

# MICROSCOPIC CALCULATIONS OF FISSION BARRIERS IN THE ACTINIDE REGION

Koh Meng Hock (UTM, CENBG),  
L. Bonneau, P. Quentin (CENBG), H. Wagiran (UTM)

Fission Experiments and Theoretical Advances  
Santa Fe, New Mexico

10-12 September 2014



# INTRODUCTION

## AIM OF RESEARCH

- calculation of fission barrier heights of **odd-mass** nuclei in the actinide region
- dependence of the inner barrier height on the  $K^\pi$  quantum numbers (assuming  $K$  is conserved along fission process)
- study of the energy spectra in the ground-state and fission-isomeric wells and transition (discrete) states at the top of the first barrier

# MICROSCOPIC APPROACH TO ODD NUCLEI

## HARTREE-FOCK (HF) PLUS PAIRING (BCS)

- breaking of **time-reversal symmetry** (due to the addition of one unpaired nucleon)
  - proper account of the effect through **self-consistent blocking (SCB)** calculations
  - vs. the Equal Filling Approximation (EFA); see e.g.
    - F. de la Iglesia, V. Martin, S. Perez Martin and L. M. Robledo, AIP Conf. Proc. **1175**, 199 (2009) :  $^{239}\text{Pu}$
    - S. Perez Martin and L. M. Robledo, Int. J. Mod. Phys. E **18** 788-797 (2009):  $^{235}\text{U}$
    - Koh Meng Hock, L. Bonneau and P. Quentin, EPJ Web of Conferences **62**, 04004 (2013)
- blocking of a single-particle state with specific  $K^\pi$  quantum numbers, and taking the lowest-energy solution

# MICROSCOPIC APPROACH TO ODD NUCLEI

## DEFINING A PAIR STATE IN BCS SCHEME

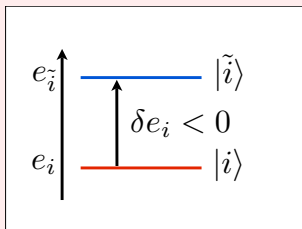
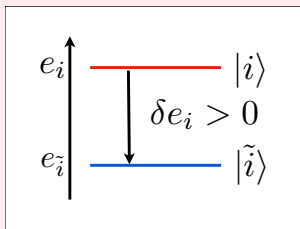
- Definition of a “Kramers quasi-pair” ( $|i\rangle, |\tilde{i}\rangle$ ):

$$\hat{h}_{\text{HF}}|i\rangle = e_i|i\rangle \quad \hat{J}_z|i\rangle = \Omega_i|i\rangle \quad \text{with } \Omega_i > 0$$

$$\hat{h}_{\text{HF}}|\tilde{i}\rangle = e_{\tilde{i}}|\tilde{i}\rangle \quad \hat{J}_z|\tilde{i}\rangle = \Omega_{\tilde{i}}|\tilde{i}\rangle \quad \text{with } \Omega_{\tilde{i}} = -\Omega_i < 0$$

$|\langle \tilde{i} | i \rangle|$  maximum (close to 1 in practice)

- Energy splitting in a Kramers quasi-pair:  $\delta e_i = e_i - e_{\tilde{i}}$



# MICROSCOPIC APPROACH TO ODD NUCLEI

## NUMERICAL PARAMETERS

- effective nucleon-nucleon interaction : Skyrme **SkM\*** force
- the single-particle states are expanded on a **cylindrical harmonic-oscillator basis** with a basis size,  $N_0 = 14$
- seniority force with pairing strength in the BCS scheme where the pairing strengths were fitted to the **odd-even binding energy differences** of some actinide nuclei, with retained values of  $G_0(\text{neutron}) = G_0(\text{proton}) = -16.0 \text{ MeV}$ 
  - pairing window up to  $\epsilon_F + 6.0 \text{ MeV}$  with a diffuseness parameter of  $0.2 \text{ MeV}$

