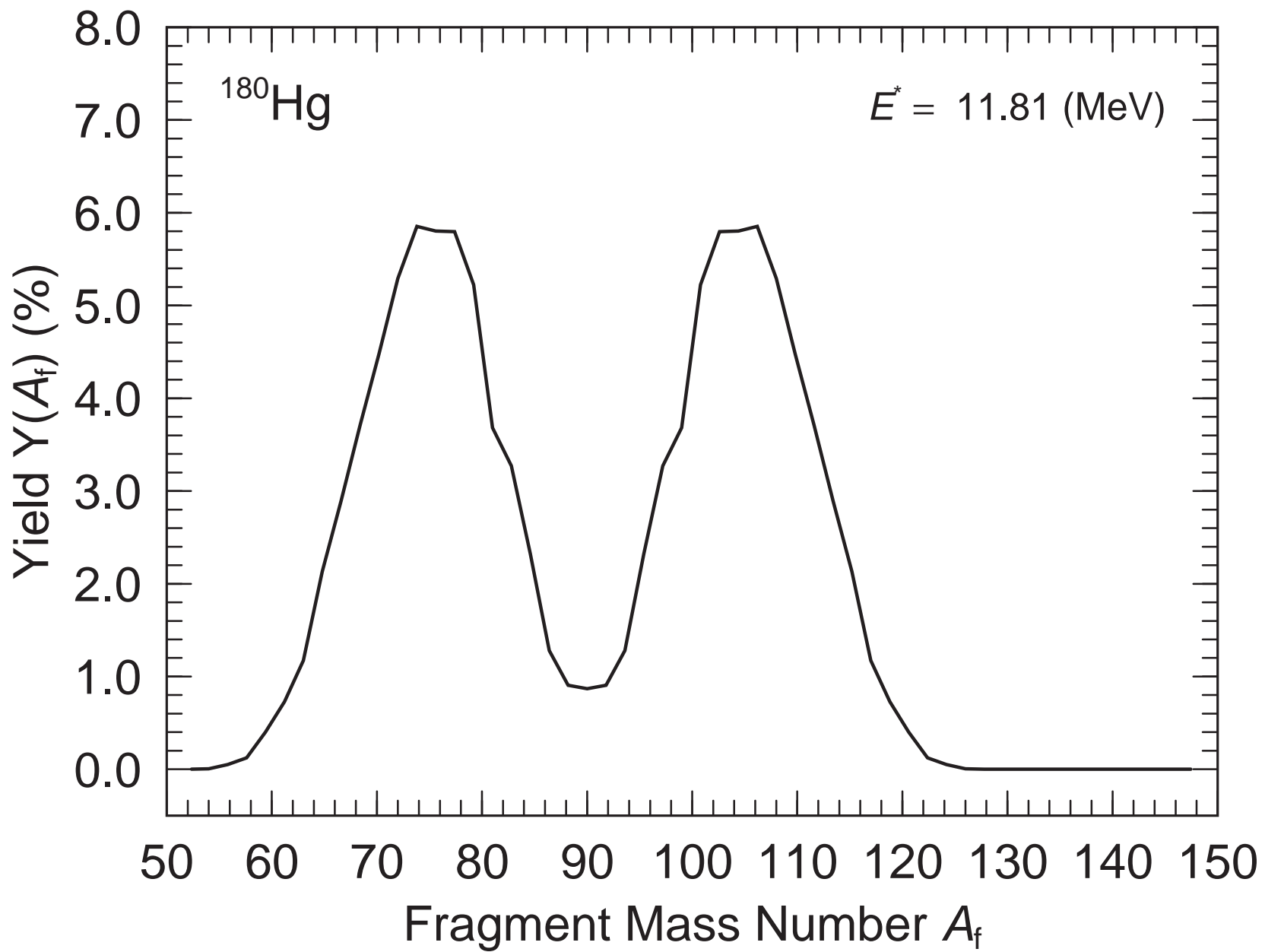




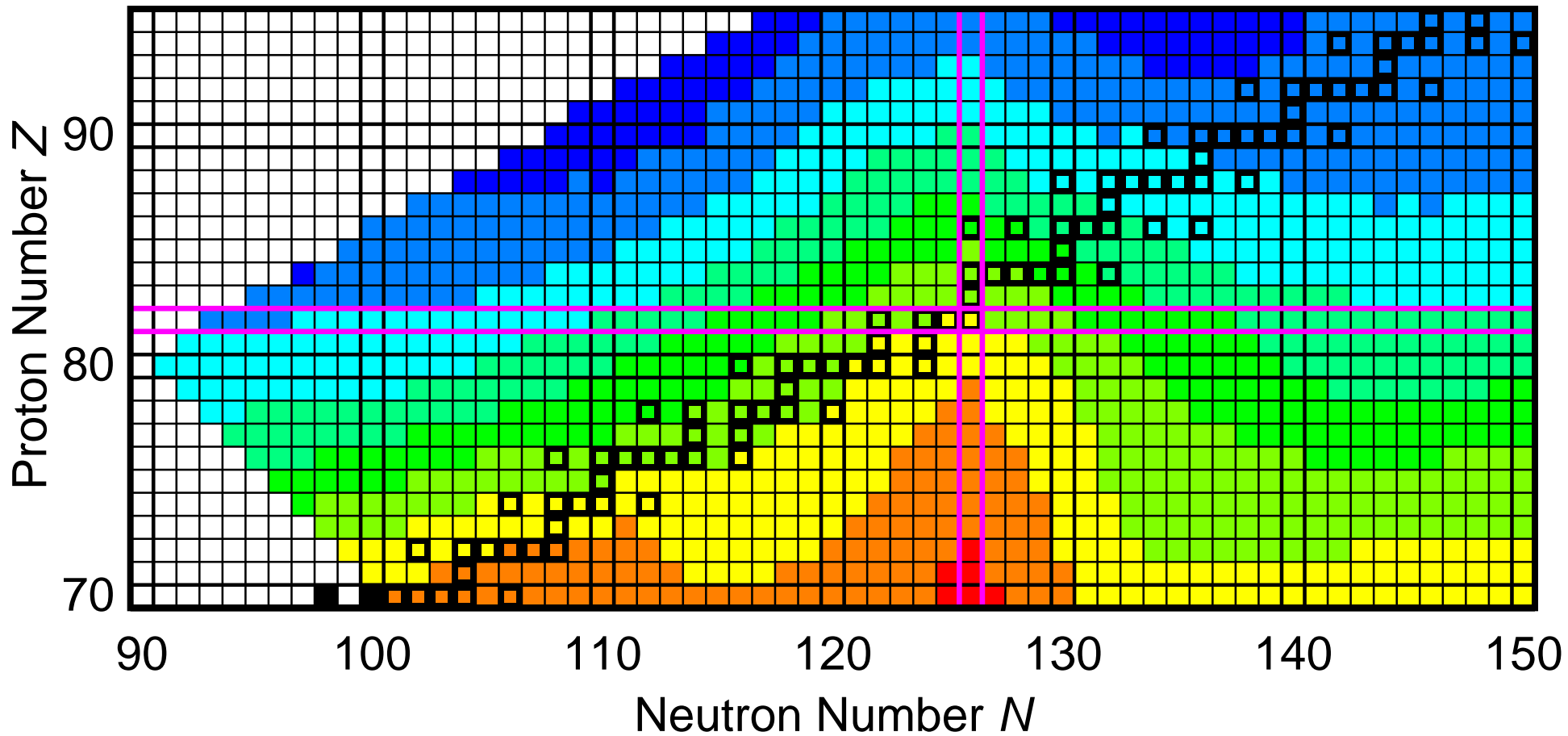
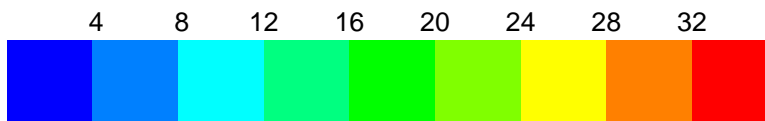