# MICROSCOPIC APPROACH TO ODD NUCLEI

## UNIFIED MODEL PICTURE

- HFBCS solution  $|\Psi_K\rangle$  as intrinsic state
- rotational correction to intrinsic energy
- Coriolis coupling for  $K = \frac{1}{2}$  states
- energy of the  $J^\pi$  member of the  $K^\pi$  rotational band:

$$E_J = E_{K=J} + \frac{\hbar^2}{2\mathcal{I}} [J(J+1) - K(K+1)]$$

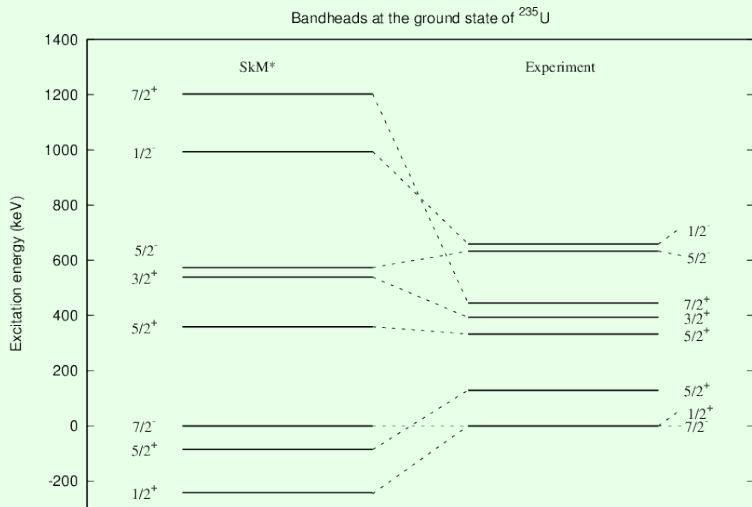
with

$$E_{J=K} = \underbrace{\langle \Psi_K | \hat{H} | \Psi_K \rangle}_{\text{intrinsic energy}} - \underbrace{\frac{\hbar^2}{2\mathcal{I}} (\langle \Psi_K | \hat{\mathbf{J}}^2 | \Psi_K \rangle - K(K+1))}_{\text{rotational correction}} - \underbrace{\frac{\hbar^2}{2\mathcal{I}} \delta_{K\frac{1}{2}} (-)^{J+\frac{1}{2}} (J + \frac{1}{2}) a}_{\text{Coriolis coupling}}$$

$\mathcal{I}$  is the **moment of inertia** calculated for the even-even core (preliminary)