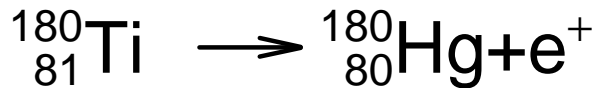


Fission-Barrier Height (MeV)



Folded-Yukawa potential

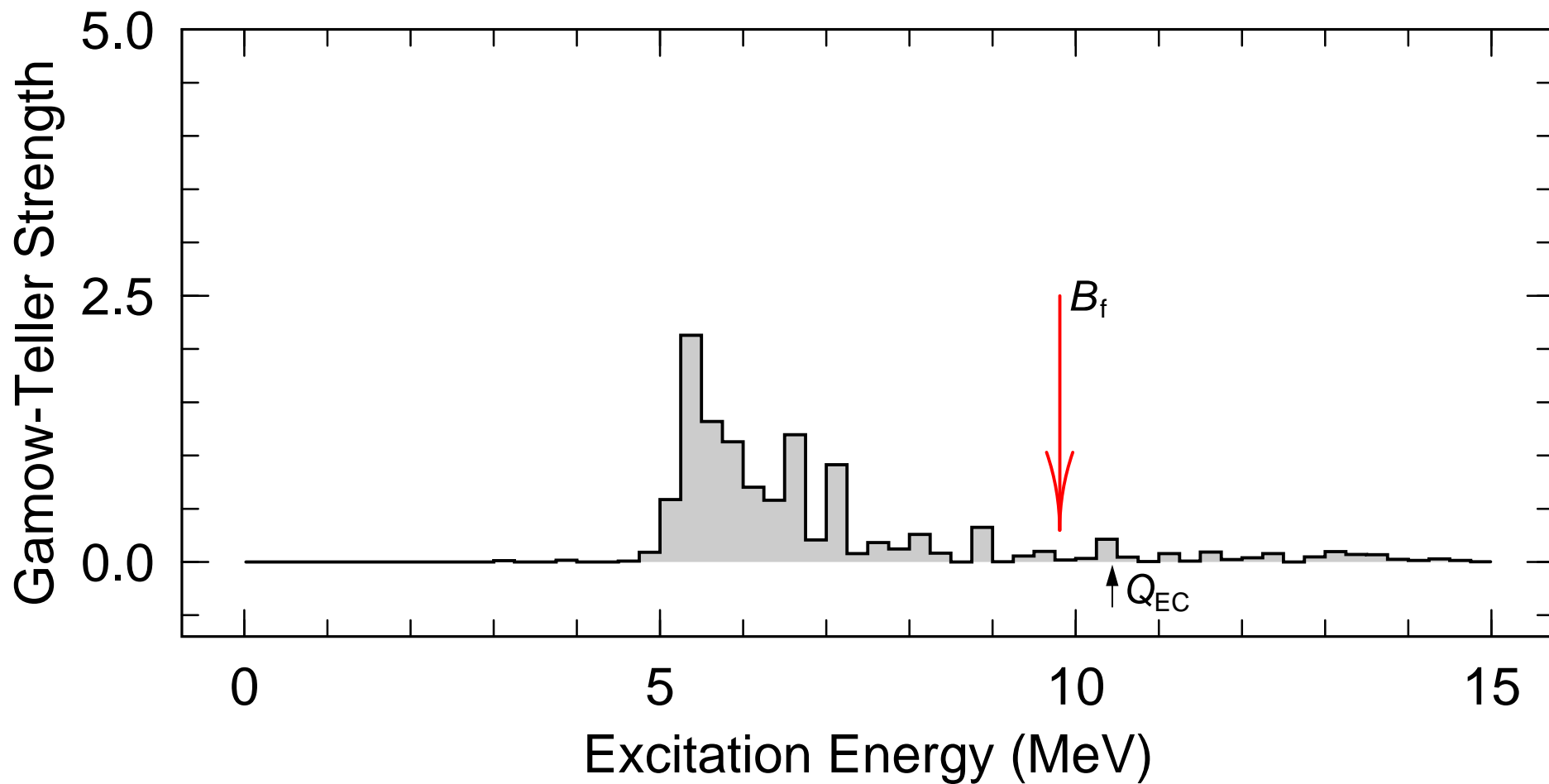
$T_{1/2} = 1.74$ (s)



$\varepsilon_2 = -0.130$ $\Delta_n = 0.99$ MeV $\lambda_n = 34.88$ MeV

$\varepsilon_4 = 0.010$ $\Delta_p = 0.51$ MeV $\lambda_p = 32.50$ MeV

$\varepsilon_6 = 0.010$ (L-N) $a = 0.80$ fm



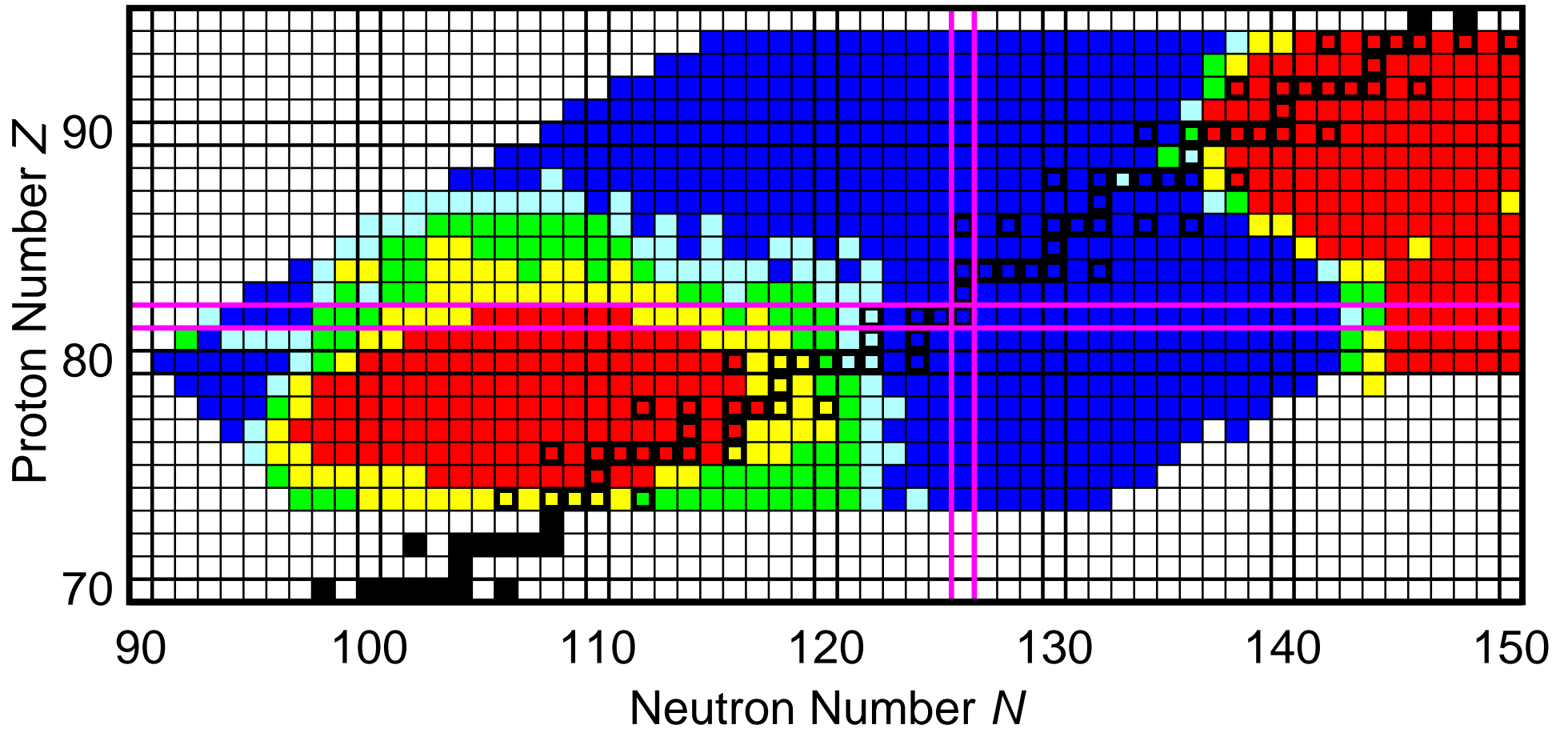