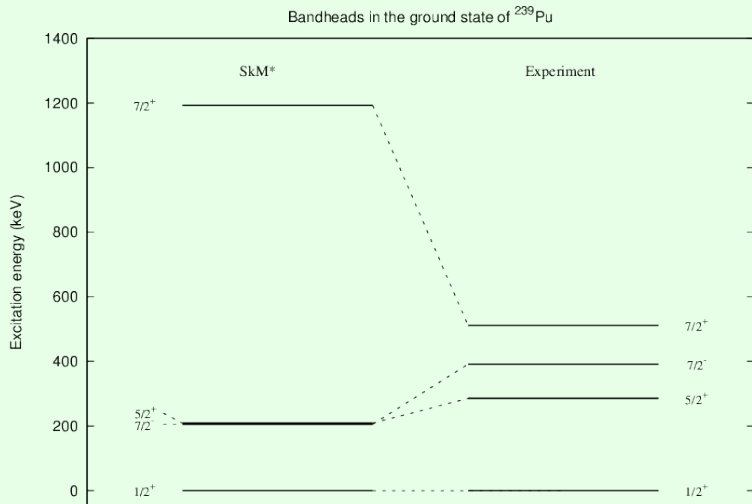
# RESULTS

## ONE-QUASIPARTICLE BANDHEADS IN $^{235}\text{U}$ (GS WELL)



# RESULTS

## ONE-QUASIPARTICLE BANDHEADS IN $^{239}\text{Pu}$ GS WELL

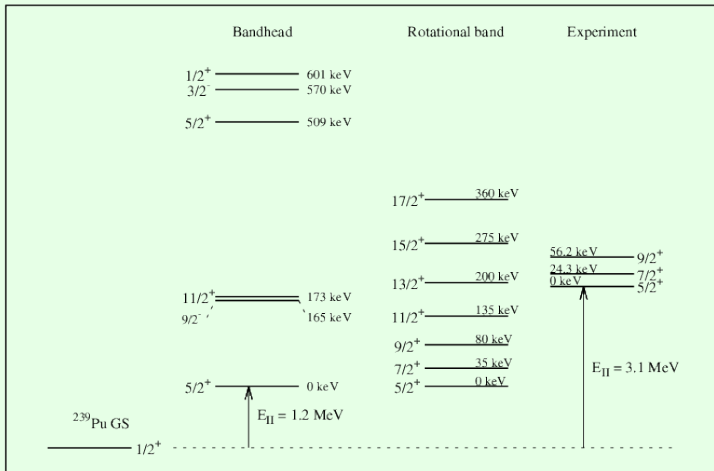




# RESULTS

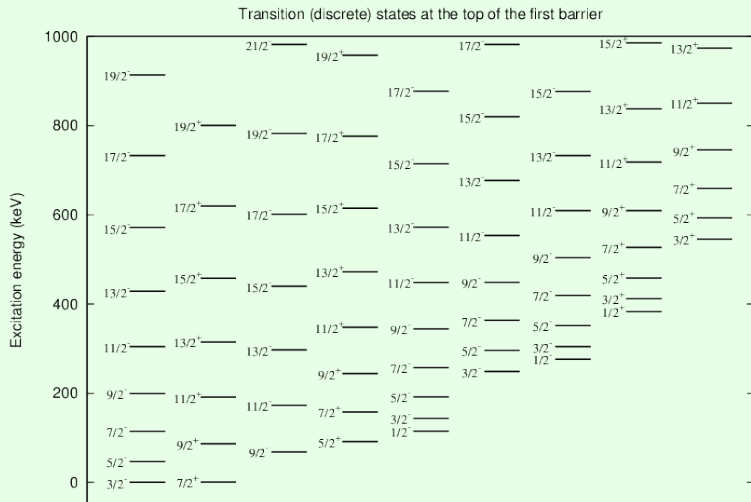
## ONE-QUASIPARTICLE BANDHEADS IN $^{239}\text{Pu}$ SD WELL

Fission-isomeric (SD) well of  $^{239}\text{Pu}$



# RESULTS

## ROTATIONAL BANDS AT $^{239}\text{Pu}$ FIRST SADDLE

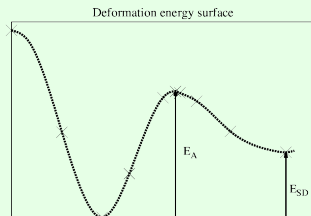


# RESULTS

## FISSION BARRIERS FOR FIXED $K^\pi$

Relative energies of first saddle point and second (SD) minimum with respect to GS minimum for various  $K^\pi$ :

$^{235}\text{U}$			$^{239}\text{Pu}$	
$K^\pi$	$E_A$	$E_{SD}$	$E_A$	$E_{SD}$
$1/2^+$	6.6	2.6	7.4	1.7
$7/2^-$	6.8	2.5	7.9	2.5
$5/2^+$	5.8	1.4	7.0	0.9
$7/2^+$	—	—	5.9	1.6



# RESULTS

## EFFECT OF TIME-REVERSAL SYMMETRY BREAKING

Difference between first-fission-barrier heights without (EFA) and with (SCB) time-reversal symmetry breaking in the selfconsistent blocked HFBCS solution for various  $K^\pi$ :

$K^\pi$	$E_A(\text{SCB}) - E_A(\text{EFA})$ (keV)
$1/2^+$	20
$7/2^-$	20
$5/2^+$	0

# CONCLUSIONS

- ① The calculated spectra compare favorably with experimental data in GS and SD wells  
⇒ reasonable class-I, class-II and transition states of rotational character

# CONCLUSIONS

- ① The calculated spectra compare favorably with experimental data in GS and SD wells  
⇒ reasonable class-I, class-II and transition states of rotational character
- ② The inner barrier height can vary significantly with  $K^\pi$

# CONCLUSIONS

- 1 The calculated spectra compare favorably with experimental data in GS and SD wells  
⇒ reasonable class-I, class-II and transition states of rotational character
- 2 The inner barrier height can vary significantly with  $K\pi$
- 3 EFA seems justified for calculations of fission-barrier heights (not for spectroscopic properties like magnetic moments)

# PERSPECTIVES

- Extend study to second saddle point (outer fission barrier)
- Improve moment of inertia for the core:
  - core polarization
  - pairing quenching because of unpaired nucleon (blocking)
- Restore particle-number symmetry broken by BCS  
⇒ Highly Truncated Diagonalization Approach (HTDA)  
≈ highly truncated shell model based on a mean-field solution
- Account for vibrational degrees of freedom, for example in the HTDA approach
- Extension to odd-odd compound nuclei



Fission-isomeric (SD) well of  $^{235}\text{U}$