Fission-Yield Valley-to-Peak Ratio

0.2 0.4 0.6 0.8

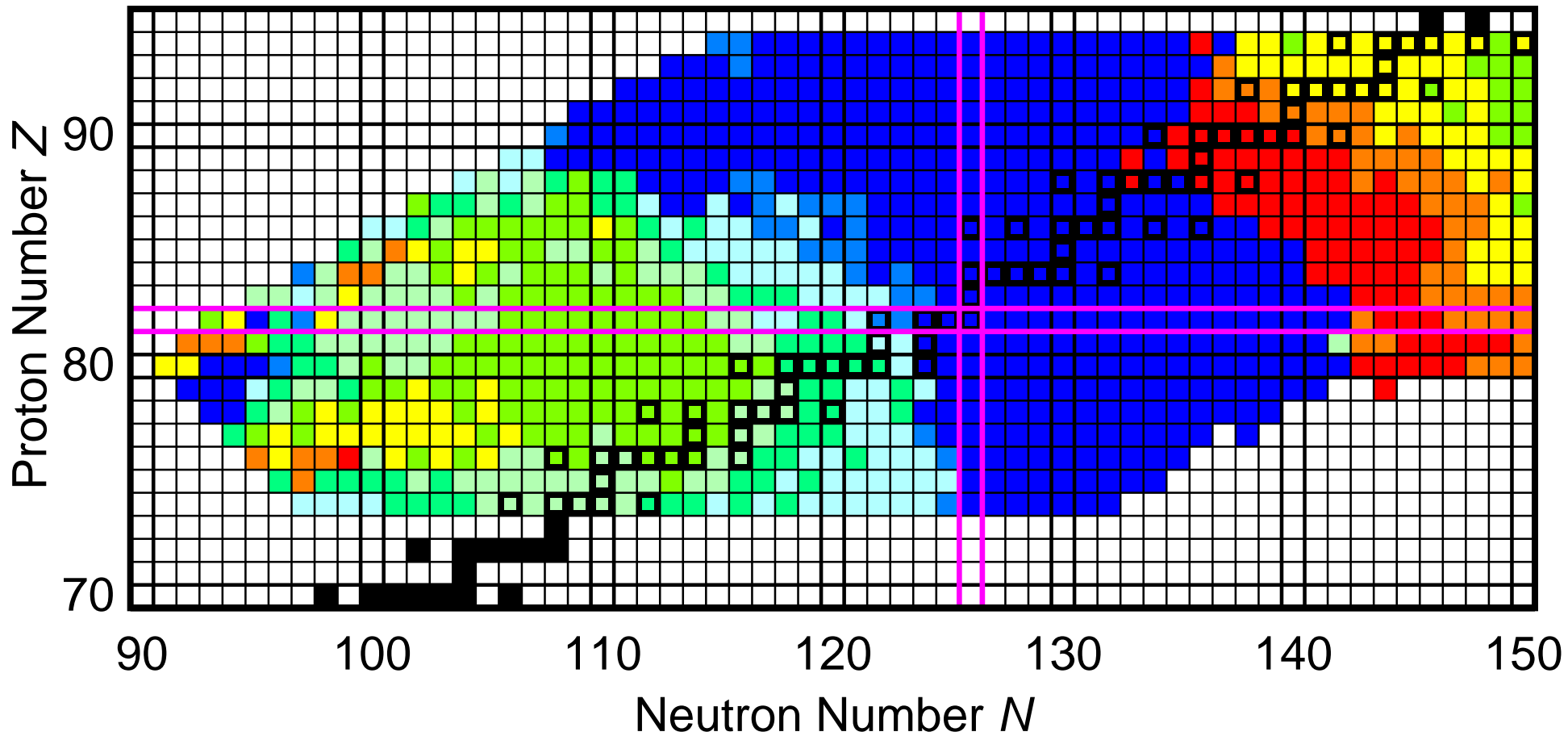
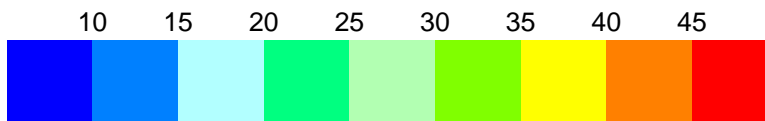
Asymmetric



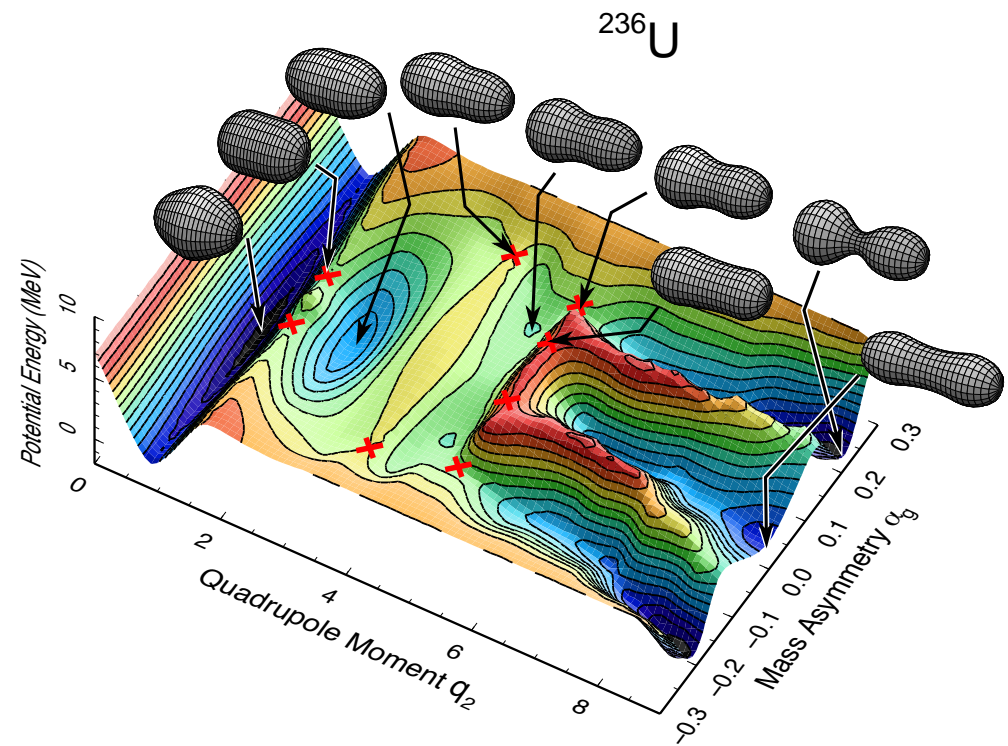
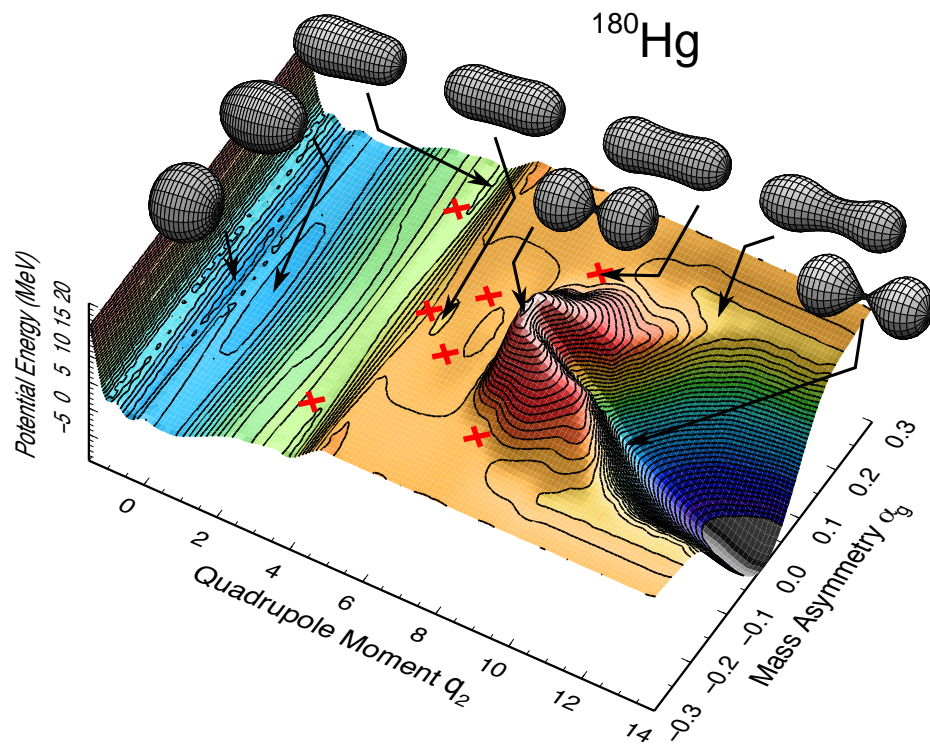
Symmetric

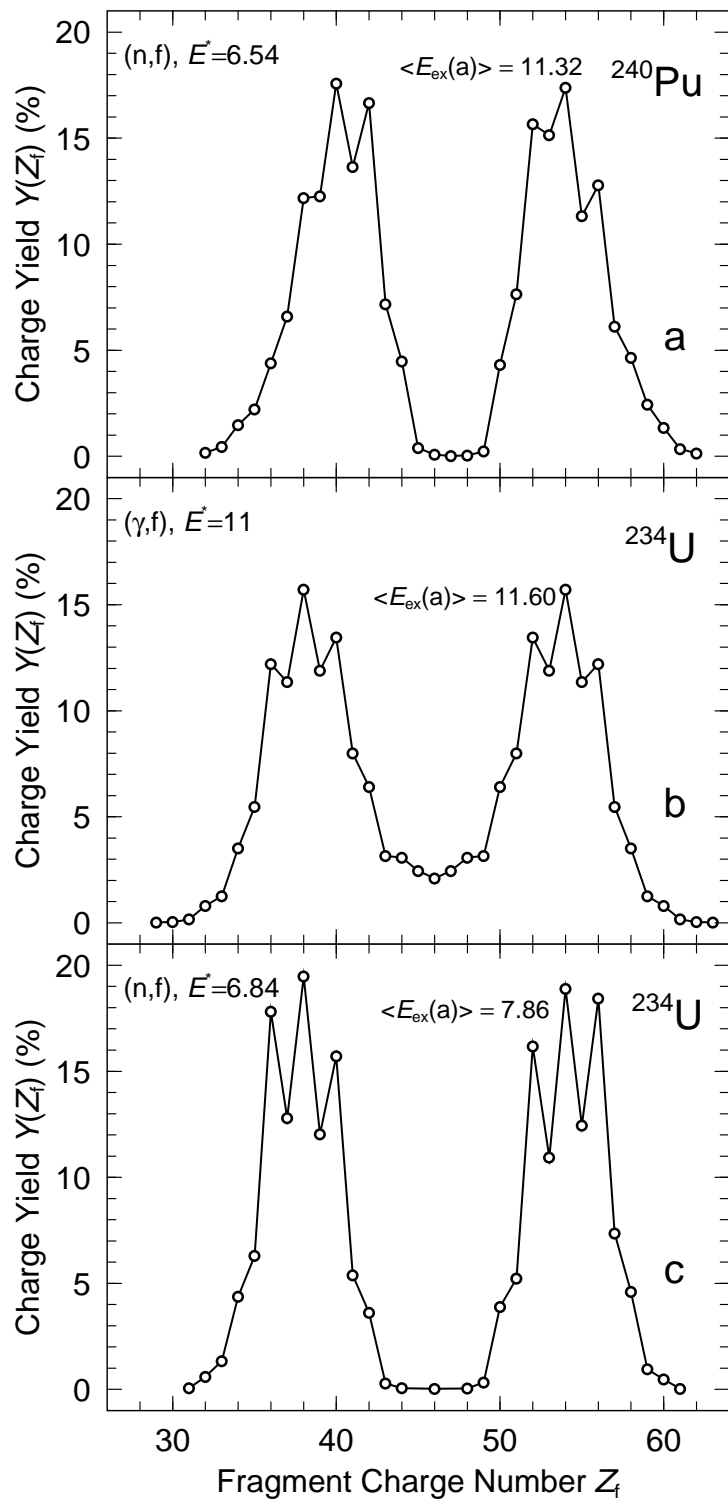


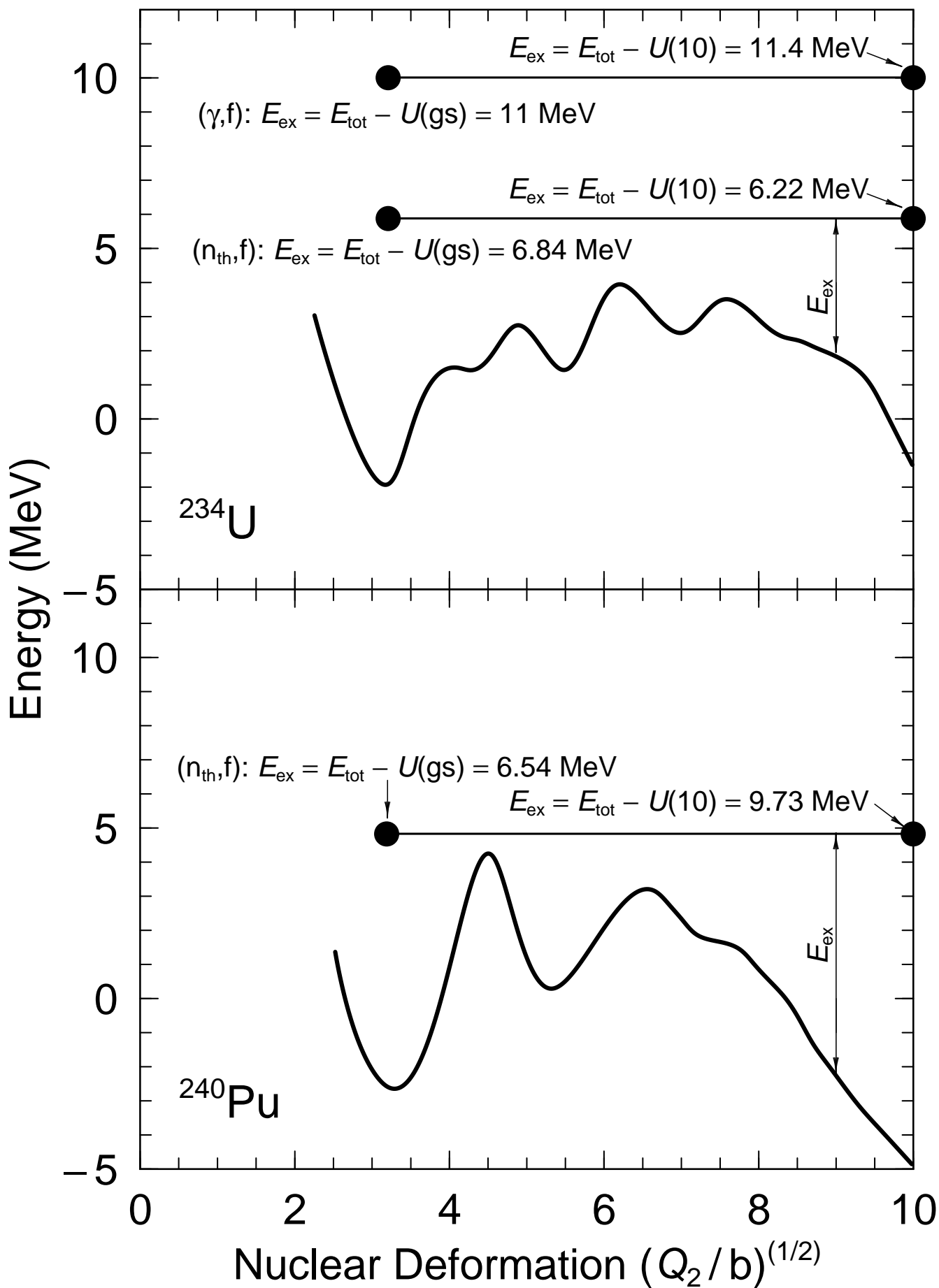
Fission-Fragment Mass Division (MH-ML)

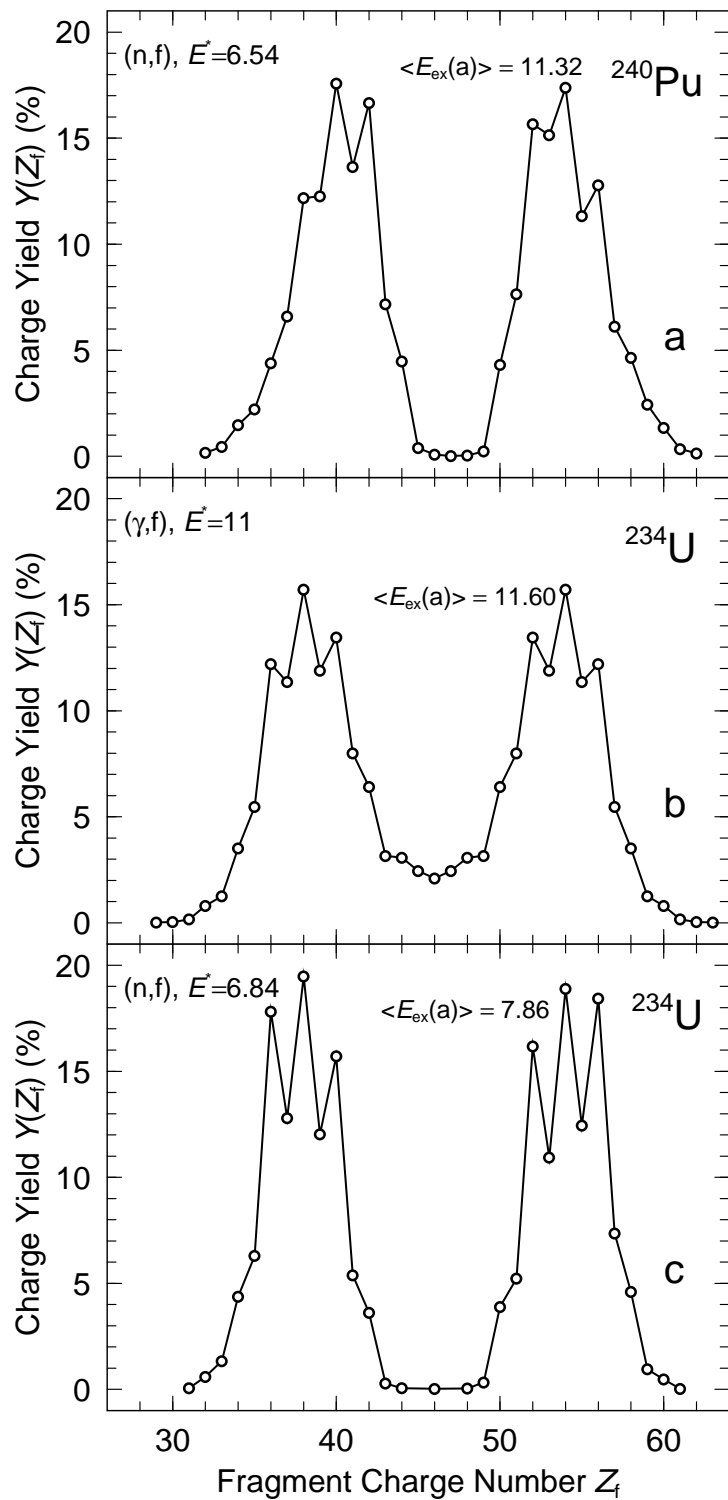


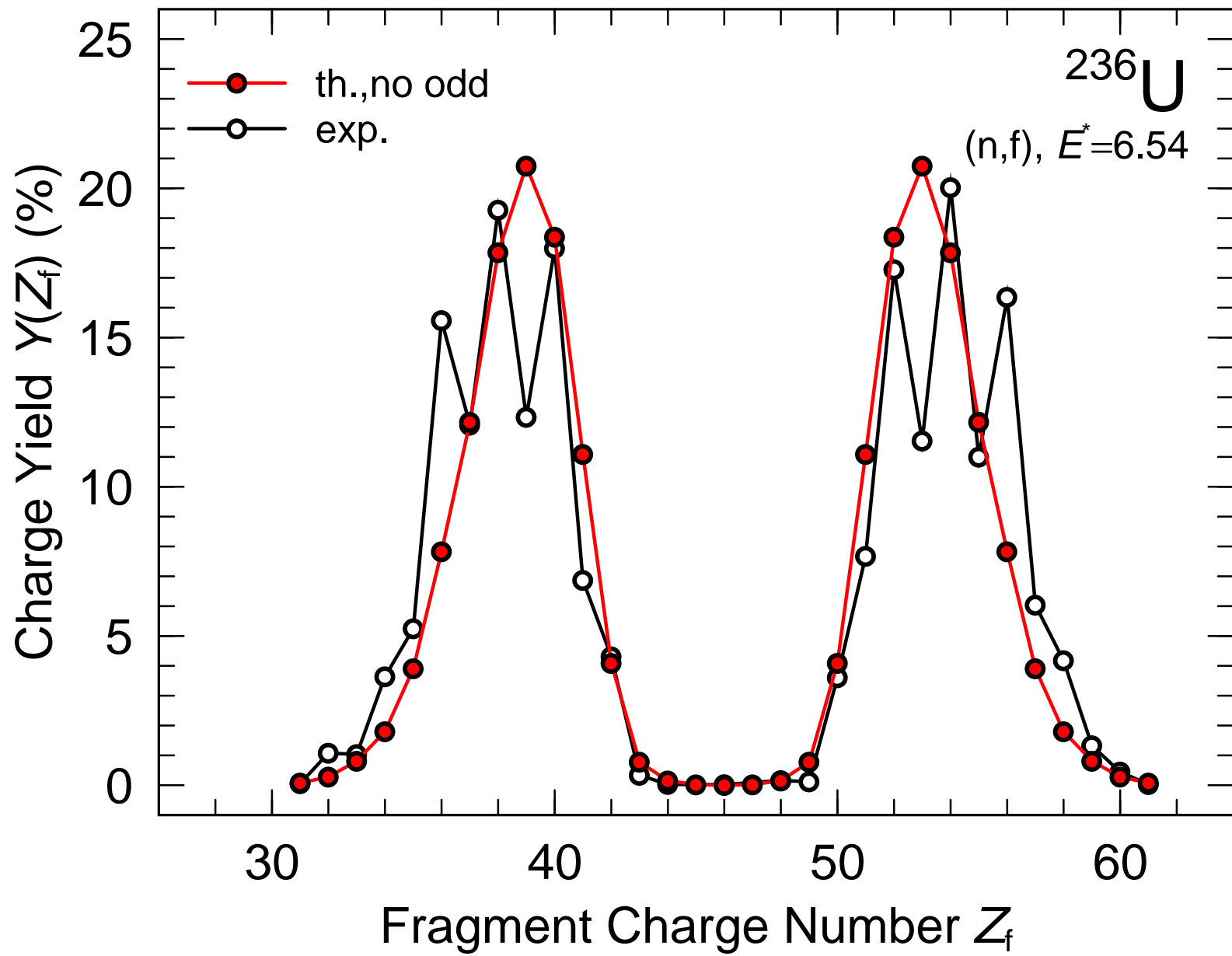
Contrasting Fission Potential-Energy Surfaces $\text{Hg} \leftrightarrow \text{U}$









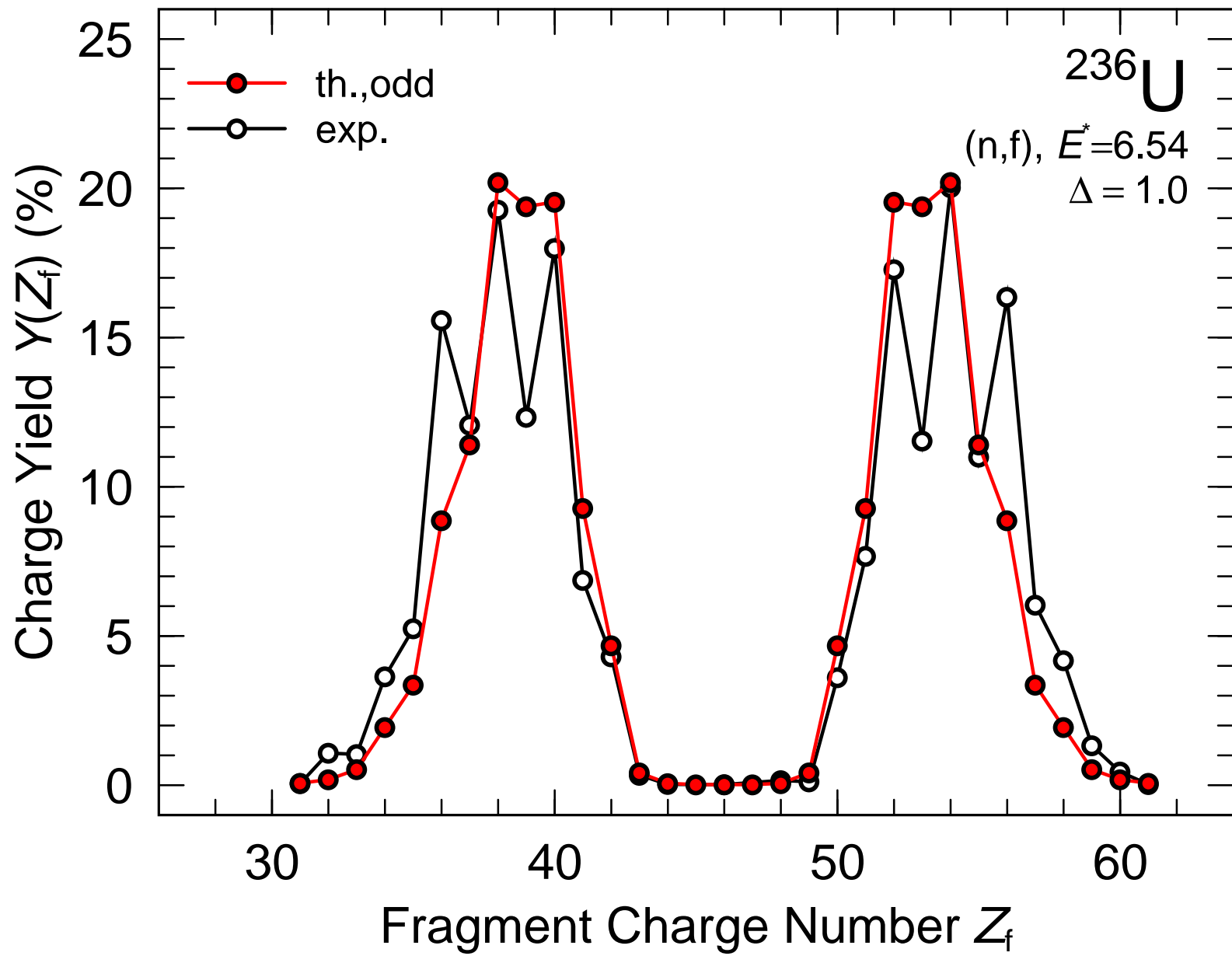


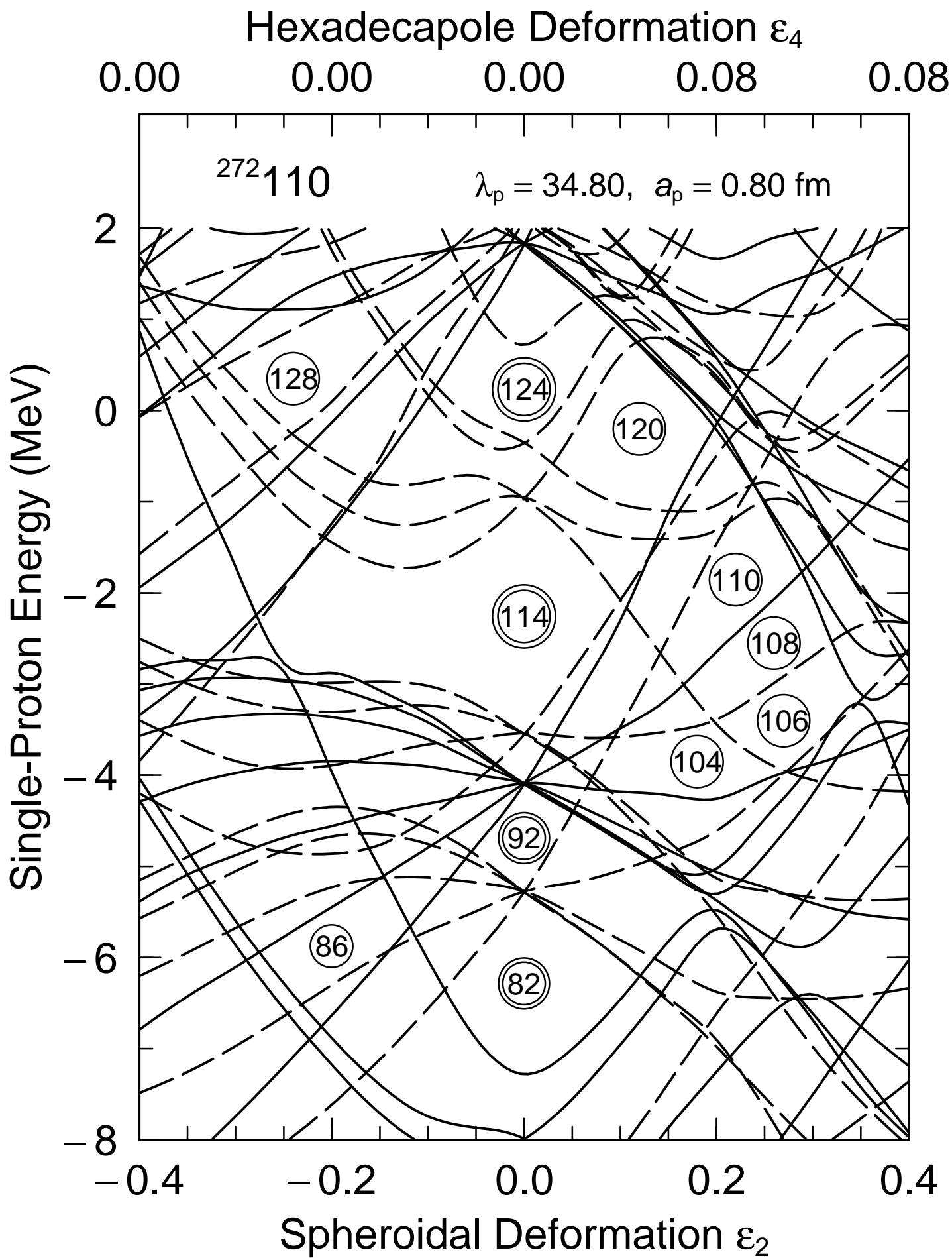
```
READ(LU,'(2f10.3)') r,rw
```

```
idiv =(N+1)/2
```

```
if(N+1 .eq. 2*idiv) r = r + (rw-1 +0.01)*2*1.0
```

```
E(I,J,K,L,N) = r
```





Next Track-Point Candidates

